Shape Descriptor/Feature Extraction Techniques

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UCI iCAMP 2011

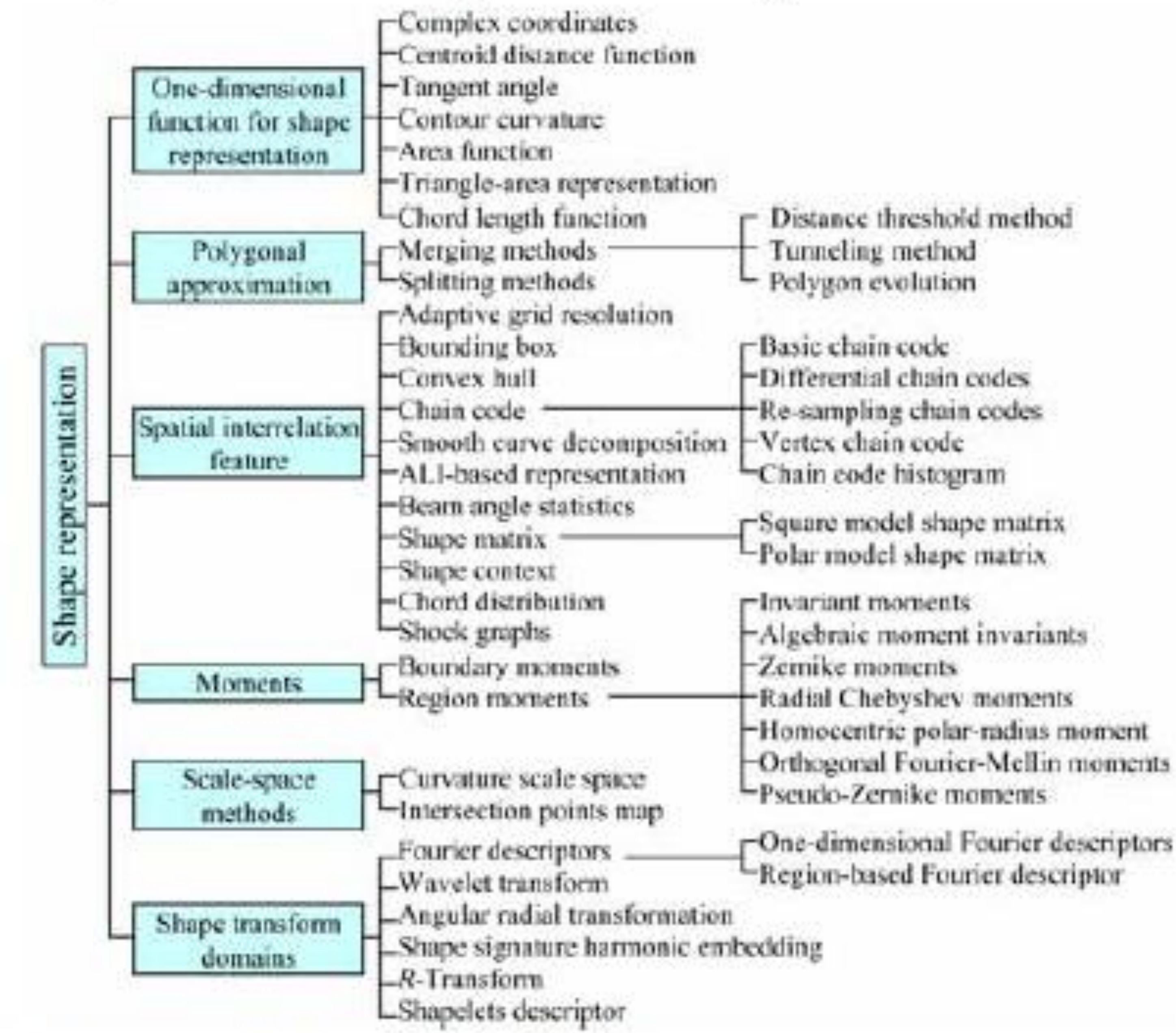
Outline

1. Overview and Shape Representation
2. Shape Descriptors: Shape Parameters
3. Shape Descriptors as 1D Functions (Dimension Reducing Signatures of shape)

Efficient shape features must have some essential properties such as:

