Shape Descriptor/Feature Extraction Techniques

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Outline

1. Overview and Shape Representation
   Shape Descriptors: Shape Parameters

2. Shape Descriptors as 1D Functions
   (Dimension Reducing Signatures of shape)

3. Polygonal Approx, Spatial Interrelation, Scale
   Space approaches, and Transform domains
Efficient shape features must have some essential properties such as:

- **Identifiability**: shapes which are found perceptually similar by human have the same features that are different from the others.

- **Translation, rotation and scale invariance**: the location, the rotation and the scaling changing of the shape must not affect the extracted features.

- **Affine invariance**: the affine transform performs a linear mapping from coordinates system to other coordinates system that preserves the "straightness" and "parallelism" of lines. Affine transform can be constructed using sequences of translations, scales, flips, rotations and shears. The extracted features must be as invariant as possible with affine transforms.

- **Noise resistance**: features must be as robust as possible against noise, i.e., they must be the same whichever be the strength of the noise in a give range that affects the pattern.

- **Occultation invariance**: when some parts of a shape are occulted by other objects, the feature of the remaining part must not change compared to the original shape.

- **Statistically independent**: two features must be statistically independent. This represents compactness of the representation.

- **Reliability**: as long as one deals with the same pattern, the extracted features must remain the same.
Overview of Descriptors

- One-dimensional function for shape representation
  - Complex coordinates
  - Centroid distance function
  - Tangent angle
  - Contour curvature
  - Area function
  - Triangle-area representation
  - Chord length function
  - Merging methods
  - Splitting methods
  - Adaptive grid resolution
  - Bounding box
  - Convex hull
  - Chain code
  - Smooth curve decomposition
  - ALI-based representation
  - Beam angle statistics
  - Shape matrix
  - Shape context
  - Chord distribution
  - Shock graphs
  - Boundary moments
  - Region moments

- Spatial interrelation feature
  - Distance threshold method
  - Tunneling method
  - Polygon evolution
  - Basic chain code
  - Differential chain codes
  - Resampling chain codes
  - Vertex chain code
  - Chain code histogram
  - Square model shape matrix
  - Polar model shape matrix
  - Invariant moments
  - Algebraic moment invariants
  - Zernike moments
  - Radial Chebyshev moments
  - Homocentric polar-radius moment
  - Orthogonal Fourier-Mellin moments
  - Pseudo-Zernike moments

- Moments
  - One-dimensional Fourier descriptors
  - Region-based Fourier descriptor

- Scale-space methods
  - Intersection points map
  - Fourier descriptors
  - Wavelet transform
  - Angular radial transformation
  - Shape signature harmonic embedding
  - R-Transform
  - Shapelets descriptor

Fig. 1. An overview of shape description techniques
Geometric Features for Shape Descriptors

- Measure similarity bet. Shapes by measuring simil. bet. Their features
- In General, simple geom. features cannot discriminate shapes with large distances e.g. rectangle vs ellipse
- Usual combine with other complimentary shape descriptors and also used to avoid false hits in image retrieval for ex.
- Shapes can be described by many aspects we call shape parameters:
  center of gravity/centroid, axis of least inertia, digital bending energy, eccentricity, circularity ratios, elliptic variance, rectangularity, convexity, solidity, Euler number, profiles, and hole area ratio.
Shape Representation

View as a binary function

\[ f(x, y) = \begin{cases} 
1 & \text{if } (x, y) \in D \\ 
0 & \text{otherwise} 
\end{cases} \]

\( D \) is the domain of the binary shape.

The centroid \((g_x, g_y)\) is:

\[
\begin{align*}
  g_x &= \frac{1}{N} \sum_{i=1}^{N} x_i \\
  g_y &= \frac{1}{N} \sum_{i=1}^{N} y_i
\end{align*}
\]

\( N \) is the number of points in the shape, \((x_i, y_i) \in \{(x_i, y_i) \mid f(x_i, y_i) = 1\}\)
View in Parametric form

\[ \Gamma(n) = (x(n), y(n)) \]

where \( n \in [0, N - 1] \),
\( \Gamma(N) = \Gamma(0) \)
Center of Gravity/Centroid

2.1 Center of gravity

Fixed in relation to shape

$A$ is the contour's area given by

$$ A = \frac{1}{2} \left| \sum_{i=0}^{N-1} (x_i y_{i+1} - x_{i+1} y_i) \right| $$

why? See explanation in class.

