

UC IRVINE MATH CEO

Community Educational Outreach



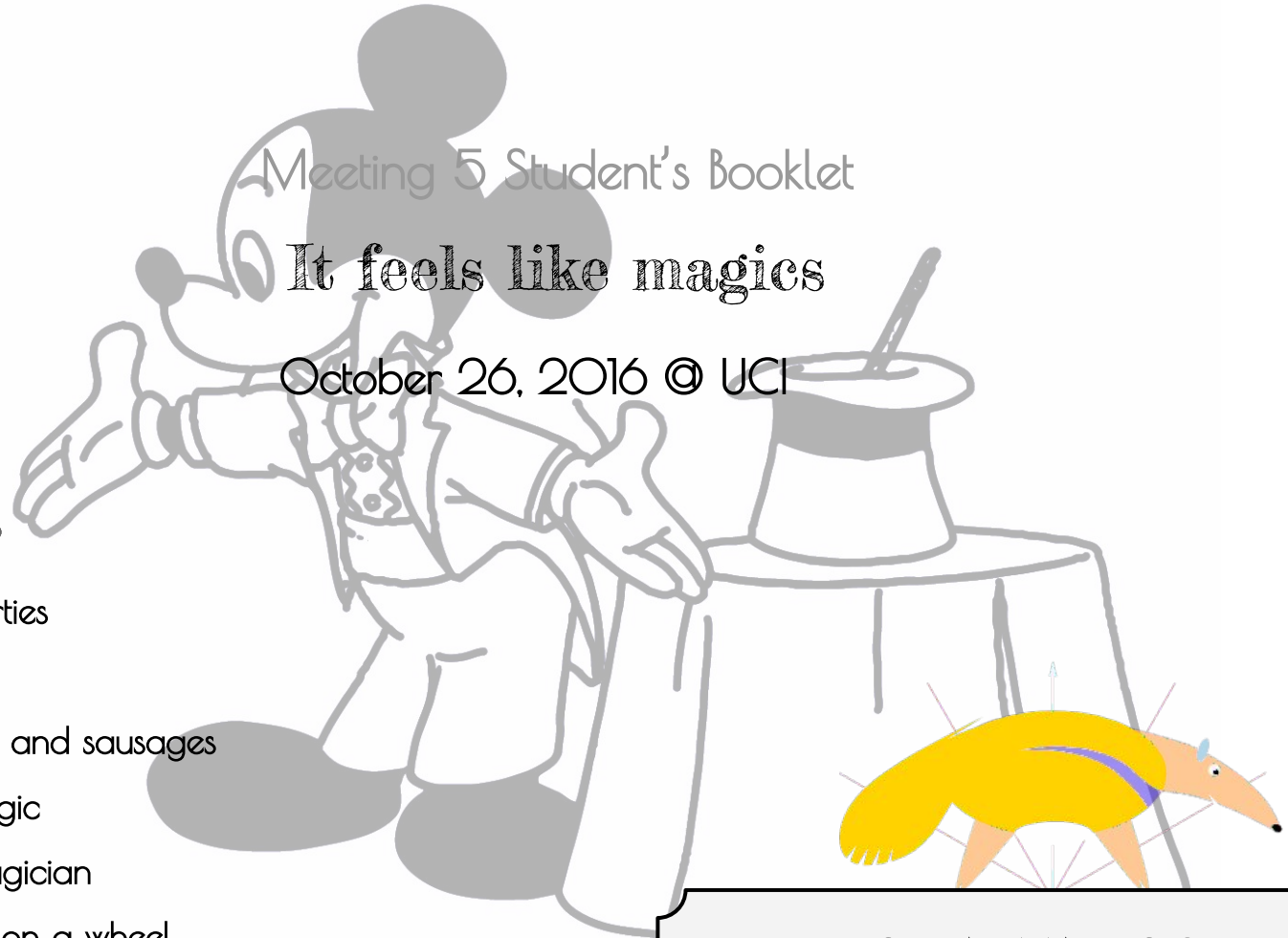
Meeting 5 Student's Booklet

It feels like magics

October 26, 2016 @ UCI

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UC IRVINE MATH CEO
<http://www.math.uci.edu/mathceo/>

THE MATHEMAGICIAN



The mathemagician insisted on doing his next trick. He pointed to a child in the audience.

