Shape Classification and Cell Movement in 3D Matrix Tutorial (Part I)

Fred Park

UCI iCAMP 2011
Outline

1. Motivation and Shape Definition
2. Shape Descriptors
3. Classification
4. Applications: Shape Matching, Warping, Shape Prior Segmentation
5. Cell Movement in 3D Matrix Overview and Set-up
Outline

1. Motivation and Shape Definition
2. Shape Descriptors
3. Classification
4. Applications: Shape Matching, Warping, Shape Prior Segmentation
5. Cell Movement in 3D Matrix Overview and Set-up
Shape

The term “Shape” is commonly used to describe the overall appearance of an object.
Humans: Understanding and learning shapes requires cognitive perception and some good old trial and error.
Example of unsuccessful shape sorting

Note:
What is the notion of Shape?

Shapes

- octagon
- square
- triangle
- circle
- parallelogram
- cube
- oval
- trapezoid
- sphere
- rectangle
- pentagon
- cylinder
Example: Airplanes

- Tapered leading edge, Straight trailing edge
- Tapered leading and trailing edge
- Delta wing
- Sweptback wing
- Straight leading and trailing edges
- Straight leading edge, tapered trailing edge
- low wing
- mid wing
- high wing
- inverted gull
- gull wing
- dihedral wing
- Anhedral wing
Example: Biology
Example: Biology

Bacterial Shapes

- Spirilla
- Bacilli
- Cocci

Bacterial Shapes
Example: Biology

Shark Tails
The Diversity of Form and Function
Example: Biology

Dolphins: raked fin

Sharks: more upright fin
Example: Biology

Shark or Dolphin?
Shape Classes
Computer Vision: MPEG-7

Vehicle shape classes: (a) Sedan, (b) Pickup, (c) Minivan, (d) SUV.

Airplane shape classes: (a) Mirage, (b) Eurofighter, (c) F-14 wings closed, (d) F-14 wings opened, (e) Harrier, (f) F-22, (g) F-15

Shape Definition

Mathematician and Statistician David George Kendall defined shape as: "Shape is all the geometric information that remains when location, scale, and rotational effects are filtered out from an object"
Shape Definition cont’d.

Shape is geometrical information that remains when location, scale, and rotational effects are filtered out from an object.
Scale Issues

• Dryden&Mardia/Kendall: “Sometimes we are also interested in retaining scale as well as shape”

• Filtering out rotation and translation: Shape and Size Information remains
Shape Registration

Also known as shape matching or Shape alignment

Early transformation grids of human profiles
Outline

1. Motivation and Shape Definition
2. Shape Descriptors
3. Classification
4. Applications: Shape Matching, Warping, Shape Prior Segmentation
5. Cell Movement in 3D Matrix Overview and Set-up
Shape Representation in 2D

• Implicit: View Shape as zero level set of a 3D function (surface)
• Explicit: View Parametric Representation of Shape
• In General, in either case, should be able to view shape as a binary function (image)
Shape Descriptors

• How Does one Mathematically Quantify Shape?
• What are the Challenges?
• What are the Tools?
Shape Descriptors

Geometric Information:
• Edges
• Corners
• Lines
• Curves
• Areas
• Distances
• Etc.
Useful Descriptor Properties

- Invariance to translation
- Invariance to rotation
- Scalability
Simply connected, convex and non-convex shapes

Simply Connected Shapes

Concave

Convex

Non Simply Connected Shape
Useful Descriptor Properties Cont’d.

• Unique Identification of certain types of shapes. E.g. Convex Shapes/Polygons.
• In general non-convex/non-simply connected shapes are more challenging.
• Easy and natural incorporation into classification, matching, warping, or segmentation models
Non-uniqueness of Descriptor Example

Geodesic distances on 2D shapes.

