The Galaxy

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1 Abstract

The game Galaxy has yet to be solved and the optimal strategy is unknown. Solving the game boards would contribute new ideas to game theory and mathematical methods that may not have been used before. Our solution allows us to know which player has the optimal strategy to n number of stars, with minimum/maximum edges applied, and the number of game states each board will have. With this information, we hope to continue our research by creating an algorithm that will allow us to discover which player has the optimal strategy on n number of stars with m number of edges applied.

2 Introduction

Galaxy, developed by James Ernest under Cheapass Games in 2000, is a game consisting of a board and two players. On the official board for this
game, there are 20 stars and 38 edges, connecting two stars to each other in a random order. Player 1 begins the game by marking one of these stars with their initials. Then the players alternate turns marking a star until every star has one of the players initials. Players want to obtain the highest amount of points by the end of the game in which case they are declared the winner. The player obtains a point for every of the opponents stars adjacent to the star just marked by the said player. The game of Galaxy can be classified as determinate, non-zero sum, symmetric, perfect information and sequential. However, this game is neither normal nor misere as players want to obtain the highest number of points by the end of the game. The game of Galaxy, though similar to other games, does not seem to have any pre-existing information such as who has the winning strategy and what is the winning strategy and other similar questions. It was relatively difficult to find a copy of this game let alone any analysis that might have been done previously on this game. We begin our analysis not only for the defined board of Galaxy but also for any number of stars and edges.

3 Problem

Galaxy is a simple pen and paper game with qualities similar to many classic combinatorial games. Though this game was created by James Ernest in 2000, there appears to be a lack of any previous research on the game. Many questions, such as who has the winning strategy, what is the optimal strategy,
is the game unfair, how should a player approach this game, etc., remain as open questions for research. We begin to explore this game by looking at various smaller game boards in hope of seeing some pattern that may lead us to answer some of the questions previously mentioned.

4 Our Idea

In this section I will describe the plan we have attempted to follow throughout the course of our research. During the first week, we sought to play the game a massive amount of times in an effort to produce conjectures and possible research questions for further exploration. Our results of such are as follows:

- If possible, avoid marking stars (or vertices) with multiple connections (edges) as this could potentially give your opponent an advantage, due to the fact there are many possible vertices that could award them points.

- Which player has the winning strategy for boards with ‘n’ number of vertices and ‘m’ number of edges connecting those vertices? (This proved to be an incredibly broad question)

- The number of edges, rather than the number of vertices more closely determines the winner of a board with ‘n’ vertices. Player 2 has the winning strategy (This was hypothesized due to the fact that the second player’s first move will always yield a point for that player).
The second step in our plan became an effort to create a human vs human and human vs random computer simulation of the game. Once that was finalized, the next step was to begin working on any combinatorial questions related to the Galaxy such as the number of game states and begin development on a computer vs computer adaptive learning program. Lastly, we began to look at games similar to Galaxy such as Snort and Col and began drawing analysis from our adaptive learning program. With this done, we answered our research questions and came to our conclusions.

5 The Details

Combinatorial Questions and Answers

In the Galaxy, the order in which the vertices (or stars) were visited (marked) is the most important factor in defining a game state. This is due to the nature in which point values are assigned to each player; a board configuration may appear similar to another at first glance, but the scores for each player may be completely different due to the order in which the vertices were visited. As such, we can consider the number of total possible game states given a board of size ‘n’ stars, from turn 1 to turn ‘n’, to be a series of permutations. To begin, we look at the number of ways we may obtain an ordered subset of ‘k’ elements from a from a larger set of ‘n’ elements. This number can be derived from the formula:
\[ nP_k = n!/(n - k)! \]

[2]

Where \( n \) is the number of vertices, and \( k \) is the current turn number (number of vertices that have been visited). For example, to find the number of total possible game states for a 5-star game, the formula would be as follows:

\[ \frac{5!}{4!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{1!} + \frac{5!}{1!} = 325 \]

We use \( nP_k \) for every turn. On the last turn, we use the same number of game states as the previous turn, due to the fact that there is exactly one more move that could be made from each of those game states, thus one more game state per each of the previous states.

Once that has been calculated, we face the issue of having to calculate how many times the game would have to be played randomly in order to encounter all the possible game states. For this, we look to the Coupon Collector’s problem, and use the formula:

\[ n \times H_n \]

[1]
Where n is the possible number of game paths (estimated by multiplying the number of possible last moves by the number of moves in the game). To figure out how many simulations to run with our adaptive learning program, we multiply the result of this equation by roughly 2 or 3. This is because the computer would need to first view the game state, then revisit it to change the weights of the particular game state (more on our adaptive program later in this section). Minimally Connected and Maximally Connected Board Configurations

We define a minimally connected board configuration as one which contains n-1 edges, where n is the number of stars, and the difference between the vertex with the greatest number of adjacent vertices and the one with the least number of adjacent vertices is no larger than 1. A maximally connected board configuration is one in which each vertex is adjacent to all other vertices. The result of games played with these board configurations is always a tie when there is an odd number of vertices and a win for player 2 with an even number of vertices. On the minimally connected board, player 2 has a chance of winning if player 1 does not use the optimal strategy—that is, player 1’s first move should be to mark a vertex which contains the least amount of adjacent vertices. The result for the maximally connected board configuration will be the same regardless of strategy.