In general for polygons centroid $C$ is:

$$ C = \frac{\sum C_i A_i}{\sum A_i} $$

In general for a polygon, let $X_1, X_2, \ldots, X_n$ be triangles partitioning the polygon

$$ \frac{\sum C_i A_i}{\sum A_i} = \frac{1}{A} \sum \left( \frac{\vec{x}_i + \vec{x}_{i+1}}{3} \right) \frac{(x_i y_{i+1} - x_{i+1} y_i)}{2} $$

Centroid of triangle

Area of triangle

$\vec{x}_i = (x_i, y_i)$
2D Centroid Formula

\[
\frac{\sum C_i A_i}{\sum A_i} = \frac{1}{A} \sum \left( \frac{\vec{x}_i + \vec{x}_{i+1}}{3} \right) \frac{(x_i y_{i+1} - x_{i+1} y_i)}{2}
\]

\[\vec{x}_i = (x_i, y_i)\]

Thus formula for centroid \(C = (g_x, g_y)\) is given below:

\[
\begin{align*}
g_x &= \frac{1}{6A} \sum_{i=0}^{N-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \\
g_y &= \frac{1}{6A} \sum_{i=0}^{N-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)
\end{align*}
\]
Centroid Invariance to boundary point distribution

Fig. 2. Centroid of contour. The dots are points distributed on the contour uniformly (a) and non-uniformly (b). The star is the centroid of original contour and the inner dot is the centroid of sampled contour.
2.2 Axis of least inertia

ALI: unique ref. line preserving orientation of shape
Passes through centroid

Line where shape has easiest way of rotating about

ALI: Line $L$ that minimizes the sum of the squared distance from it to the boundary of shape

\[ \text{Denotes centroid} \]
Axis of Least Inertia

ALI defined by: \[ I(\alpha, S) = \int \int_S r^2(x, y, \alpha) \, dx \, dy \]

Here, \( r(x, y, \alpha) \) is the perpendicular distance from the pt \((x, y)\) to the line given by \( X \sin \alpha - Y \cos \alpha = 0 \).

We assume that the coordinate \((0, 0)\) is the location of the centroid.
Average Bending Energy

2.3 Average bending energy

The Average Bending Energy is defined as

$$BE = \frac{1}{N} \sum_{s=0}^{N-1} K(s)^2$$

where $K(s)$ denotes the curvature of the shape parametrized by arclength.

One can prove that the circle is the shape with the minimum Bending Energy.

General Def’n. of Curvature

$$\kappa = \left\| \frac{dT}{ds} \right\|.$$  

For Plane Curve $\Gamma(t) = (x(t), y(t))$

$$\kappa = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{3/2}},$$
Eccentricity is the measure of aspect ratio.

It’s ratio of length of major axis to minor axis (think ellipse for example).

Calculated by principal axes method or minimum bounding rectangular box.
Eccentricity: Principal Axes Method

2.4.1 Principal axes method

Principal Axes of a shape is uniquely def’d as:
two segments of lines that cross each other perpendicularly through the centroid
representing directions with zero cross correlation

Covariance Matrix C

\[ C = \frac{1}{N} \sum_{i=0}^{N-1} \begin{pmatrix} x_i - g_x \\ y_i - g_y \end{pmatrix}^T \begin{pmatrix} x_i - g_x \\ y_i - g_y \end{pmatrix} = \begin{pmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{pmatrix} \]

where

\[ c_{xx} = \frac{1}{N} \sum_{i=0}^{N-1} (x_i - g_x)^2 \]
\[ c_{xy} = \frac{1}{N} \sum_{i=0}^{N-1} (x_i - g_x)(y_i - g_y) \]
\[ c_{yx} = \frac{1}{N} \sum_{i=0}^{N-1} (y_i - g_y)(x_i - g_x) \]
\[ c_{yy} = \frac{1}{N} \sum_{i=0}^{N-1} (y_i - g_y)^2 \]

G\((g_x, g_y)\) is the centroid of the shape

Lengths of the two principal axes equal the
eigenvalues \( \lambda_1 \) and \( \lambda_2 \) of the Covariance Matrix C