“Pick a number”, the said.

- “Double it.
- Add 7.
- Multiply by 5,
- Subtract the number you started with.
- Remove any non-zero digit from the answer.
- Now tell me the remaining digits in any order.”

The child said: “6 and 8”.

“Then the digit you removed is a 3,” announced the mathemagician.

He was correct. How did he know?

From “Problem solving through recreational mathematics.”

1 Sausage parties

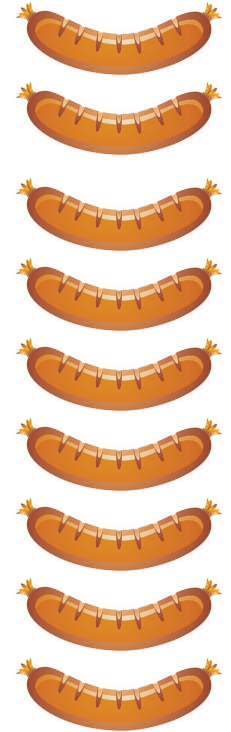
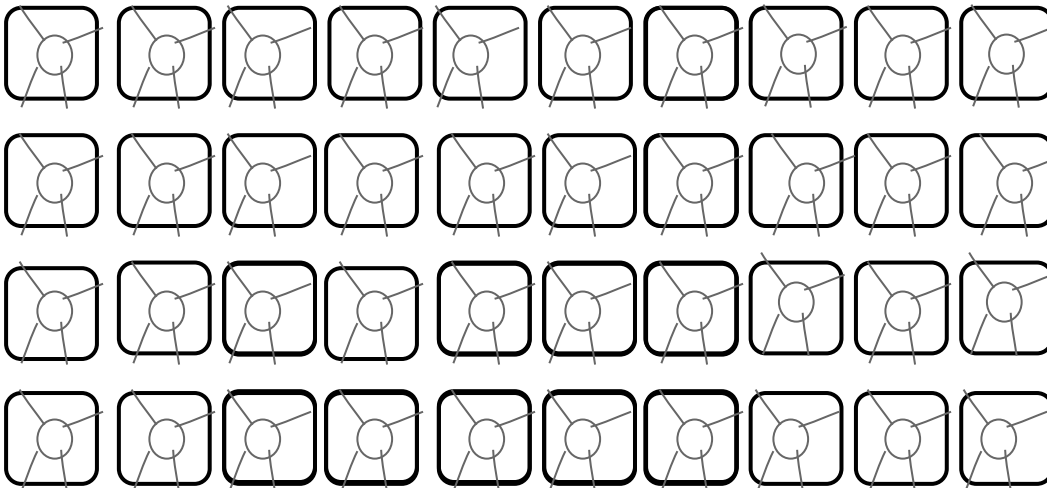
Pancho's store sells sausages in **packages of 9**. Buns, on the other hand, are individually wrapped so you can always buy just as many buns as you need.

You invite **42 friends** for a birthday party and buy a bun for each.

Some of the friends may be vegetarian and you certainly do not want to have any sausages left over.



You decide to buy the largest number of packages that allows you to have no leftover sausages. How many friends will be left with no sausage?



Your answer:

In order to have no sausage left over, you buy _____ packages of sausages.

There will be exactly _____ friends with no sausage.

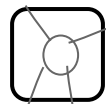
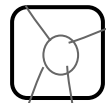
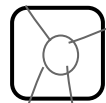
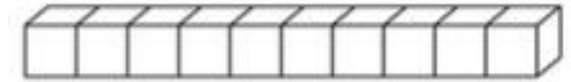


Splitting buns in blocks of tens

Write $42 = 40 + 2$ as the sum of (4) tens and (2) units.
Take out a group of 9 from each pile of ten...
This gives you 4 groups of 9 buns (matched with 4 packages of sausages). There are 6 buns left.

There is no way to take out another group of 9 from the remaining 6 buns. We conclude that:

- We should buy 4 boxes of sausages.
- There will be 6 friends without sausage.