**Adaptive Learning Program Description**

As board sizes and configurations get larger and more complex, attempting to analyze each of them individually becomes a daunting task. In order
to cope with this challenge, we created an adaptive learning program (written in the Python programming language). The idea is that with every game play, the computer learns which moves it shouldn’t make and thus slowly learns the winning strategy for a certain board configuration. The main components of program include the board configuration, game states and matchboxes. A 'game state' contains a list of 'stars.' These stars are represented as indices, at each of these indices there is a number, either 0, 1, or 2. A value of 0 signifies that the particular star has not been marked. A value of 1 or 2 signifies that the particular star has been marked by player 1 or 2, respectively. A game state also contains player 1 and 2 scores. A sample initial game state of a game of 3 stars: 
\[
[[0,0,0,0],[0,0]]
\] . Note that the first index is always omitted as we never assume there is exists a star 0. A board configuration contains information on which star is adjacent to which in the form of a list of lists. Index 1 contains a list of integers which represent all the stars adjacent to star 1. A sample board configuration of 3 stars is as follows:
\[
[[],[2,3],[1,3],[1,2]]
\]. Again, the first index is omitted as there is no star 0. Matchboxes are perhaps the most important factor in the program. A Matchbox contains a game state, a list of legal moves, and a weight associated with each of the moves named 'beads.' Each time the computer encounters a new game state, it creates a new matchbox and adds it to the list of Matchboxes. By default, each of the legal moves is given a weight of 1. When the computer punishes a losing move, the weight becomes 0. The following flow chart shows the mechanics of the program:
bf Adaptive Learning Program Results

Due to the immense volume of possible board configurations, we decided on an ‘average’ board configuration per number of stars. We were able to calculate this balanced average by adding the least amount of edges in a minimally connected board with the amount of a maximally connected board and dividing by 2. From then on, we decided to apply these edges to stars in the most balanced manner possible. We did this by making sure that the difference between the vertex with the greatest number of adjacent vertices
and the one with the least number of adjacent vertices was no larger than 1, as with a minimally connected configuration. Results for 5, 6, 7, and 8 star games with these properties are as follows: 5 Star Game - 7 edges applied, neither player has a winning strategy as a tie can be forced 6 Star Game - 10 edges applied, player 2 has the winning strategy 7 Star Game - 13 edges applied, neither player has a winning strategy as a tie can be forced 8 Star Game - 17 edges applied, player 2 has the winning strategy

6 Related Works

The combinatorial games Col and Snort are map coloring games that share many properties with Galaxy. Col is a two player game played on a map that has regions not colored in. The two players take turns shading in the regions, but may not shade a region next to one he/she may have already shaded. For example, player one may not shade a region next to one he/she may have already shaded. The game is over when the next player has no legal moves available. The winner is the last player to shade in a region. Snort has a similar concept as Col, but the players may not shade next to a region that has already been shaded. For example, player two may not shade next to a region that the other player has shaded. The player that has the winning strategy for these two games depends on how many regions are available to the players, and their geographic map. Therefore, Col and Snort have been classified as being unfair due to the asymmetrical game boards, which does
not allow each player the equal opportunity to a certain move. Galaxy is related to these games because players take turns claiming stars, however there are no restrictions as to which star a player may claim and the game is based on a point system rather than being normal or misre. The point system changes the way Galaxy is analyzed compared to Col and Snort because the amount of points awarded is based on how many edges are connecting the stars. The number of edges, along with the number of stars, determine which player will have the winning strategy. Because of the way the stars are connected and how many connections one star may have, the analysis differs from Col and Snort. However the number of edges connected to a star does not change the number of game states the game board has. Meaning, the edges do not affect all the possible moves that player is able to make for the game board. Although Galaxy is analyzed differently from Col and Snort, similar strategies can be used when playing a game of Galaxy. A similar strategy between the games, Col and Galaxy, is to claim the region/star that has the least edges connecting it to another region/star. This strategy works for both of the games, because the strategy allows players to minimize their chances of winning. In Col, the strategy allows more legal moves for the player himself/herself. While in Galaxy it decreases the amount of points the opponent is able to receive. When comparing the strategies to Snort and Galaxy, the strategies are opposite. The strategies are opposite because of the different rules the games have. In Snort, the player wants to claim the region that connects to most of the others because claiming the region with
the most connections regions causes the opponent to have less options, just as claiming the star with the least connections/edges does in Galaxy.

7 Conclusion and Future Work

Galaxy, like many other two player pen and pencil game, requires players to develop a unique strategy in order to win this game. Though we were unable to solve the actual game board of 20 stars and 38 edges, we have uncovered many patterns to the game and come to the conclusion that the game is indeed unfair. In the boards we have looked at, it appears that player two has the optimal strategy but on even sized boards, while player one can force a tie on odd sized boards. Future works include solving the actual game board and refining the adaptive learning program so that the matchbox list is structured in such a way that it will be faster to search through (perhaps through the use of a hashing function).

References