Cross correlation: sliding dot product

\[ (f \ast g)(t) \overset{\text{def}}{=} \int_{-\infty}^{\infty} f^*(\tau) \ g(t + \tau) \ d\tau, \]
Eccentricity: Principal Axes Method

Lengths of the two principal axes equal the eigenvalues $\lambda_1$ and $\lambda_2$ of the Covariance Matrix C

So the eigenvalues $\lambda_1$ and $\lambda_2$ can be calculated by

$$\det(C - \lambda_{1,2} I) = \det \begin{pmatrix} c_{xx} - \lambda_{1,2} & c_{xy} \\ c_{yx} & c_{yy} - \lambda_{1,2} \end{pmatrix} = (c_{xx} - \lambda_{1,2})(c_{yy} - \lambda_{1,2}) - c_{xy}^2 = 0$$

So

$$\begin{cases} 
\lambda_1 = \frac{1}{2} \left[ c_{xx} + c_{yy} + \sqrt{(c_{xx} + c_{yy})^2 - 4(c_{xx}c_{yy} - c_{xy}^2)} \right] \\
\lambda_2 = \frac{1}{2} \left[ c_{xx} + c_{yy} - \sqrt{(c_{xx} + c_{yy})^2 - 4(c_{xx}c_{yy} - c_{xy}^2)} \right]
\end{cases}$$

Then, eccentricity can be calculated:

$$E = \frac{\lambda_2}{\lambda_1}$$

What is the eccentricity of a circle?
Eccentricity

2.4.2 Minimum bounding rectangle

Minimum bounding rectangle (minimum bounding box):
Smallest rectangle containing every pt. in the shape

Eccentricity: \( E = \frac{L}{W} \)
L: length of bounding box
W: width of bounding box

Elongation: \( Elo = 1 - \frac{W}{L} \)
Elo \( \in [0,1] \)
Circle of square (symmetric): Elo = 0
Shape w/ large aspect ratio: Elo close to 1

Fig. 3. Minimum bounding rectangle and corresponding parameters for elongation
Circularity Ratio

2.5 Circularity ratio

Circularity ratio: How similar to a circle is the shape
3 definitions:

**Circularity ratio 1:** \( C_1 = \frac{A_s}{A_c} = \frac{\text{(Area of a shape)}}{\text{(Area of circle)}} \)
where circle has the same perimeter

\[
A_c = \frac{p^2}{4\pi} \quad \text{thus} \quad C_1 = \frac{\frac{4\pi A_s}{p^2}}{\frac{p^2}{4\pi}} \\
\text{since} \ 4\pi \text{ is a constant,} \quad C_2 = \frac{A_s}{p^2}
\]

**Circularity ratio 2:** \( C_2 = \frac{A_s}{p^2} \) (\( p \) = perim of shape)
Area to squared perimeter ratio.

**Circularity ratio 3:** \( C_{va} = \frac{\sigma_R}{\mu_R} \)
\( \mu_R \) : mean of radial dist. from centroid to shape bndry pts
\( \sigma_R \) : standard deviation of radial dist. from centroid to bndry pts

\[
\mu_R = \frac{1}{N} \sum_{i=1}^{N-1} d_i \quad \text{and} \quad \sigma_R = \sqrt{\frac{1}{N} \sum_{i=1}^{N-1} (d_i - \mu_R)^2}
\]
where \( d_i = \sqrt{(x_i - g_x)^2 + (y_i - g_y)^2} \).
Ellipse Variance

Ellipse Variance $Eva$:
Mapping error of shape to fit an ellipse with same covariance matrix as shape: $C_{ellipse} = C_{shape}$
(Here $C = C_{shape}$)

$$V_i = \begin{pmatrix} x_i - g_x \\ y_i - g_y \end{pmatrix}$$

$$d_i' = \sqrt{V_i^T \cdot C_{ellipse}^{-1} \cdot V_i}$$

$$\mu_R' = \frac{1}{N} \sum_{i=1}^{N-1} d_i'$$ and $$\sigma_R' = \sqrt{\frac{1}{N} \sum_{i=1}^{N-1} (d_i' - \mu_R')^2}$$

$$Eva = \frac{\sigma_R'}{\mu_R'}$$

di’: info about shape and ellipse variance of radial distances

Ellipse Variance

2.6 Ellipse variance
Rectangularity

2.7 Rectangularity

Rectangularity represents how rectangular a shape is, i.e. how much it fills its minimum bounding rectangle:

\[ \text{Rectangularity} = \frac{A_S}{A_R} \]

where \( A_S \) is the area of a shape; \( A_R \) is the area of the minimum bounding rectangle.