- **identifiability:** shapes which are found perceptually similar by human have the same features that are different from the others.
- **translation, rotation and scale invariance:** the location, the rotation and the scaling changing of the shape must not affect the extracted features.
- **affine invariance:** the affine transform performs a linear mapping from coordinates system to other coordinates system that preserves the "straightness" and "parallelism" of lines. Affine transform can be constructed using sequences of translations, scales, flips, rotations and shears. The extracted features must be as invariant as possible with affine transforms.
- **noise resistance:** features must be as robust as possible against noise, i.e., they must be the same whichever be the strength of the noise in a give range that affects the pattern.
- **occultation invariance:** when some parts of a shape are occulted by other objects, the feature of the remaining part must not change compared to the original shape.
- **statistically independent:** two features must be statistically independent. This represents compactness of the representation.
- **reliability:** as long as one deals with the same pattern, the extracted features must remain the same.

Overview of Descriptors



ig. 1. An overview of shape description techniques

Geometric Features for Shape Descriptors

- Measure similarity bet. Shapes by measuring simil. bet. Their features
- In General, simple geom. features cannot discriminate shapes with large distances e.g. rectangle vs ellipse
- Usual combine with other complimentary shape descriptors and also used to avoid false hits in image retrieval for ex.
- Shapes can be described by many aspects we call shape parameters:

  center of gravity/centroid, axis of least inertia, digital bending energy, eccentricity, circularity ratios, elliptic variance, rectangularity, convexity, solidity, Euler number, profiles, and hole area ratio.

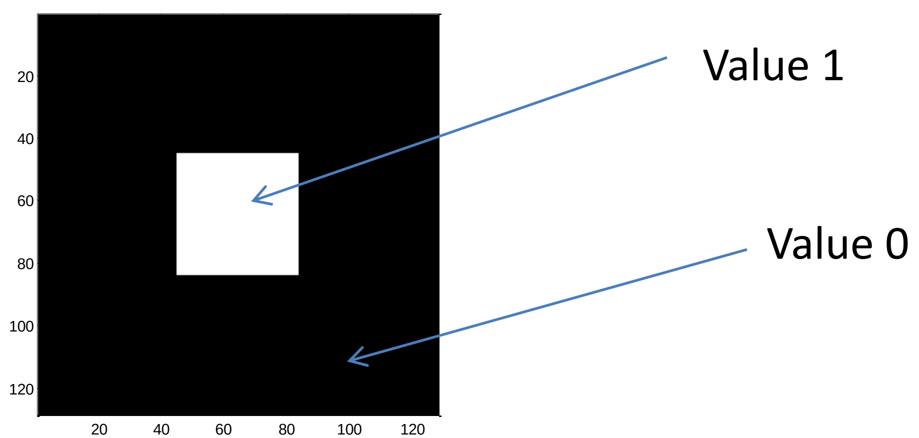
Shape Representation

View as a binary function $$f(x,y) = \begin{cases} 1 & if\ (x,y) \in D \\ 0 & otherwise \end{cases}$$

D is the domain of the binary shape.

centroid (g_x, g_y) is: $$\begin{cases} g_x = & \frac{1}{N}\sum_{i=1}^{N} x_i \\ g_y = & \frac{1}{N}\sum_{i=1}^{N} y_i \end{cases}$$

N is the number of point in the shape, $(x_i, y_i) \in \{(x_i, y_i) \mid f(x_i, y_i) = 1\}$