6 buns with no sausage

4x9 buns with sausage

More sausage parties

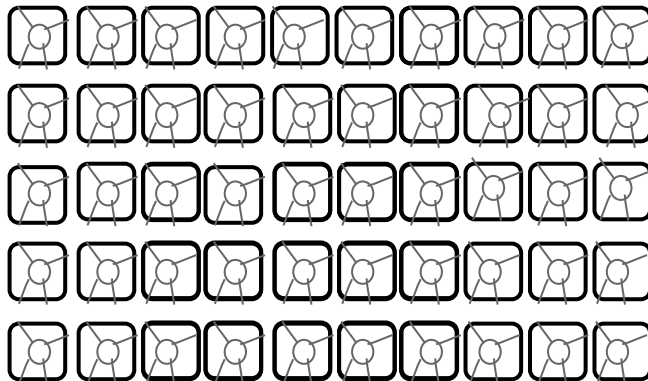
Remember the deal: Sausages come in packages of 9; buns are individually wrapped.

When you plan a party, you need to make sure that

- Everybody gets a bun (so always buy as many buns as guests)
- You have no sausages left over (so buy as many packages of sausage as you can, but make sure the number of sausages never exceeds the number of guests).

Your job is to find out how many friends will be without sausages at each of these parties.

53 people
at the party



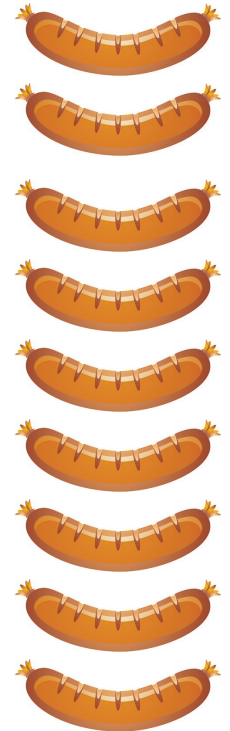
53 buns

$$53 = 5(\text{tens}) + 3(\text{units})$$

Circle the groups of 9
... you have 1 group of 9
in every "ten".

Color the remaining buns.

Make more groups of 9
if possible,

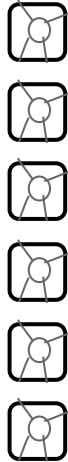


Number of packages of
sausages
to buy: _____

Number of friends left
without sausages:



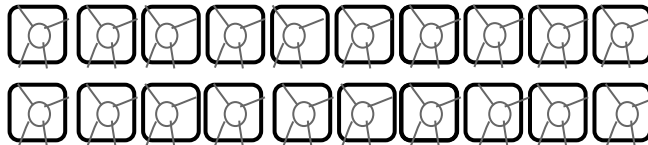
*26 people
at the party*



Number of packages of
sausages to buy: _____

Number of friends left
without sausages: _____

26 buns



$26 = 2(\text{tens}) + 6(\text{units})$

Circle the groups of 9
... you have 1 group of 9
in every "ten".

Color the remaining buns.

Make more groups of 9
if possible,

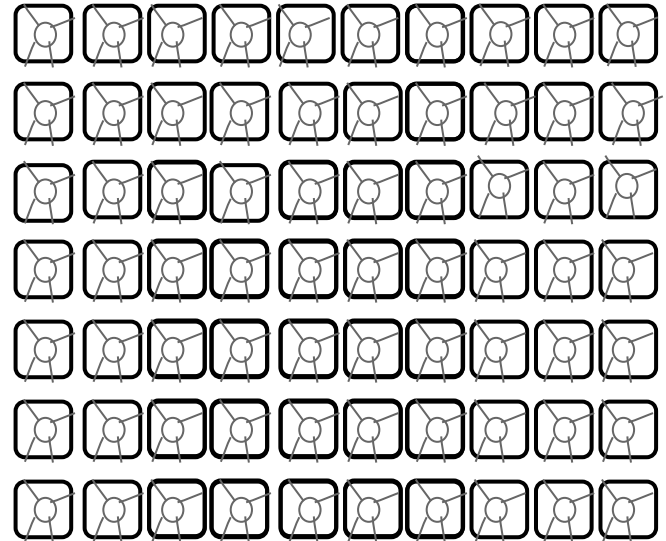


*75 people
at the party*



Number of packages of
sausages to buy: _____

Number of friends left
without sausages: _____



75 buns

Let's try to generalize.