Using the geodesic distances along the contours, the two shapes are indistinguishable.
Descriptor Examples

• Fourier: shape info in low freq. components
• Moments: mean, variance, skew, kurtosis
• Centered/Centroid Distance
• Cliques: Intervertex Distances
• Inner Distance
• Landmarks and Statistical Signatures
• Many others, still very active research topic
Descriptor Examples

• Fourier
• Moments
• Centered/Centroid Distance
• Inner Distance
• Cliques
• Landmarks and Statistical Signatures
• Many others, still very active research topic
Statistical Moments

Given a 1D discrete function $f(x)$

First moment: mean

$$\mu = \frac{\sum_{x=1}^{N} x f(x)}{\sum_{x=1}^{N} f(x)}$$

Second moment: variance

"how spread out is function"

$$\sigma^2 = \frac{\sum_{x=1}^{N} (x - \mu)^2 f(x)}{\sum_{x=1}^{N} f(x)}$$

Third moment: skew

"how symmetric is function"

$$\text{skew} = \frac{\sum_{x=1}^{N} (x - \mu)^3 f(x)}{\sum_{x=1}^{N} f(x)}$$

$n$-th moment about mean: 

"$n$-th central moment"

$$\mu_n = \frac{\sum_{x=1}^{N} (x - \mu)^n f(x)}{\sum_{x=1}^{N} f(x)}$$
Statistical Moments

Fourth central moment: Kurtosis
“how peaky is the function”

Kurtosis = $\mu_4$
Statistical Moments in 2D

\[ i j \text{th moment about zero} \quad m_{ij} = \frac{\sum_{x=1}^{N} \sum_{y=1}^{N} x^i y^j f(x, y)}{\sum_{x=1}^{N} \sum_{y=1}^{N} f(x, y)} \]

\[ m_{10} \text{ is the } x \text{ component } \mu_x \text{ of the mean and } m_{01} \text{ is the } y \text{ component } \mu_y \text{ of the mean.} \]

\[ \text{central moments} \quad \mu_{ij} = \frac{\sum_{x=1}^{N} \sum_{y=1}^{N} (x - \mu_x)^i (y - \mu_y)^j f(x, y)}{\sum_{x=1}^{N} \sum_{y=1}^{N} f(x, y)} \]
2D Second Order Moments

\[ \mu_{ij} = \frac{\sum_{x=1}^{N} \sum_{y=1}^{N} (x - \mu_x)^i(y - \mu_y)^j f(x, y)}{\sum_{x=1}^{N} \sum_{y=1}^{N} f(x, y)} \]

moments \( \mu_{20} \) and \( \mu_{02} \) are the variances of \( x \) and \( y \) respectively.

moment \( \mu_{11} \) is the covariance between \( x \) and \( y \).

The covariance matrix \( C \)
\[ C = \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} \]

• By finding the Eigenvalues and Eigenvectors of \( C \), we can determine how elongated a shape is (eccentricity) by looking at the ratio of the eigenvalues.
• Direction of this eccentricity is given by the eigenvector with largest absolute value eigenvalue.
Centroid Distance Signature

- Centroid-based “time series” representation

- All extracted time series are further standardised and resampled to the same length
Motivation:
Bending Invariant Signatures
Elad and Kimmel

Intervetex Distances

\[ \sum_{i,j} \left( \| p_i - p_j \|^2 - \| r_i - r_j \|^2 \right)^2 \]

\( r_i \): pts. lying on reference shape
\( p_i \): pts. lying on evolving contour

Incorporation into:
• Geodesic Active Contours (Snakes)
• Polygonal Implementation of the P.W. Constant Mumford Shah Segmentation Model
Landmarks (Dryden & Mardia)

- **Landmark** is a pt. of correspondence on each object that matches between and within populations.
- An **Anatomical Landmark** is a pt. assigned by an expert that corresponds between objects of study in a way of meaningful in the context of the disciplinary context.
- **Mathematical Landmarks** are points located on an object according to some mathematical or geometrical property of the figure.
Landmarks

• Much Shape Information is Embedded in Landmarks
• Basis for Statistical Shape Descriptors
Landmarks

Three landmark points along a line for simple shape comparison

Cf. G. Small, The Statistical Theory of Shape

FIGURE 1.2. Side view of skulls. From top to bottom: modern human, Neanderthal, australopithecine, chimpanzee. The skull profiles are redrawn from Figure 3.59 of [131].
Inner Distance Definition:

- $x$ and $y$ are landmark points
- Dashed line shows shortest path between $x$ and $y$
Fig. 4. Computation of the inner-distance. Left, the shape with the sampled silhouette landmark points. Middle, the graph built using the landmark points. Right, a detail of the right top of the graph. Note how the inner-distance captures the holes.
Outline

1. Motivation and Shape Definition
2. Shape Descriptors
3. Classification
4. Applications: Shape Matching, Warping, Shape Prior Segmentation
5. Cell Movement in 3D Matrix Overview and Set-up
Shape Classification Definition

Shape Classification: What are the shapes of a given type up to equivalence.
Shape Classification: 2 Parts

(1) Shape Descriptors

(2) Classification from Descriptors (Learning Algorithms)
Problem formulation

- Object recognition systems dependent heavily on the accurate identification of shapes

- Learning the shapes without supervision is essential when large image collections are available
What is Unsupervised Learning?