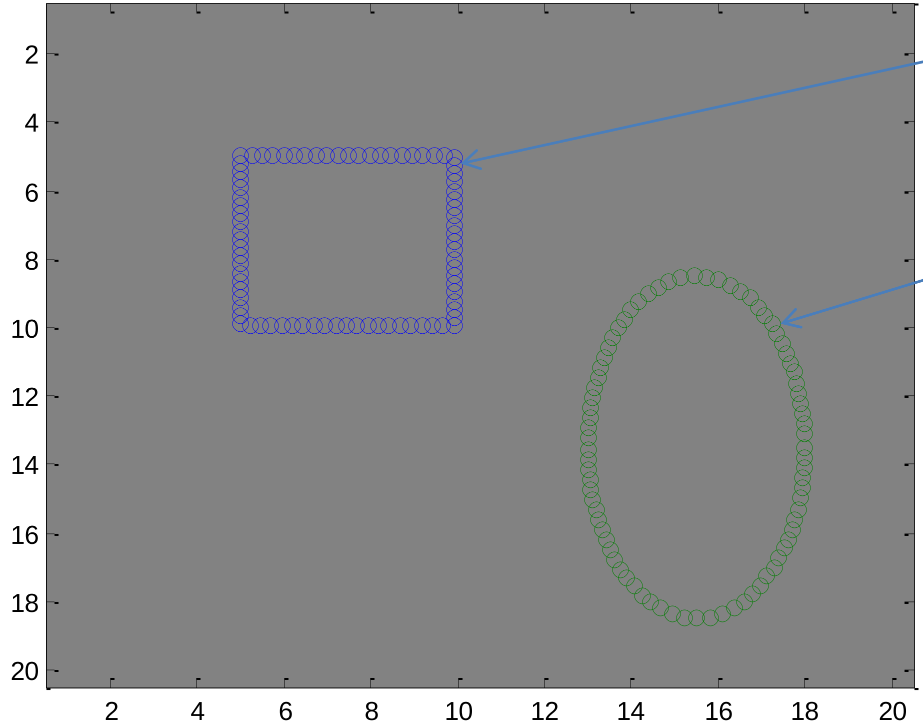


Shape Representation Cont’d

View in Parametric form $\Gamma(n) = (x(n), y(n))$

where $n \in [0, N - 1]$;

$\Gamma(N) = \Gamma(0)$



Γ(i) = ((x(i),y(i))

Γ(j) = ((x(j),y(j))

Center of Gravity/Centroid

2.1 Center of gravity

Fixed in relation to shape

A is the contour's area given by $$A = \frac{1}{2}\left|\sum_{i=0}^{N-1}(x_i y_{i+1} - x_{i+1} y_i)\right|$$

why? See explanation in class.

In general for polygons centroid C is: $$C = \frac{\sum C_i A_i}{\sum A_i}.$$

In general for a polygon, let $X_1, X_2, \ldots, X_n$ be triangles partitioning the polygon

$$\frac{\sum C_i A_i}{\sum A_i} = \frac{1}{A}\sum\left(\frac{\vec{x}_i + \vec{x}_{i+1}}{3}\right)\frac{(x_i y_{i+1} - x_{i+1} y_i)}{2}$$

$$\vec{x}_i = (x_i, y_i)$$

Centroid of triangle

Area of triangle

2D Centroid Formula

$$\frac{\sum C_i A_i}{\sum A_i} = \frac{1}{A} \sum \left(\frac{\vec{x}_i + \vec{x}_{i+1}}{3} \right) \frac{(x_i y_{i+1} - x_{i+1} y_i)}{2}$$

$$\vec{x}_i = (x_i, y_i)$$

Centroid of triangle

Area of triangle

Thus formula for centroid C = (g_x, g_y) is given below:

$$\begin{cases} g_x = \frac{1}{6A} \sum_{i=0}^{N-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \\ g_y = \frac{1}{6A} \sum_{i=0}^{N-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \end{cases}$$