What is rectangularity for a square? Circle? Ellipse?
2.8 Convexity

Convexity is defined as the ratio of perimeters of the convex hull $O_{\text{Convexhull}}$ over that of the original contour $O$ [7]:

\[
\text{Convexity} = \frac{O_{\text{Convexhull}}}{O}
\]  

(13)

The region $R^2$ is a convex if and only if for any two points $P_1, P_2 \in R^2$, the entire line segment $P_1P_2$ is inside the region. The convex hull of a region is the smallest convex region including it. In Figure 6, the outline is the convex hull of the region.

Examples of convex and non-convex based on above definition?

Illustration of convex hull
Solidity

2.9 Solidity

Solidity describes the extent to which the shape is convex or concave [8] and it is defined by

$$Solidity = \frac{A_s}{H}$$

where, $A_s$ is the area of the shape region and $H$ is the convex hull area of the shape. The solidity of a convex shape is always 1.

Examples of 2 shapes that have solidity 1 and less than one? Can you create a shape with solidity $= \frac{1}{2}$?
2.10 Euler number

Euler number describes the relation between the number of contiguous parts and the number of holes on a shape. Let $S$ be the number of contiguous parts and $N$ be the number of holes on a shape. Then the Euler number is:

$$Eul = S - N$$

For example,

```
  3 B 9
```

Euler Number equal to 1, -1 and 0, respectively.
2.11 Profiles
The profiles are the projection of the shape to $x$-axis and $y$-axis on Cartesian coordinates system. We obtain two one-dimension functions:

$$Pro_x(i) = \sum_{j=j_{min}}^{j_{max}} f(i, j) \quad \text{and} \quad Pro_y(j) = \sum_{i=i_{min}}^{i_{max}} f(i, j)$$

where $f(i, j)$ represents the region of shape Eq. 1. See Figure 7.
Hole Area Ratio

2.12 Hole area ratio

Hole area ratio $HAR$ is defined as

$$HAR = \frac{A_h}{A_s}$$

where $A_s$ is the area of a shape and $A_h$ is the total area of all holes in the shape. Hole area ratio is most effective in discriminating between symbols that have big holes and symbols with small holes [9].

HAR is the ratio: (area of the holes)/(area of shape)

Can you think of a shape with HAR equal to 0,1, arbitrarily large?
Shape Descriptors Part II
Shape Signatures
Exercises (To be done in Matlab. See demos)

• For a binary image given in matlab (see demos), find the center of mass any way you wish
• Find area of a shape rep’d. by a binary image from demo
• For a polygonal shape, find the area & center of mass. Also, calculate the distance to centroid shape signature
• Can you write code to calculate the perimeter for a given polygonal shape.
• Write code to compute the average curvature for a polygonal shape.
• Can you think of a way to denoise a given shape that has either a binary representation or parametric?
3. 1D Function for Shape Representation -Shape Signatures-

• Shape Signature: 1D function derived from shape boundary coord’s.
• Captures perceptual features of shape
• Dimension reduction. 2D shape \(\rightarrow\) 1D function Rep.
• Often Combined with some other feature extraction algorithms. E.g. Fourier descriptors, Wavelet Descriptors

• Complex Coordinates
• Centroid Distance Function
• Tangent Angle
• Curvature Function
• Area Function
• Triangle Area Representation (TAR)
• Chord Length Function
Complex Coordinates

3.1 Complex coordinates

Let \( P_n(x(n), y(n)) \) boundary pts of a Shape
\( g = (g_x, g_y) \) centroid

Complex Coordinates function is:
\[
z(n) = [x(n) - g_x] + [y(n) - g_y]i
\]

Main Idea:
Transform shape in \( \mathbb{R}^2 \) to one in \( \mathbb{C} \)
Can use additional transforms
like conformal mapping once in Complex Plane

Complex Coordinates Transform is Invariant to Translation. Why?
3.2 Centroid distance function

- Centroid-based “time series” representation

- All extracted time series are further standardised and resampled to the same length

Centroid Distance Transform is Invariant to Translation. Why?

$$r(n) = |\langle x(n) - g_x, y(n) - g_y \rangle|$$
3.3 Tangent angle

Let \( \mathbf{r}(t) = \langle x(t), y(t) \rangle \) be a parametric representation of a shape.

