Non-Blind Barcode Deconvolution by Gradient Projection

Christine Lew, Dheyani Malde

Supervisors: Ernie Esser, Yifei Lou

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1 Abstract

This research examines methods of implying deconvolution to blurry barcode signals with noise. Our goal is to take these signals and reconstruct them, using Yu Mao’s method of Gradient Projection, to be as clear as possible. This research examines the work of Yu Mao [5], alumni from the University of Minnesota. Our research is motivated by Yu Mao’s findings for how to reconstruct binary functions and shapes from incomplete frequency information. The findings are significantly interesting because we are able to understand how a binary function can be reconstructed with a simple convex optimization, while only partial frequency information is available.

2 Introduction

A barcode is a optical machine readable representation of data. Barcodes represent data with various widths and spacing of parallel lines. In this research, we focused on Universal Product Code (UPC) barcode signals, which are usually

Figure 1: Standard UPC Barcode
Figure 2: Binary Representation in One-Dimension of Barcode

one-dimensional and generally in a rectangular form, as illustrated in Figure 1.
The clean barcode signal is binary, represented in digits 0 or 1 seen in Figure 2.

Barcodes were first used to label railroad cars, but they were not publicly useful
until they were used to automate supermarket checkout systems. This became
their universal use. Their use has continued to develop and become moderately
more involved with databases and automatic identification. The very first scan-
ning UPC barcode was on a pack of Wrigley’s chewing gum in 1974. Since then,
there have been many applications of commercial barcodes.

Conventionally, barcodes were scanned by special scanners called barcode
readers; as technology has progressed more intelligent and equipped software
have been made to scan barcodes, such as desktop printers and smart phones.
Often UPC barcodes become damaged due to noise and blur from malfunc-
tioning scanners and/or movement. This is a problem because it corrupts the data,
destroying the readability of the signal. This issue is seen in transferring in
organization large amounts of data which could affect a large amount of people,
in setting such as healthcare, retail, and corporations.
2.1 Mathematical Modeling

We assume that the recorded barcode signal $b$ can be expressed in the following way,

$$b = k \ast u + e$$  \hspace{1cm} (1)

where $u$ is a clean barcode, $k$ is a blur kernel and $e$ is random noise. \ast denotes the convolution operator, which is a mathematical operation on two functions to produce a third function that is more modified than the two original functions. In the case of barcode degradation, the convolution is equivalent to a weighted averaging of the clean signal $u$ where the weights are defined by the blurring kernel $k$. In reality, a good approximation for such blurring kernel is the family of Gaussian functions as seen in Figure 3. We add blur to a clear barcode signal by using a Gaussian kernel (also known as point spread function). We add noise by convolving $u$ with the Gaussian kernel. The Gaussian kernel is a symmetric bell curve. Gaussian blurring is also known as Gaussian smoothing, used to reduce image noise and detail.

Given the recorded signal $b$, our goal is to reconstruct a clean barcode $\hat{u}$ that is as close to the original signal $u$ as possible. This is the inverse of convolution, which is referred to as deconvolution. If we have no or little knowledge on the kernel $k$, it is often called “blind” deconvolution; if $k$ is completely known as $a priori$, then it becomes “non-blind” deconvolution.

Figure 3: Generating Barcode with Noise and Blur using Convolution with a Blur Kernel
Convolution?

This is the mathematical approach of two functions, $f$ and $g$, to produce a third function that is more modified than the two original functions. The area overlapped by the original two functions is represented as a function of the amount that one of the original functions is translated. There are two types of convolution. Discrete convolution is for periodic functions. Circular convolution is when one of the two functions is convolved in the normal way, with a periodic summation of the other function.

Deconvolution?

The object of deconvolution is to find the solution of a convolution equation of the form:

$$f \ast g = h$$  \hspace{1cm} (2)

$h$ is the recorded signal and $f$ is the signal we want to recover. $g$ has been convolved with signal $g$.

- $f$: blur kernel
- $g$: clear barcode
- $e$: noise
- $h$: barcode with noise

The lower the signal to noise ratio, the worse our estimate of the deconvolved signal will be.

3 Introduction

This paper compares several methods of reconstructing clean barcode signals from blurred barcodes with noise. The methods used in this paper are Yu Mao's
method of Gradient Projection and the Wiener Filter. Yu Mao’s approach involves a non-blind method, in which we are knowledgeable about the blur kernel.