Centroid Invariance to boundary point distribution

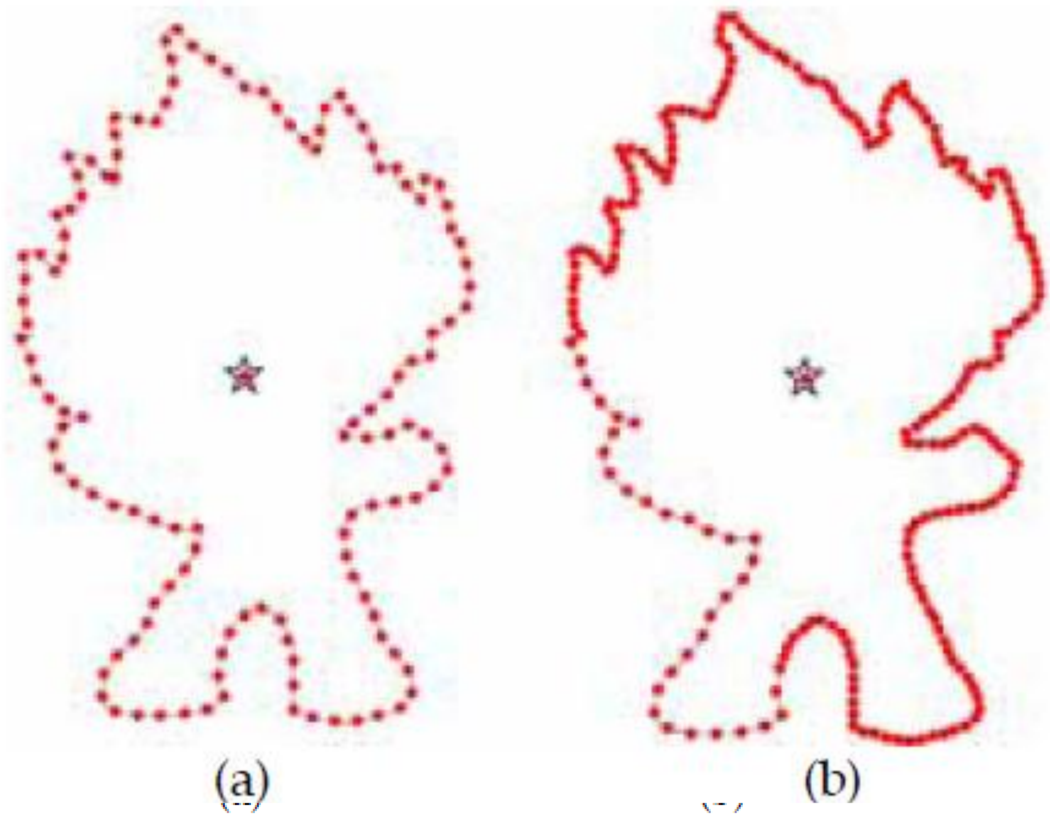


Fig. 2. Centroid of contour. The dots are points distributed on the contour uniformly (a) and non-uniformly (b). The star is the centroid of original contour and the inner dot is the centroid of sampled contour.

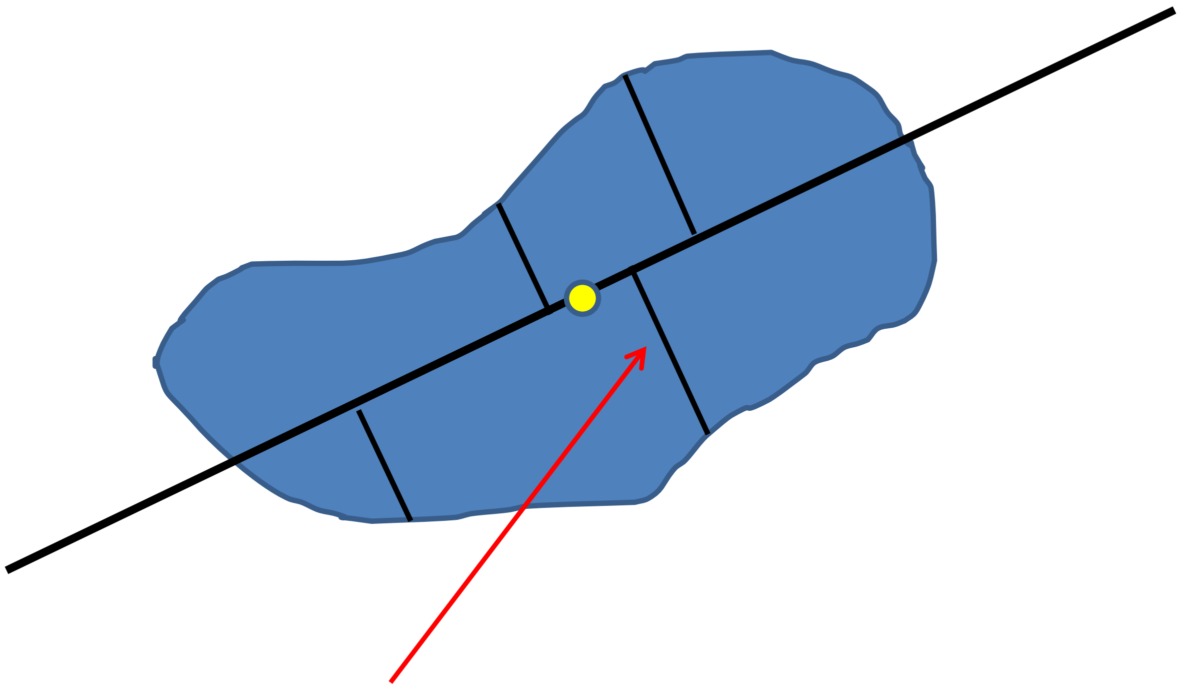
Axis of Least Inertia

2.2 Axis of least inertia

ALI: unique ref. line preserving orientation of shape

Passes through centroid

Line where shape has easiest way of rotating about



ALI: Line L that minimizes the sum of the squared distance from it to the boundary of shape