Tangent vector: \( \mathbf{r}'(t) = \langle x'(t), y'(t) \rangle \)

Tangent angle: \( \theta = \tan^{-1} \left( \frac{y'(t)}{x'(t)} \right) \)

The Tangent Angle Function at pt. \( P_n(x(n), y(n)) \) is def’d:

\[
\theta(n) = \theta_n = \tan^{-1} \left( \frac{y(n)-y(n-w)}{x(n)-x(n-w)} \right)
\]

‘\( w \)’ is a small window to calc. \( \theta(n) \) more accurately

2 issues with Tangent angle function:
1. Noise Sensitivity (contour usually filtered beforehand)
2. Discontinuity of tangent angle. \( \theta \in [-\pi, \pi] \) or \( [0, 2\pi] \). Thus discontinuity of size \( 2\pi \)
Tangent Angle

Angle only allowed between 0 and $2\pi$ causes some issues.

$\theta = \pi$

$\theta = 3\pi/2$

$\theta = \pi/2$

$\theta = \pi/4$

$\theta = 2\pi \rightarrow 0$

*Discontinuity*

Fix for Discontinuity:

Cumulative Angular Fctn. : $\varphi(n) = [\theta(n) - \theta(0)]$
Tangent Space

Tangent Space rep. based on Tangent Angle
• Shape bndry C simplified via polygon evolution
• C is then represented in space by graph of step function
• X-axis: arclength coord’s of pts. in C
• Y-axis: direction of line segments in decomp of C i.e. Tangent angle

Fig. 8. Digital curve and its step function representation in the tangent space

Is this method robust to noise? Does it capture small scale features well? What’s best way to choose polygonal evolution? How can this be improved? Any variational models you can think for the evolution?
Contour Curvature

3.4 Contour curvature

Curvature: important feature for human’s to judge similarity between shapes.
Salient perceptual characteristics

$$K(n) = \frac{\dot{x}(n)\ddot{y}(n) - \dot{y}(n)\ddot{x}(n)}{(\dot{x}(n)^2 + \dot{y}(n)^2)^{3/2}}$$

If $n$ is the normalized arc length parameter $s$,

$$K(s) = \dot{x}(s)\ddot{y}(s) - \dot{y}(s)\ddot{x}(s)$$

Invariant under rotations and translations
Scale dependent

$$K'(s) = \frac{K(s)}{\frac{1}{N} \sum_{s=1}^{N} |K(s)|}$$

Normalize by mean absolute curvature for scale independence depending on imp. of scale
Contour Curvature

Fig. 9. Curvature function (a) Contour normalized to 128 points; the dot marked with a star is the starting point on the contour; (b) curvature function; the curvature is computed clockwise.

What are some advantages and disadvantages of the contour curvature as a shape descriptor?
Area Function

\[ P_n = (x(n), y(n)) \]

As boundary points change, the area \( S(n) \) of the triangle formed by triplet: \((P_n, P_{n+1}, g)\)
where \( g = (g_x, g_y) \) is centroid
Area Function

• Advantages/Disadvantages?
• Translation Invariant
• Rotation invariant up to parametrization with dense enough boundary sampling
• Area function is Linear under Affine Transformation (for shapes samples at same vertices)
Triangle Area Representation (TAR)

3.6 Triangle-area representation

TAR signature computed directly from area of triangles formed by pts. on shape boundary

Let \( n \in [1, N] \) and \( t_s \in [1, N/2 - 1] \) (\( N:\text{even} \))

For each 3 points:
\[
P_{n-t_s}(x_{n-t_s}, y_{n-t_s}), \ P_n(x_n, y_n), \ P_{n+t_s}(x_{n+t_s}, y_{n+t_s})
\]

The (signed) Area of Triangle Formed by them is:
\[
TAR(n, t_s) = \frac{1}{2} \begin{vmatrix}
x_{n-t_s} & y_{n-t_s} & 1 \\
x_n & y_n & 1 \\
x_{n+t_s} & y_{n+t_s} & 1
\end{vmatrix}
\]
Triangle Area Representation

- Contour traversed counter-clockwise
- Positive TAR = Convex pts.
- Negative TAR = Concave pts.
- Zero TAR = straight line pts.