If you invite ab friends. Here “ a ” is the tens digit and “ b ” is the units digit, so $ab = a \times 10 + b$.

How many packages of sausages do you buy if we want to have no left-overs?

How many friends at the party will have no sausage?

We need to count how many groups of 9 are there in the number ab .

Use blocks of tens to draw a picture of the number ab .

Although a sample picture is provided, the answers to the questions below should be in terms of a and b .

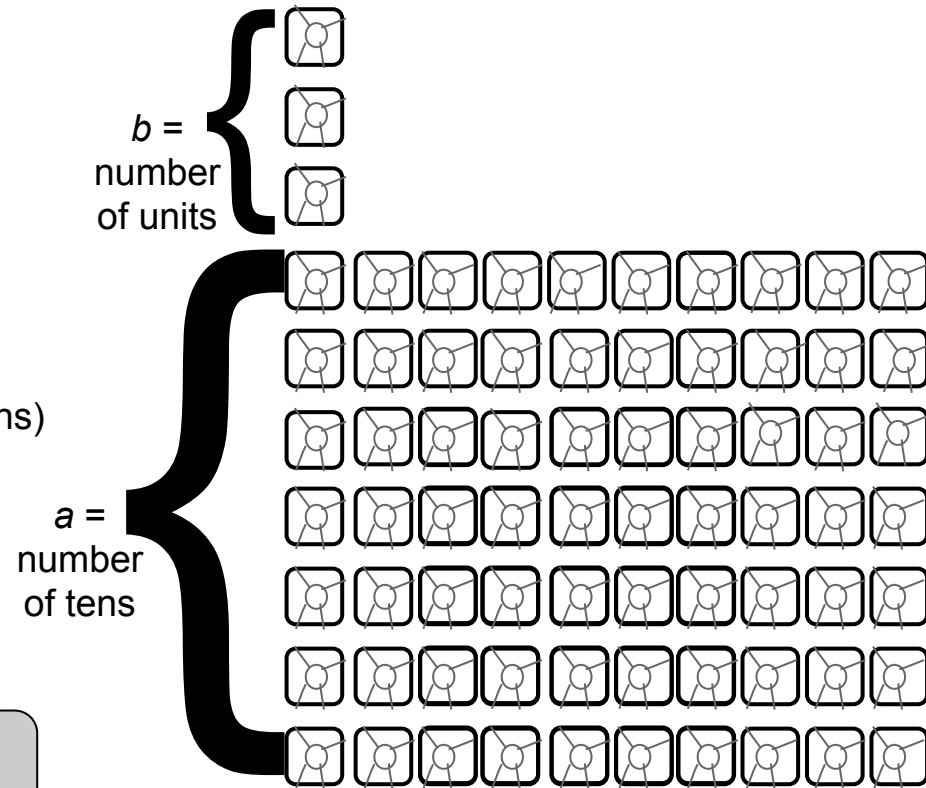
- How many groups of 9 in one ten? _____
- How many tens in the number ab ? _____
- How many groups of 9 so far? _____

Circle all the groups of 9 you got so far (from all the tens) and color the remaining buns.