: Denotes centroid

Axis of Least Inertia

ALI defined by: $$I(\alpha, S) = \int_S \int r^2(x, y, \alpha)\,dx\,dy$$

Here, $r(x, y, \alpha)$ is the perpendicular distance from the pt (x, y) to the line given by $X \sin\alpha - Y \cos\alpha = 0$.

We assume that the coordinate $(0, 0)$ is the location of the centroid.

Average Bending Energy

2.3 Average bending energy

The Average Bending Energy is defined as

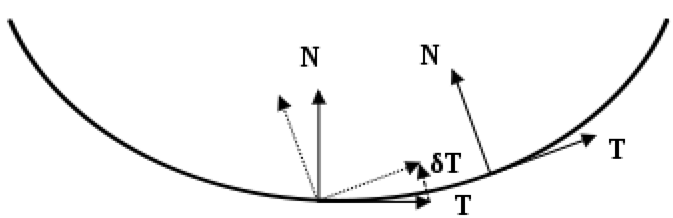
$$BE = \frac{1}{N} \sum_{s=0}^{N-1} K(s)^2$$

where $K(s)$ denotes the curvature of the shape parametrized by arclength

One can prove that the circle is the shape with the minimum Bending Energy

General Def'n. of Curvature

$$\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\| .$$



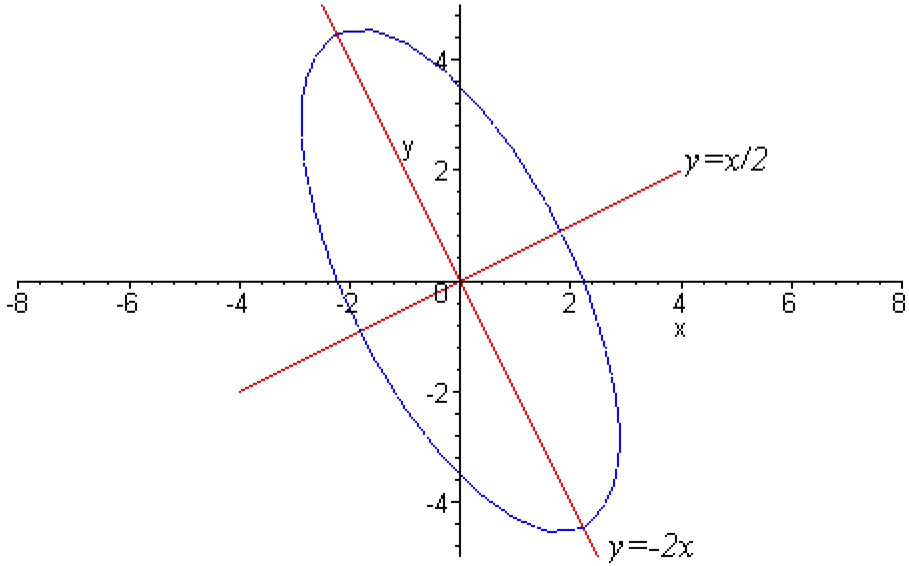
For Plane Curve $\Gamma(t) = (x(t), y(t))$

$$\kappa = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{3/2}},$$

Eccentricity

2.4 Eccentricity

- Eccentricity is the measure of aspect ratio
- It's ratio of length of major axis to minor axis (think ellipse for example)
- Calculated by principal axes method or minimum bounding rectangular box



Eccentricity: Principal Axes Method

2.4.1 Principal axes method

Principal Axes of a shape is uniquely def'd as:
two segments of lines that cross each other perpendicularly through the centroid representing directions with zero cross correlation

