Image Processing and Shape Tutorial
For Malignant Tumor Classification
Part I

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Choose Optimal Shape Descriptors that correlate highly to Tumors that are malignant with high concentrations of HGF.

Train on 2/3 of data.

Based on shape descriptors, classify remaining 1/3 of data. classify Tumors with high concentrations of HGF based purely on descriptor info.
50ng/ml HGF

Sphere 2
Well 4

Day 1
Day 2
Day 3
Day 4

Day 5
Day 6
*Day 7
DS
*Day 8
LS

*Day 9
LS + M
Day 10
Day 11
Day 12
Image Representation

- Binary Image
- 1 bit per sampled pixel. Takes only values 0 or 1.
- \(0 = \text{black}, \ 1 = \text{white}\)

\[
f(x,y) = 0 \text{ or } 1
\]

\((x,y): \text{pixel location}\)

\((x,y) \rightarrow f(x,y) \in \{0,1\}\)
Image Representation: Grayscale

- Grayscale Digital Image
- 8 bits per sampled pixel = 256 different intensities from 0 to 255. i.e. 256 shades of gray
- Value 255 = white, 0 = black. Anything else is a shade of gray

Decreasing resolution (dpi)
A Digital Image and Graph of the Function that Represents It

Digital Image

Graph of f(x)
Image Representation: Color

- **Channels:**
  - **R: red**
  - **G: green**
  - **B: blue**

- **Pixel → [R, G, B] vector**
- **8 bits per channel**
- **24 bit color image**

- **Red Dress:** high intensity in R channel
- **Green Grass:** high intensity in G channel
Some Examples of Image Processing

Denoising

Deblurring
Some Examples of Image Processing Cont’d

Super-resolution (AKA Zooming)

Zoom in by Factor of 4
Some Examples of Image Processing Cont’d

Inpainting

Segmentation
Image Processing: Motivation

Recall the Los Angeles Riots of 1992
Image Processing: Some Motivation

High Point of Riots: Reginald Denny Beaten Mercilessly on Nat’l. TV

Public Outrage!
Perpetrators at large!
Calculus Based Image Processing Used to Enhance Footage
Cognitech and UCLA Image Processing Group Help LAPD

THE WALL STREET JOURNAL

California Company Uses Calculus to Pin The Crime on the Criminal Who Did It

By Robert Langreth
Staff Reporter of The Wall Street Journal

The two men were on trial for murder. Convictions might have been easy: A gas station security camera had filmed the whole tussle, culminating in fatal gunshots. But the videotape was so blurry that no one could really tell who attacked whom. The two argued self-defense, and the Los Angeles County jury hung.

So local detectives turned to Cognitech Inc., a tiny company armed with a powerful new technique for enhancing fuzzy images. Cognitech’s improved video clearly showed the suspects pinning the victim face down against the ground and firing into his skull. Both defendants eventually pleaded guilty.

In the past two years, analyzing crime and accident videotapes has blossomed into a full-time business for Cognitech, based in Santa Monica, Calif. It is among a handful of companies applying sophisticated mathematics to clearing up crime and accident videotapes.

Before these companies existed, police trying to enhance poor videos had to buy commercial “Photoshop” software, which generally processes one frame at a time and is limited to simple operations such as improving contrast. Or they could send their work is done on computer workstations at employees’ desks.

On a typical day, Mr. Rudin prowls the office looking over employees’ shoulders as they analyze videotapes, asking questions and making suggestions. For particularly stubborn videotapes, the indefatigable Mr. Rudin often stays late into the night adapting computer programs to do that type of image better.

Cognitech isn’t alone in the expanding video-enhancement field. Another small company, Trec Inc., in Huntsville, Ala. sells software for enhancing videotapes to the FBI and other law enforcement agencies. And Aerospace Corp., a nonprofit military-research agency, recently started a small unit to analyze crime videotapes. It now handles a couple of dozen cases per year.

Neither the FBI nor Trec will comment on their enhancement techniques. Aerospace, for its part, says a variety of standard mathematical methods for improving images serve it just fine. “Standard image enhancement is a whopping field,” agrees Massachusetts Institute of Technology electrical engineer Alan Willsky. He adds that “the jury’s still out on the overall impact” of Cognitech’s method.

In any case, all sides expect their caseloads to increase. As more businesses turn sophisticated video...
It started out as just a speck on a photograph of a man who threw a brick at truck driver Reginald Denny at Florence and Normandie avenues in the opening hours of the 1992 Los Angeles riots.

But when Leonid Rudin subjected it to a complicated computer algorithm and a slew of complex mathematical equations, that speck—originally less than 1/6,000th the size of the total photograph—was revealed to be a rose-shaped tattoo on the arm of the man, later identified in court as Damian Monroe Williams.
either identify those workstation bottlenecks that can be improved, automated, or augmented by the ACI algorithms, or to introduce new exploitation aids tailored to digital libraries.