Exercise: Check via vector calculus why TAR sign matches convexity/concavity

Fig. 11. Three different types of the triangle-area values and the TAR signature for the hammer shape
Triangle Area Representation

- Increasing length of Triangle Sides i.e. considering farther pts. TAR fct. Rep’s. longer variations along contour
- TAR’s with diff. triangle sides $\rightarrow$ different scale spaces
- Total TAR’s, $t_s \in [1,N/2-1]$, compose multi-scale space TAR
- TAR relatively invariant to affine transform
- Robust to rigid transform
Chord Length Function

- Chord Length derived sans ref. point
- For each boundary pt P, Chord Length function CL:
  - Shortest distance between P and another boundary pt P’ s.t. PP’ ⊥ tangent vector at P
  - Chord Length function Invariant to translation and overcomes biased reference pt. problems (centroid biased to boundary noise or deformations)
- CL very sensitive to noise. Chord length can increase or decrease significantly even for smoothed boundaries
Summary of Shape Signatures

- Shape Signature 1D function Derived from shape contour
- For translation invariance: defined by relative values
- For Scale invariance: normalization needed
- Orientation changes: shift matching
- Occultation: Tangent angle, contour curvature, and triangle area rep have invariance
  (why are centroid dist, area function and chord length not robust to occultation?)
- Shape signatures are computationally simple
- Unfortunately Sensitive to noise
- Slight changes in boundary cause large errors in matching procedure
- Usually further processing is needed to increase robustness
- Ex. Shape signature can be quantized into a signature histogram which is rotationally invariant. However, some level of detail in matching is lost
Shape Descriptors (Part III)
Shape Matrix

- Two types: square and polar model
- Idea: Create an MxN matrix to represent shape features
Square Shape Matrix

• Let $S$ be a shape.
• Construct square centered at centroid $G$ of $S$
• Size of each side is $2L$, $L = \text{Max Euclid. Distance between centroid } G \text{ and pt. } M \text{ on boundary of } S$
• Pt. $M$ lies in center of one side
• Line $GM$ is $\perp$ to side $M$ lies on

Divide the square into $N \times N$ subsquares and denote $S_{kj}$, $k, j = 1, \ldots, N$, the subsquares of the constructed grid. Define the shape matrix $SM = [B_{kj}]$,

$$B_{kj} = \begin{cases} 1 & \iff \mu(S_{kj} \cap S) \geq \mu(S_{kj})/2 \\ 0 & \text{otherwise} \end{cases},$$

where $\mu(F)$ is the area of the planar region $F$. 
What if there are multiple maximum lengths from G to pt on boundary M?
Q: What if there are multiple maximum lengths from G to pt on boundary M?

A: Use multiple shape matrices to describe the shape
Polar Shape Matrix

• Let $S$ be a shape. $G$: centroid, $GA$: maximum radius of shape
• Construct $n$ circles centered at centroid $G$ with equally spaced radii
• Starting with $GA$, counterclockwise draw radii that divide circles into $m$ equal arcs
• Values of matrix are same as in square model. i.e. value 1 if shape occupies more than $\frac{1}{2}$ the area of the polar rectangle
• Example below for $n=5$ and $m=12$
Polar Shape Matrix

• Polar model shape matrix is simpler than square since only one matrix
• Shape matrix in general is invariant under rigid transformation and scaling. Why?
• Shape of the object can be reconstructed from shape matrix. Accuracy given by the size of grid cells.
Shape context
Shape context is a powerful tool for object recognition tasks. Often used to find corresponding features between model and image.

Shape Context for a Point
From a point, P, measure distance and angle to all other points. Histogram it. That histogram is the shape context for that point.