• In **supervised learning** we were given attributes & targets (e.g. class labels). In **unsupervised learning** we are only given attributes.

• Our task is to discover **structure** in the data.

• Example I: the data may be structured in clusters:

• Example II: the data may live on a lower dimensional manifold:
Why Discover Structure?

• **Data compression**: If you have a good model you can encode the data more cheaply.

• Example PCA: To encode the data I have to encode the x and y position of each data-case. However, I could also encode the offset and angle of the line plus the *deviations* from the line. Small numbers can be encoded more cheaply than large numbers with the same precision.

• This idea is the basis for model selection: The complexity of your model (e.g. the number of parameters) should be such that you can encode the data-set with the fewest number of bits (up to a certain precision).

• Clustering: represent every data-case by a cluster representative plus deviations.

• ML is often trying to find semantically meaningful representations (abstractions). These are good as a basis for making new predictions.
Bullseye Test for shape clustering

• There is a frequently used test in shape retrieval and enables us to compare our algorithm against many of the best performing ones.

• A single shape is presented as a query and the top forty matches are retrieved. The task is to repeated for each shape and the number of correct matches (out of a maximum possible 20) are noted, a perfect performance results in 1400*20 matches
Manifold clustering of shapes

- Vision data often reside on a nonlinear embedding that linear projections fail to reconstruct.

- We apply Isomap to detect the intrinsic dimensionality of the shapes data.

- Isomap moves further apart different clusters, preserving their convexity.
Manifold Learning

- Assume data lives on low level manifold
- Two very different objects may have smaller Euclidean distance than geodesic distance
Outline

1. Motivation and Shape Definition
2. Shape Descriptors
3. Classification
4. Applications: Shape Matching, Shape Warping, Shape Prior Segmentation
5. Cell Movement in 3D Matrix Overview and Set-up
Shape Warping

- Definition: Deformation of one given shape to another
- Often done through some sort of gradient flow of an associated energy
- Move a curve according to some normal velocity that minimizes a shape comparison energy (standard practice)
Shape Warping Example

Min over functions “$\phi$”

$\psi$: reference shape

$\phi$: evolving contour/surface

Implicit Shape Descriptors

$$S(\phi) = \frac{1}{2} \int_{\mathbb{R}^+} |A_{\phi}(\mu) - A_{\psi}(\mu)|^2 d\mu$$
Implicit shape warping: disocclusion

Observed noisy image (white), occluded region (green)  shape prior (blue), initial level set (red)
Implicit shape warping: disocclusion

contour of disoccluded image (red)
Shape Prior Segmentation and Disocclusion

• Again cliques

\[ \sum_{i,j} \left( \| p_i - p_j \|^2 - \| r_i - r_j \|^2 \right)^2 \]

- \( r_i \): pts. lying on reference shape
- \( p_i \): pts. lying on evolving contour
Shape Prior Segmentation

(a) Image to be Segmented
(b) Image to be Segmented w/ Learned Reference Shape
(a) Initial Contour
Shape Prior Segmentation

(b) Proposed Model w/ Shape = 0.1
(d) Proposed Model w/ Shape = 0.5
Proposed Model w/ Shape = 1.0
Shape Prior Segmentation

Proposed Model w/ Shape = 1.0

Standard PPWCMS (No Shape)
Figure 12: Dissocclusion of a Maple Leaf
(a) Clean Image and Learned Shape (Note Scale Differential)

(b) Initial Contour

(c) Disocclusion

(d) Disocclusion with Mask Removed
Outline

1. Motivation and Shape Definition
2. Shape Descriptors
3. Classification
4. Applications: Shape Matching, Warping, Shape Prior Segmentation
5. Cell Movement in 3D Matrix Overview and Set-up
Cells
cells
Bio Apps

• Viable Project: Segmenting 2-D Cell slice data with possibly shape priors
• Long term calculate velocity field both inside cell and boundary as well as outside (ECM) in 3-D
• Long Term also 3D segmentation of volumetric cells
Cell Protrusion