Outcome: All 3 Criminals Convicted!
Example: License Plate Recognition in Low Light
Example: Elevator Video Enhancement

degraded

reconstructed
The Image Restoration Problem

A given Observed image $f$
Related to True Image $u$
Through Blur $K$
And Noise $\eta$

$$f = K \ast u + \eta$$

Initial Blur Blur+Noise

Inverse Problem: restore $u$, given $K$, and statistics for $\eta$.

Keeping edges sharp and in the correct location is a key problem!

See Matlab functions: conv2 and filter2
Total Variation Regularization

\[ \text{TV}(u) = \int_\Omega |\nabla u| \, dx \]

- Measures “variation” of \( u \) w/o penalizing discontinuities

- 1D: If \( u \) is monotonic on \([a,b]\), then \( \text{TV}(u) = |u(b)-u(a)| \), regardless of whether \( u \) is discontinuous or not

- Coarea Formula:

\[ \int_\Omega f |\nabla u| \, dx = \int_{-\infty}^{\infty} \left( \int_{\{u=r\}} f \, ds \right) \, dr \]

- Thus TV controls both size of jumps and geometry of boundaries

- TV norm can be thought of as ‘Jump x Perimeter’ of a feature

- Extends to vector valued images. e.g. Color

- For Binary Images, TV reduces to Perimeter
TV Norm vs $H^1$ Norm

$$TV(u) = \int_{\Omega} |\nabla u| \, dx$$

Preserves Edges doesn’t penalize gradients too heavily

$$H^1(u) = \int_{\Omega} |\nabla u|^2 \, dx$$

Tends to Penalize High Gradients More than the TV Norm

Zooming Example

TV norm: Sharper edges

$H^1$: More smoothed edges
Calculus of Variations

$$\min_u g(u) = \int_\Omega |\nabla u| \, dx$$

Functional: Function of functions

- How does one minimize a functional? Depends on functional
- Special case: convex function
- Take derivative and set equal to zero like in calculus
- Derivative wrt to a functional is called ‘variation’

When differentiate $g(u)$ and set $= 0$ you obtain:

$$-\nabla \cdot \frac{\nabla u}{|\nabla u|} = 0$$

Which can be solved by time marching:

$$u_t = \nabla \cdot \frac{\nabla u}{|\nabla u|}$$

First variation:

$$\delta J(y)(h) = \lim_{\varepsilon \to 0} \frac{J(y + \varepsilon h) - J(y)}{\varepsilon} = \left. \frac{d}{d\varepsilon} J(y + \varepsilon h) \right|_{\varepsilon=0}.$$
\[ TV(u) = \int_{\Omega} |\nabla u| \, dx \]

Min TV equiv to solving:

\[ u_t = \nabla \cdot \left( \frac{1}{|\nabla u|} \nabla u \right) \]

Anisotropic Diffusion:
When gradient big = smaller diffusion

\[ H^1(u) = \int_{\Omega} |\nabla u|^2 \, dx \]

Min \( H^1 \) equiv to solving Heat Equation:

\[ u_t = \nabla \cdot \nabla u = \Delta u \]

Diffusion of heat source in all directions

Observed noisy image
Heat Flow
TV Flow
Total Variation Restoration

Regularization: \( TV(u) = \int_{\Omega} |\nabla u| \, dx \)

Variational Model:

\[
\min_u f(u) = \frac{1}{2} \int_{\Omega} (Ku - f)^2 \, dx + \lambda \int_{\Omega} |\nabla u| \, dx
\]

- First proposed by Rudin-Osher-Fatemi ‘92.
- Allows for edge capturing (discontinuities along curves).
- TVD schemes popular for shock capturing.

Gradient

Flow: \( u_t = -g(u) = \lambda \nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) - (K^*Ku - K^*f) \)

\( \frac{\partial u}{\partial n} = 0 \)

anisotropic diffusion data fidelity
Variational Image Processing

Examples of Restoration: TV Denoising

![Original](image1)
![Noisy image](image2)
![Denoised image](image3)

![noisy](image4)
![TV denoising](image5)
![more TV denoising](image6)
Image Inpainting (Masnou-Morel; Sapiro et al ‘99)

Disocclusion

Graffiti Removal

Hello! We are Penguin A and B. You guys must think that in many words have made a large amount of image information lost. Is this true? We disagree. We are more optimistic. The TV model can restore us. See ya!
Unified TV Restoration & Inpainting model

Main Idea:
- Fit only where have data
- Regularize everywhere

\[ J[u] = \int_{E \cup D} |\nabla u| \, dx \, dy + \frac{\lambda}{2} \int_{E} |u - u^0|^2 \, dx \, dy, \]

\[-\nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) + \lambda_e (u - u^0) = 0,\]

\[ \lambda_e = \lambda, \quad z \in E; \quad 0, \quad z \in D. \]
TV Inpainting: disocclusion
Examples of TV Inpaintings

Where is the Inpainting Region?
TV Zoom-in

Inpaint Region: high-res points that are not low-res pts
**Image Segmentation**

- Problem: Detect Salient regions & their boundaries in a given image
- Often: P.W. Smooth (or cartoonish) regions & sharp boundaries
- Useful for object location and recognition, medical imaging (tumor detection etc.), face recognition, tracking, … and more
Application: “active contour”

- giving an image $f : \Omega \rightarrow \mathbb{R}$
- evolve a curve $C$ to detect objects in $f$
- the curve has to stop on the boundaries of the objects
Basic idea in classical active contours

Curve evolution and deformation (internal forces):

\[ \text{Min } \text{Length}(C) + \text{Area}(\text{inside}(C)) \]

Boundary detection: what is it? What is stopping criteria for curve?