No, of course it’s more complicated than that. The angle is relative to the local tangent. And the measurements are logs of distance, but that’s the gist of it.
• Take N samples from edge of shape (can be internal or external contours)
• Form vectors originating from a point to all other sample points on shape
• These vectors express the appearance of entire shape relative to the reference point (hence the name: shape context). Calculate the magnitude of these vectors (distances to ref pt from bdnry pts)
• Calculate the relative angle given by angle between vector emanating from ref point and tangent vector at the ref point

\[ h_i(k) = \#\{Q \neq P_i : (Q - P_i) \in \text{bin}(k)\} \]

Shape context: histogram of relative polar coord’s of all other points
Shape context for 3 different points. (d) for circle pt in shape (a) 
(e) For diamond pt in shape (b), and (f) for triangle pt in shape (a) 
Dark is large value. 5 bins for polar direction and 12 bins for angular 

Rich Descriptor since as N gets large, the representation for the shape becomes exact
Shape Context

Shape context matching often used to find corresponding points on two shapes. Applied to many object recognition problems.

• Invariance properties include:
  • Translation due to it being based on relative point locations
  • Scaling by normalizing the radial distances by the mean or median distance between all the pt. pairs
  • Rotation by rotating the coordinate system at each pt so positive x-axis aligns with tangent vector
  • Shape variation: shape context is robust against slight shape variations
  • Few outliers: points with final matching cost larger than a threshold value are class’d as outliers. Can introduce additional dummy pts to decrease effects of outliers
5.10 Chord distribution
The basic idea of chord distribution is to calculate the lengths of all chords in the shape (all pair-wise distances between boundary points) and to build a histogram of their lengths and orientations [42]. The “lengths” histogram is invariant to rotation and scales linearly with the size of the object. The “angles” histogram is invariant to object size and shifts relative to object rotation. Figure 29 gives an example of chord distribution.

Fig. 29. Chord distribution (a) Original contour; (b) chord lengths histogram; (c) chord angles histogram (each stem covers 3 degrees).
Scale Space Approaches

• Curvature scale space
• Many others
This approach is based on multi-scale representation and curvature to represent planar curves. The nature parametrization equation is shown as following:

\[ \Gamma(\mu) = (x(\mu), y(\mu)) \] (17)

An evolved version of that curve is defined by

\[ \Gamma_{\sigma}(\mu) = (X(\mu, \sigma), Y(\mu, \sigma)) \]

where \( X(\mu, \sigma) = x(\mu) \ast g(\mu, \sigma) \) and \( Y(\mu, \sigma) = y(\mu) \ast g(\mu, \sigma) \), \( \ast \) is the convolution operator, and \( g(\mu, \sigma) \) denotes a Gaussian filter with standard deviation \( \sigma \) defined by

\[ g(\mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) \]
Curvature Scale Space

Functions $X(\mu, \sigma)$ and $Y(\mu, \sigma)$ are given explicitly by

$$X(\mu, \sigma) = \int_{-\infty}^{\infty} x(v) \frac{1}{\sigma \sqrt{2\pi}} \exp\left( \frac{-(\mu - v)^2}{2\sigma^2} \right) dv$$

$$Y(\mu, \sigma) = \int_{-\infty}^{\infty} y(v) \frac{1}{\sigma \sqrt{2\pi}} \exp\left( \frac{-(\mu - v)^2}{2\sigma^2} \right) dv$$

The curvature of is given by

$$\kappa(\mu, \sigma) = \frac{X_\mu(\mu, \sigma)Y_{\mu\mu}(\mu, \sigma) - X_{\mu\mu}(\mu, \sigma)Y_\mu(\mu, \sigma)}{(X_\mu(\mu, \sigma)^2 - Y_\mu(\mu, \sigma)^2)^{3/2}}$$

Note that $\sigma$ is also referred to as a scale parameter. The process of generating evolved versions of $\Gamma_\sigma$ as $\sigma$ increases from 0 to $\infty$ is referred to as the evolution of $\Gamma_\sigma$. This technique is suitable for removing noise and smoothing a planar curve as well as gradual simplification of a shape.
The function defined by \( k(\mu, \sigma) = 0 \) is the CSS image of \( \Gamma \). Figure 33 is a CSS image examples.

Fig. 33. Curvature scale-space image (a) Evolution of Africa: from left to right \( \sigma = 0 \) (original), \( \sigma = 4 \), \( \sigma = 8 \) and \( \sigma = 16 \), respectively; (b) CSS image of Africa.
it captures the main features of a shape, enabling similarity-based retrieval;

it is robust to noise, changes in scale and orientation of objects;

it is compact, reliable and fast;

it retains the local information of a shape. Every concavity or convexity on the shape has its own corresponding contour on the CSS image.