Covariance Matrix C of a contour:

$$C = \frac{1}{N}\sum_{i=0}^{N-1}\begin{pmatrix} x_i - g_x \\ y_i - g_y \end{pmatrix}\begin{pmatrix} x_i - g_x \\ y_i - g_y \end{pmatrix}^T = \begin{pmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix}$$

where

$$c_{xx} = \frac{1}{N}\sum_{i=0}^{N-1}(x_i - g_x)^2$$
$$c_{xy} = \frac{1}{N}\sum_{i=0}^{N-1}(x_i - g_x)(y_i - g_y)$$
$$c_{yx} = \frac{1}{N}\sum_{i=0}^{N-1}(y_i - g_y)(x_i - g_x)$$
$$c_{yy} = \frac{1}{N}\sum_{i=0}^{N-1}(y_i - g_y)^2$$

$G(g_x, g_y)$ is the centroid of the shape

Lengths of the two principal axes equal the eigenvalues λ_1 and λ_2 of the Covariance Matrix C

Cross correlation: sliding dot product

$$(f \star g)(t) \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} f^*(\tau)\ g(t+\tau)\,d\tau,$$

Eccentricity: Principal Axes Method

Lengths of the two principal axes equal the eigenvalues λ_1 and λ_2 of the Covariance Matrix C

So the eigenvalues λ_1 and λ_2 can be calculated by

$$det(C - \lambda_{1,2} I) = det \begin{pmatrix} c_{xx} - \lambda_{1,2} & c_{xy} \\ c_{yx} & c_{yy} - \lambda_{1,2} \end{pmatrix} = (c_{xx} - \lambda_{1,2})(c_{yy} - \lambda_{1,2}) - c_{xy}^2 = 0$$

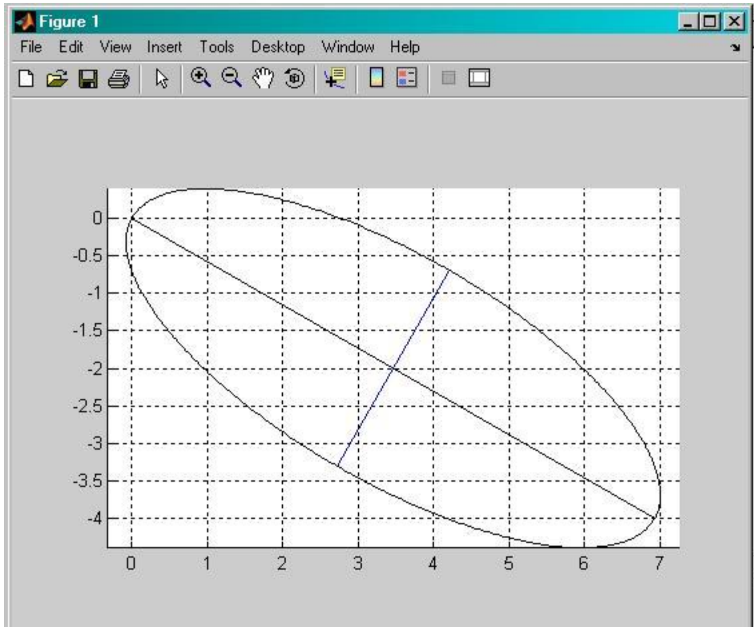
So

$$\begin{cases} \lambda_1 = \frac{1}{2}\left[c_{xx} + c_{yy} + \sqrt{(c_{xx} + c_{yy})^2 - 4\left(c_{xx}c_{yy} - c_{xy}^2\right)}\right] \\ \lambda_2 = \frac{1}{2}\left[c_{xx} + c_{yy} - \sqrt{(c_{xx} + c_{yy})^2 - 4\left(c_{xx}c_{yy} - c_{xy}^2\right)}\right] \end{cases}$$

Then, eccentricity can be calculated:

$$E = \lambda_2 / \lambda_1$$

What is the eccentricity of a circle?