Initial Curve $\rightarrow$ Evolutions $\rightarrow$ Detected
Basic idea in classical active contours

Curve evolution and deformation (internal forces):

\[ \text{Min } \text{Length}(C) + \text{Area(inside}(C)) \]

Boundary detection: stopping edge-function (external forces)

\[ g \geq 0, \quad g \downarrow, \quad \lim_{t \to \infty} g(t) = 0 \]

Example:

\[ g(|\nabla u_0|) = \frac{1}{1 + |\nabla G_\sigma \ast u_0|^p} \]

Snake model (Kass, Witkin, Terzopoulos 88)

\[ \inf_{C} F(C) = \int_{0}^{1} |C'(s)|^2 ds + \lambda \int_{0}^{1} g(|\nabla I(C(s))|) ds \]

Geodesic model (Caselles, Kimmel, Sapiro 95)

\[ \inf_{C} F(C) = 2 \int_{0}^{1} |C'(s)| g(|\nabla I(C(s))|) ds \]
Sometimes edge detectors find the boundary pretty well.

\[ g(|\nabla u_0|) = \frac{1}{1 + |\nabla G_\sigma * u_0|^p} \]

Strong Stopping Criteria
Sometimes it’s not enough.

Fuzzy Stopping Criteria

$$g(|\nabla u_0|) = \frac{1}{1 + |\nabla G_\sigma * u_0|^p}$$
Limitations

- detects only objects with sharp edges defined by gradients
- the curve can pass through the edge
- smoothing may miss edges in presence of noise
- not all can handle automatic change of topology

Examples
Snakes: Active Contour Models

- Snakes or Active Contours pose the segmentation as an energy minimization problem.

- Kass, Witkins & Terzopoulos.

\[ E_{\text{Snake}} = \int_C E_{\text{ext}} dp + \int_C E_{\text{int}} dp \]
Local Minima

- One major drawback of Active Contour model is the tendency to get stuck in “Local minima” caused by subtle irrelevant edges and image features.

Initialization

Final Segmentation
Data Fitting Term

\[
\int_{\text{inside}(C)} |f - c_1|^2 \, dx \, dy + \int_{\text{outside}(C)} |f - c_2|^2 \, dx \, dy
\]

where

\[c_1 = \text{average}(f) \, \text{inside } C\]

\[c_2 = \text{average}(f) \, \text{outside } C\]

Fit > 0                Fit > 0                Fit > 0                Fit ~ 0

Minimize: (Fitting + Regularization)

Fitting not depending on gradient detects “contours without gradient”
Chan-Vese (CV) Model

Fitting + Regularization terms (length, area)

\[
\inf_{c_1, c_2, C} F(c_1, c_2, C) = \mu \cdot |C| + \nu \cdot \text{Area(inside}(C))
\]

\[
+ \lambda \int_{\text{inside}(C)} |u_0 - c_1|^2 \, dx\,dy + \lambda \int_{\text{outside}(C)} |u_0 - c_2|^2 \, dx\,dy
\]

\(C = \text{boundary of an open and bounded domain}\)

\(|C| = \text{the length of the boundary-curve } C\)

- P.W. Constant Version of Mumford Shah Model
- Fit constant homogeneous regions while enforcing regularity on boundary of \(C\)
- Active Contours without Edges
**Experimental Results**

**Evolution of C**

**Advantages**

- Automatically detects interior contours!
- Works very well for concave objects
- Robust w.r.t. noise
- Detects blurred contours
- The initial curve can be placed anywhere!
- Allows for automatic change of topologies
Matlab Demo

- Polygonal P.W. Constant Mumford Shah
Shape Definition

Mathematician David George Kendall defined shape as:

“Shape is all the geometric information that remains when location, scale, and rotational effects are filtered out from an object”

Silhouette of Hand

Four exact copies of same shape under different Euclidean transformations
Useful Descriptor Properties

- Invariance to translation
- Invariance to rotation
- Scalability
- Unique I.D. of certain shapes. E.g. Convex Shapes
Descriptor Examples

- Fourier: shape info in low freq. components
- Moments: mean, variance, skew, kurtosis
- Centered/Centroid Distance
- Cliques: Intervertex Distances
- Inner Distance
- Landmarks and Statistical Signatures
- Many others, still very active research topic
Future Work

- Classification Problem
- Preprocessing Data. Lots of clutter and noise.
- Segmentation of Tumor Images using Snakes/Active Contours. Difficult due to clutter.
- TV norm or other norms as a descriptor
- Meyer G norm to measure oscillation or other Negative Sobolev Norms
Thank You for Your Attention!