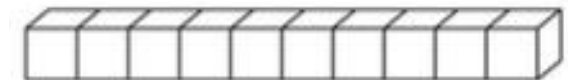
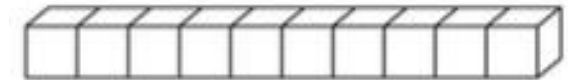
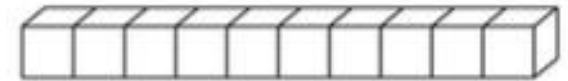
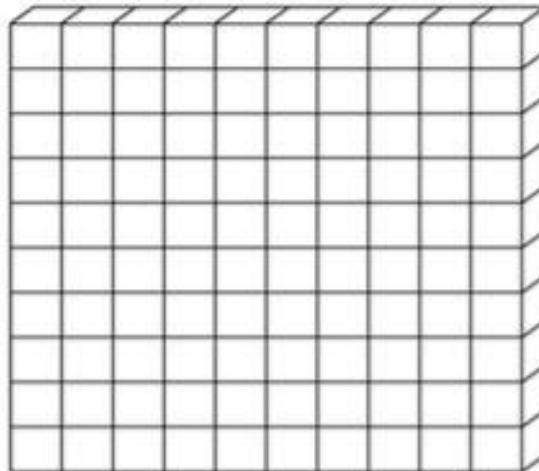
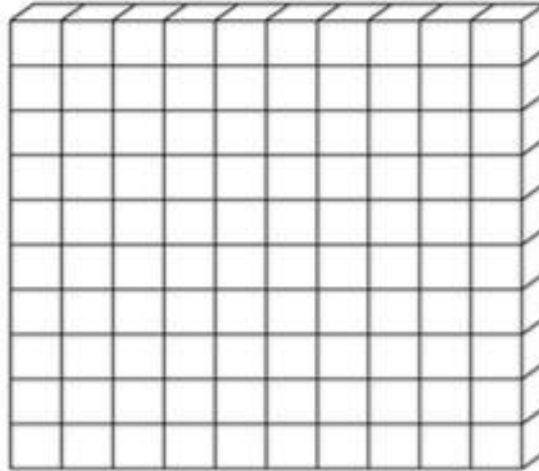
- How many buns have you colored? _____
(Express your answer in terms of a and b .)

Make more groups of 9 if possible.

True or false: *If you invite ab friends at the party, or you invite $a+b$ friends, the number of friends with no sausages will be the same.*



*232 people
at the party*



Number of packages of
sausages to buy: _____

Number of friends left
without sausages: _____

$$232 = 2(\text{hundreds}) + 3(\text{tens}) + 2(\text{units})$$

Circle the groups of 9.

*... you have 1 group of 9 in every "ten".
How many groups of 9 in a "hundred"? _____*

*Circle all the groups of 9 that come from
either a ten or a hundred.*

How many groups of 9 have you circled?

_____ (from 10s) + _____ (from 100s) = _____

Color the remaining buns.

Can you form any other group of 9?

232 buns



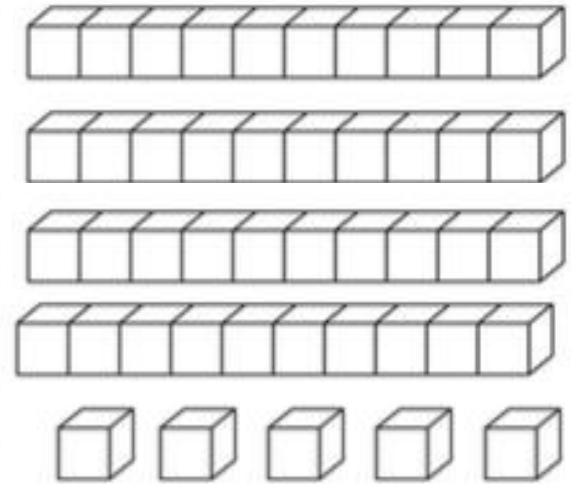
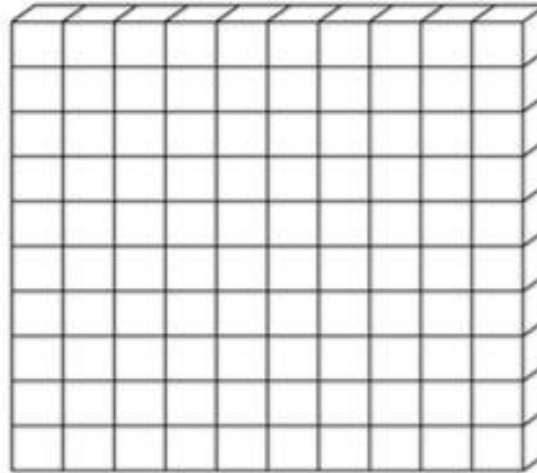
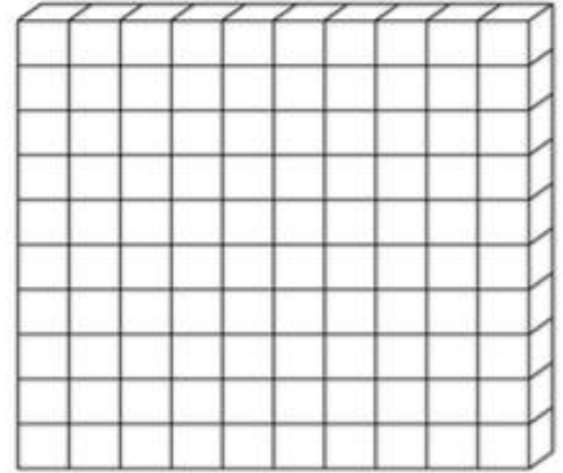
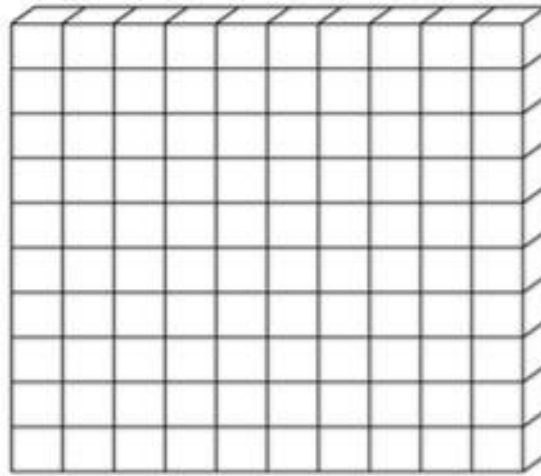
*345 people
at the party*