Eccentricity

2.4.2 Minimum bounding rectangle

Minimum bounding rectangle (minimum bounding box):
Smallest rectangle containg every pt. in the shape

Eccentricity: $E = L/W$
L: length of bounding box
W: width of bounding box

Elongation: $Elo = 1 - W/L$
$Elo \in [0,1]$
Circle of square (symmetric): $Elo = 0$
Shape w/ large aspect ratio: Elo close to 1

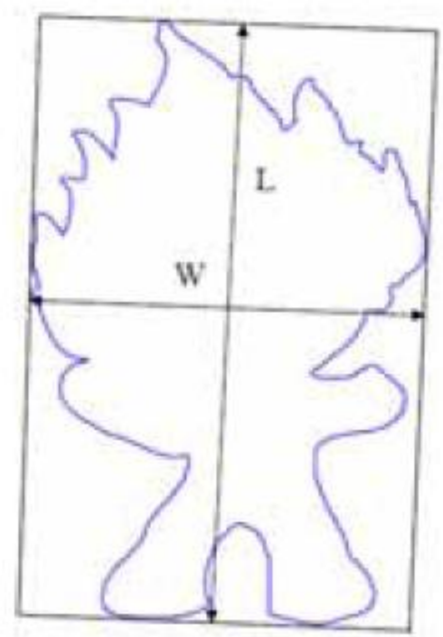


Fig. 3. Minimum bounding rectangle and corresponding parameters for elongation

Circularity Ratio

2.5 Circularity ratio

Circularity ratio: How similar to a circle is the shape
3 definitions:

Circularity ratio 1: $C_1 = A_s/A_c$ = (Area of a shape)/(Area of circle)
where circle has the same perimeter

$$A_c = p^2/4\pi \text{ thus } C_1 = \frac{4\pi A_s}{p^2}$$
$$\text{since } 4\pi \text{ is a constant, } C_2 = \frac{A_s}{p^2}$$

Circularity ratio 2: $C_2 = A_s/p^2$ (p = perim of shape)
Area to squared perimeter ratio.

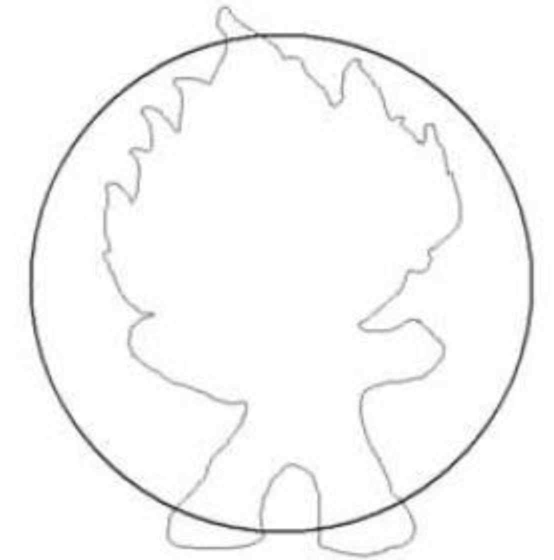
Circularity ratio 3: $Cva = \frac{\sigma_R}{\mu_R}$

μ_R : mean of radial dist. from centroid to shape bndry pts

σ_R : standard deviation of radial dist. from centroid to bndry pts

$$\mu_R = \frac{1}{N}\sum_{i=1}^{N-1} d_i \quad and \quad \sigma_R = \sqrt{\frac{1}{N}\sum_{i=1}^{N-1}(d_i - \mu_R)^2}$$

where $d_i = \sqrt{(x_i - g_x)^2 + (y_i - g_y)^2}$.



Circle variance

Ellipse Variance

2.6 Ellipse variance

Ellipse Variance Eva:
Mapping error of shape to fit an ellipse
with same covariance matrix as shape: $C_{ellipse} = C_{shape}$
(Here $C = C_{shape}$)

di': info about shape and ellipse variance of radial distances

$$V_i = \begin{pmatrix} x_i - g_x \\ y_i - g_y \end{pmatrix}$$

$$d_i' = \sqrt{V_i^T \cdot C_{ellipse}^{-1} \cdot V_i}$$

$$\mu_R' = \frac{1}{N}\sum_{i=1}^{N-1} d_i' \quad and \quad \sigma_R' = \sqrt{\frac{1}{N}\sum_{i=1}^{N-1}(d_i' - \mu_R')^2}$$

$$E_{va} = \frac{\sigma_R'}{\mu_R'}$$

Ellipse

Rectangularity

2.7 Rectangularity

Rectangularity represents how rectangular a shape is, i.e. how much it fills its minimum bounding rectangle:

$$Rectangularity = A_S / A_R$$

where A_S is the area of a shape; A_R is the area of the minimum bounding rectangle.

What is rectangularity for a square? Circle? Ellipse?