Although CSS has a lot of advantages, it does not always give results in accordance with human vision system. The main drawbacks of this description are due to the problem of shallow concavities/convexities on a shape. It can be shown that the shallow and deep concavities/convexities may create the same large contours on the CSS image. In [54, 55], the authors gave some methods to alleviate these effects.
Shape Transform Domain

- Fourier
- Wavelet
- Shapelets
- Radon
- Etc...
Fourier Shape Transform

• Can be used directly on a shape
• We will talk about using the 1D Fourier Transform in conjunction with centroid distance shape descriptor
Centroid Distance Function

3.2 Centroid distance function

- Centroid-based "time series" representation

- All extracted time series are further standardised and resampled to the same length

Centroid Distance Transform is Invariant to Translation, Rotation, and can scaled.

What is the key issue then?

\[ r(n) = \left| \langle x(n) - g_x, y(n) - g_y \rangle \right| \]
Centroid Distance Issue

- Key issue: shift in the parametrization
- i.e. starting point issue
- If have 2 same shapes and start at 2 different starting points. The distance between signatures will be large. i.e. signature doesn’t ID correctly the same shapes
Why Fourier?

- Fourier usually combined with shape signatures (prev’ly discussed)
- This example will demonstrate why
- So consider:

  Centroid Distance Signature + Fourier
Fourier Descriptor

• Fourier Descriptor (FD) obtained by applying Fourier transform to a Shape Signature.
  Note, Shape Signature is a 1D function in general
• Normalized Fourier transformed coefficients are called: Fourier Descriptor for the shape
• FD’s derived from different signatures has significant different performances on shape retrieval.
• In General FD from centroid distance $r(t)$ outperforms FD’s derived from other shape descriptors in terms of overall performance

Discrete Fourier transform for $r(t)$:

$$ a_n = \frac{1}{N} \sum_{t=0}^{N-1} r(t) \exp \left( -j \frac{2\pi nt}{N} \right) $$

for $n = 0, 1, ..., N - 1$

Since centroid distance sig. is only rotational and translational invariant. Goal: start point and scaling invariance
Fourier Descriptor

Start point is periodic. i.e. for N pts. Then \( r(0) = r(N) \)

Let \( r^{(og)}(t) = r^{(o)}(t) \) be a given original shape.
Let \( r(t) \) be the original shape under
start point shift \( \theta \) and scaling \( s \).
i.e. \( r(t) = s \, r^{(o)}(t + \theta) \)

then \( a_n = \exp(jn\tau) \cdot s \cdot a_n^{(o)} \)
with \( \tau = -2\pi\theta \) where \( \theta \) shift.
\( \tau \): angles incurred by change of start point
\( s \): scale factor

then \( a_n \) and \( a_n^{(o)} \) are Fourier coefficients for \( r \) and \( r^{(o)} \)
the transformed shape and the original one
Fourier Descriptor

Consider the following:

\[ b_n = \frac{a_n}{a_1} = \frac{\exp(jn\tau) \cdot s \cdot a_n^{(o)}}{\exp(j\tau) \cdot s \cdot a_1^{(o)}} = \frac{a_n^{(o)}}{a_1^{(o)}} \exp[j(n - 1)\tau] = b_n^{(o)} \exp[j(n - 1)\tau] \]

Here \( b_n \) and \( b_n^{(o)} \) are the normalized Fourier coeff’s of the transformed shape and the original shape respect’ly.

Ignore phase info and only use magnitude of the coeffs

Thus, \( |b_n| = |b_n^{(o)}| \)

i.e. \( |b_n| \) is invariant to translation, rotation, scaling, and shift or change in start point
FD’s

The set of magnitudes of the normalized Fourier coefficients of the shape \( \{|b_n|, 0 < n < N\} \) are used as shape descriptors, denoted as

\[
\{FD_n, 0 < n < N\}.
\]

One-dimensional FD has several nice characteristics such as simple derivation, simple normalization and simple to do matching. As indicated by [53], for efficient retrieval, 10 FDs are sufficient for shape description.
Research/Directed reading

• Triangle Area Representation. Read, implement, think about ways of improving or in specific cases where improvements can be made
• Centroid distance+Fourier Descriptor. Ditto.
• Shape Context. Ditto
• Moments. Ditto

You will need to present these next week!