$$345 = 3 \text{ (hundreds)} + 4 \text{ (tens)} + 5 \text{ (units)}$$

- $1 \text{ hundred} = 11 \text{ (groups of 9)} + 1$
→ $3 \text{ hundreds} = 33 \text{ (groups of 9)} + 3$
- $1 \text{ ten} = 1 \text{ (group of 9)} + 1$
→ $4 \text{ tens} = 4 \text{ (groups of 9)} + 4$
- $5 \text{ units} = 0 \text{ (group of 9)} + 5$

So

- $345 = (33+4) \text{ groups of 9} + (3+4+5)$
= $37 \text{ (groups of 9)} + 12$
= $37 \text{ (groups of 9)} + (9+3)$
= $38 \text{ (groups of 9)} + 3.$



345 buns



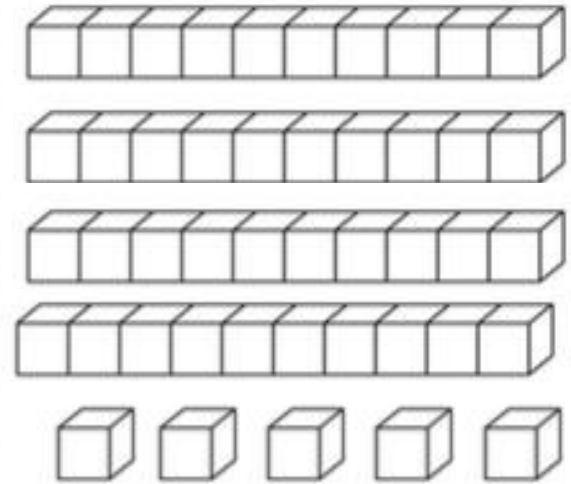
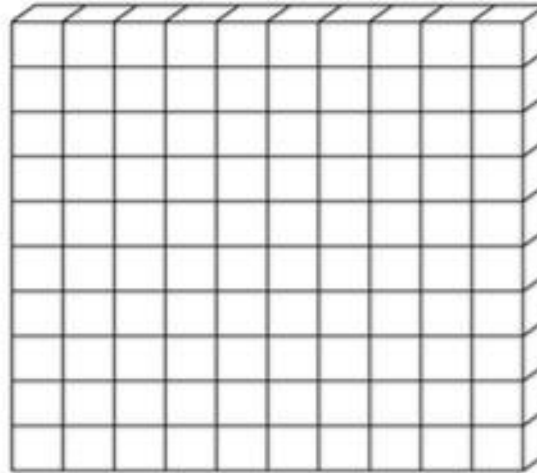
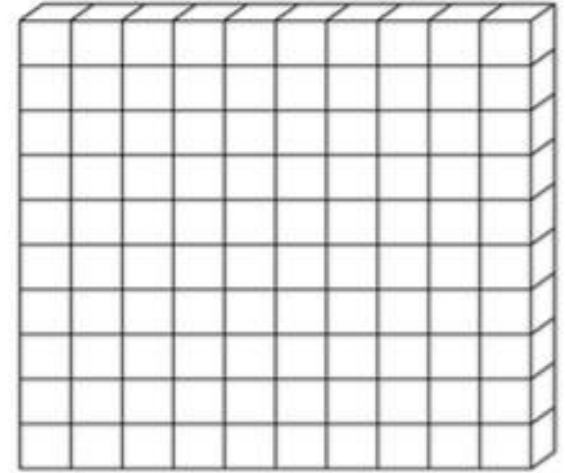
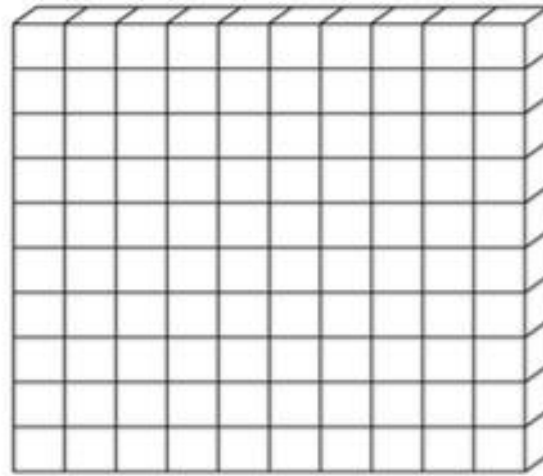
*345 people
at the party*