4 Previous Methods

Thresholding?
The simplest method is to threshold the recorded signal. In particular, we threshold the amplitude at a specific point to see whether it is closer to 0 or 1. This approach only works for the small amount of the blur and noise.

Wiener Filter?
Wiener Filter is a classical method in signal processing to reduce the amount of noise in a given signal. Its formula is

$$\min_u \|k \ast u - b\|^2 + \frac{r}{2} \|u\|^2$$  
(3)

where $r$ is a constant we added to minimize the noise even more. The lower the signal to noise ratio, the worse our estimate of the deconvolved signal will be. In signal processing, the purpose of the Wiener Filter is to reduce the amount of noise in a given signal. We compare the Wiener Filter to Yu Mao’s method of Gradient Projection, in terms of the error rate. The error rate is the difference
between reconstructed signal and ground truth (the solution).

5 Our Method

For barcode deconvolution, we want to minimize the data discrepancy under the binary constraint. Mathematically, it can be expressed as

$$\min_{u} \| k * u - b \|$$
$$s.t. \quad u \in \{0, 1\}$$  \hspace{1cm} (4)

This is a difficult problem, as the binary constraint is not convex. In [5], Mao proves that under certain conditions, the optimization problem with the binary constraint (4) can be relaxed to the box constraint, i.e.,

$$\min_{u} \| k * u - b \|$$
$$s.t. \quad u \in [0, 1]$$  \hspace{1cm} (5)

To solve (5), we apply gradient projection, which is very similar to gradient descent. Gradient descent is a first order optimization algorithm. We use gradient descent to find a local minimum of a function. We are "descending" on a graph to find the lowest point. Gradient Descent can be solved using a system of linear equations:

$$Ax - b = 0 \Rightarrow F(x) = \|Ax - b\|^2$$  \hspace{1cm} (6)
For gradient projection, we just add projection onto $[0,1]$ every iteration. We continue to gradient project until we reach the minimum, which we then use to predict the clear signal. We use the fast fourier transform in gradient projection. The fast fourier transform is an algorithm used to compute the discrete fourier transform. The FFT is a fast way to compute a sequence of values into components of different frequencies. This is a transform operation.

Formula:

$$
\min_u \|k * u - b\|^2 + \frac{r}{2} \|u\|^2
$$

$k$ is the blur kernel
$u$ is the clear barcode signal
$b$ is the noise added to the blurry signal
$r$ is a constant we added to minimize the noise even more
Note: no constraint on $u$, unlike in Yu Mao’s (restraint: $0 \leq u \leq 1$)

6 Experiments

There are many different methods offering the same conclusion of a clear barcode signal. We compare Yu Mao’s method with thresholding and the Wiener Filter, both visually and in terms of error rate. The error rate is defined as the difference between the reconstructed signal and the ground truth.

$$
\frac{1}{N} \sum ||x_i - u_i||
$$

We examined the efficiency Yu Mao’s method with thresholding with deconvoluting the corrupted barcode seen in Figure 9 and Figure 10. Using this we compared with the Wiener Filter’s results seen in Figure 7 and 8.

One way we thought about approaching this was measure the error rate by the amount of jumps generated by a reconstructed barcode. Each barcode is a representation of 0s and 1s. In a string of 10 numbers, a jump is defined when a digit changes from 0 to 1 or 1 to 0. We want the least amount of jumps for our barcode; in other words, we want the least amount of rectangles on our visual.

To begin this comparison, we made a code for calculating the number of jumps for every reconstructed barcode signal. We collected data by using a random number generator to generate different strands for various barcodes. With this, we ran our code using the different de-blurring methods. Our data
consisted of the number of jumps calculated for each de-blurring method, \( m \) with a changing sigma value. We observed that as our sigma-noise value increased we had generated less jumps. The least amount of jumps is equivalent to the least amount of error. As predicted, Yu Mao’s method of Gradient Projection calculated less error than the Wiener Filter.

Using the classical method of the Wiener Filter as a comparison, we were able to determine the efficiency of our implementation of Yu Mao’s method for Gradient Projection. First, we examined the results for a randomly generated barcode. Then further analysing the efficiency, we did a comparison of the two methods based on the number of jumps, which determine the difficulty of recovering the barcode (the less jumps, the easier recovery, and vice-versa). As a general means, we are looking for error less than or equal to 10\% , as this is considered acceptable results.