Convexity

2.8 Convexity

2.8 Convexity

Convexity is defined as the ratio of perimeters of the convex hull $\mathcal{O}_{Convexhull}$ over that of the original contour $\mathcal{O}$ [7]:

$$Convexity = \frac{\mathcal{O}_{Convexhull}}{\mathcal{O}} \quad (13)$$

The region R^2 is a convex if and only if for any two points $P_1, P_2 \in R^2$, the entire line segment P_1P_2 is inside the region. The convex hull of a region is the smallest convex region including it. In Figure 6, the outline is the convex hull of the region.

Examples of convex and non-convex based on above definition?

Illustration of convex hull

Solidity

2.9 Solidity

Solidity describes the extent to which the shape is convex or concave [8] and it is defined by

$$Solidity = A_s/H$$

where, A_s is the area of the shape region and H is the convex hull area of the shape. The solidity of a convex shape is always 1.

Examples of 2 shapes that have solidity 1 and less than one? Can you create a shape with solidity = ½?

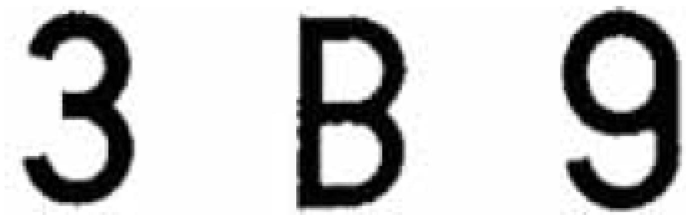
Euler Number

2.10 Euler number

Euler number describes the relation between the number of contiguous parts and the number of holes on a shape. Let S be the number of contiguous parts and N be the number of holes on a shape. Then the Euler number is:

$$Eul = S - N$$

For example



Euler Number equal to *1*, *-1* and *0*, respectively.

Profiles

2.11 Profiles

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The profiles are the projection of the shape to x-axis and y-axis on Cartesian coordinates system. We obtain two one-dimension functions:

$$Pro_x(i) = \sum_{j=j_{min}}^{j_{max}} f(i,j) \quad and \quad Pro_y(j) = \sum_{i=i_{min}}^{i_{max}} f(i,j)$$

where $f(i, j)$ represents the region of shape Eq. 1. See Figure 7.

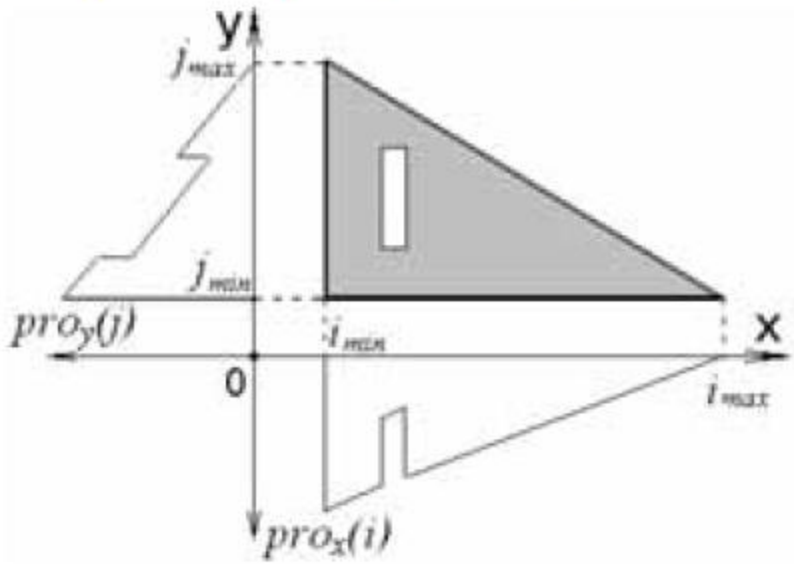

Fig. 7. Profiles

Hole Area Ratio

2.12 Hole area ratio

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Hole area ratio HAR is defined as

$$HAR = \frac{A_h}{A_s}$$

where A_s is the area of a shape and A_h is the total area of all holes in the shape. Hole area ratio is most effective in discriminating between symbols that have big holes and symbols with small holes [9].

HAR is the ratio: (area of the holes)/(area of shape)

Can you think of a shape with HAR equal to 0,1, arbitrarily large?