7 Results

In the two figures above, we attempt to recover a clear barcode, with randomly generated numbers, yielding a random barcode. From implementing Yu Mao’s Gradient Projection seen in Figure 11, we observed little blur and less, resulting with approximately 20\% -25\% error. For the Wiener Filter seen in Figure 12,
Figure 8: Deconvoluted Barcode using Wiener Filter (large blur and large noise)

Figure 9: Deconvoluted Barcode using Yu Mao’s Gradient Projection (small blur and large noise)
Figure 10: Deconvoluted Barcode using Yu Mao’s Gradient Projection (large blur and large noise)

Figure 11: Yu Mao’s with Randomly Generated Barcode
Figure 12: Wiener Filter with Randomly Generated Barcode

Figure 13: Yu Mao’s with Low Number of Jumps
Figure 14: Yu Mao’s with Medium Number of Jumps

Figure 15: Yu Mao’s with High Number of Jumps
Figure 16: Wiener Filter with Low Number of Jumps

Figure 17: Wiener Filter with Medium Number of Jumps
Figure 18: Yu Mao’s Gradient Projection with Low Number of Jumps

when there is not noise- regardless of the amount of blur- it results in 0% error. When there is more noise and blur, the Wiener Filter is drastically less effective than Yu Mao’s method.

Examining Yu Mao’s method of Gradient Projection with the small amount of jumps (4 jumps) seen in Figure 13, we see that a large portion of the graph is in the 10% error range. The majority of the visual is blue. Blue indicates least amount of error. Red indicates high amount of error. There is only one section of the visual that shows more error, which is seen in the 35% range. This error is seen with high noise and no blur.

In regards to Yu Mao’s Gradient Projection with a medium amount of jumps (22 jumps) seen in Figure 14, most of the values of noise and blur yield an error rate to the value of 25%. Similar to the above graph with 4 jumps, there is a higher error rate around 35%, with high noise and no blur.

Yu Mao’s method of Gradient Projection with a high amount of jumps (45 jumps) seen in Figure 15, yields a moderately large amount of error. Higher blur and higher noise yields, on average, 25% to 35% error.

Examining the general effect of jumps with Yu Mao’s method of Gradient
Projection, we see that as we gradually increase the amount of jumps the error rate increases slightly. There is not a drastic increase in error rate as the jumps increase, rather the error proportionately becomes worse as jumps increase.

Implementing a small amount of jumps (4 jumps) with the Wiener Filter (seen in Figure 16), regardless of the amount of blur or noise, the error rate does not exceed the 5% range.

Implementing the Wiener Filter with the medium amount of jumps (22 jumps) seen in Figure 17, the error is at most 20%, regardless of the amount of blur and noise.

Examining the Wiener Filter with a high amount of jumps (45 jumps) seen in Figure 18, we observe that the error rate became drastically worse. A large portion of the graph shows, roughly, 40% to 50% error. Only with an extremely small amount of noise (close to zero) and a relatively small amount of blur does the error rate range around 25%.

Examining the general effect of the number of jumps on the Wiener Filter, we see that with a small or medium of jumps this method gives moderately good results, with 25% being the highest error. However, when implementing the Wiener Filter with the actual number of jumps according to a standard barcode, the error rate increased. Overall, Yu Mao’s method of Gradient Projection is far more effective than the Wiener Filter.

8 Conclusion and Future Works

From our research and test runs, we concluded that Yu Mao’s method of Gradient Projection, is the most accurate approach to reconstructing binary signals. Overall, Yu Mao’s method is better than the Wiener Filter because it produces less error. In all the jump cases, Yu Mao’s method was more consistent in terms of a 20% error rate. The Wiener Filter did well with a small and medium amount of jumps. However, at 45 jumps, there was a 50% error rate. Due to this, the Wiener Filter was not consistent in error rate.

For future works, we have come across new interesting ideas to work with and research. Now that we have learned to generate our own data and apply our knowledge to deblurring barcodes, we can look at this problem through a more theoretical analysis. This summer we focused on a more factual approach. One idea for this is to find the string of numbers that corresponds to each reconstructed barcode. Generating this new data will allow us to create a dictionary of what binary sequences correspond to what numbers. This will be a different optimization problem. In Yu Mao’s paper there are many other formulas and theories mentioned, which can introduce new research topics. There are
many other theories to continue studying that Yu Mao has mentioned. Another suggestion for future research is to reduce the unknown variables. In order to do this, we would have to work with a new matrix. With a new matrix we would make sure we would not be able to find bars smaller than the minimum width. To do this, we would incorporate a linear stretch model. This would allow us to reduce the unknown variables because there would be a limit constraint on what each bar could be.

References