$$345 = 3 \text{ (hundreds)} + 4 \text{ (tens)} + 5 \text{ (units)}$$

- $1 \text{ hundred} = 11 \text{ (groups of 9)} + 1$
→ $3 \text{ hundreds} = 33 \text{ (groups of 9)} + 3$
- $1 \text{ ten} = 1 \text{ (group of 9)} + 1$
→ $4 \text{ tens} = 4 \text{ (groups of 9)} + 4$
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So

- $345 = (33+4) \text{ groups of 9} + (3+4+5)$
= $37 \text{ (groups of 9)} + 12$
= $37 \text{ (groups of 9)} + (9+3)$
= $38 \text{ (groups of 9)} + 3.$



345 buns

More sausage parties



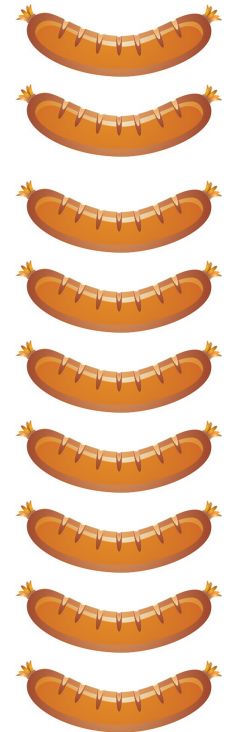
$$256 =$$

2(hundreds) + 5(tens) + 6(units)



$$756 =$$

7(hundreds) + 5(tens) + 6(units)



- $100 = 9 \times (\underline{\quad}) + \underline{\quad}$
- $200 = 9 \times (\underline{\quad}) + \underline{\quad}$

- $10 = 9 \times (\underline{\quad}) + \underline{\quad}$
- $50 = 9 \times (\underline{\quad}) + \underline{\quad}$

- $200 + 50 + 6 = 9 \times (\underline{\quad} + \underline{\quad}) + \underline{\quad}$
 $= 9 \times (\underline{\quad}) + \underline{\quad}$

Is 256 divisible by 9? Why or why not?

- $100 = 9 \times (\underline{\quad}) + \underline{\quad}$
- $700 = 9 \times (\underline{\quad}) + \underline{\quad}$

- $10 = 9 \times (\underline{\quad}) + \underline{\quad}$
- $50 = 9 \times (\underline{\quad}) + \underline{\quad}$

- $700 + 50 + 6 = 9 \times (\underline{\quad} + \underline{\quad}) + \underline{\quad}$
 $= 9 \times (\underline{\quad}) + \underline{\quad}$

Is 756 divisible by 9? Why or why not?

2 Digital roots

To compute the **digital root** of a number, you keep adding its digits until you get a number between 1 and 9:

$N = 123456$

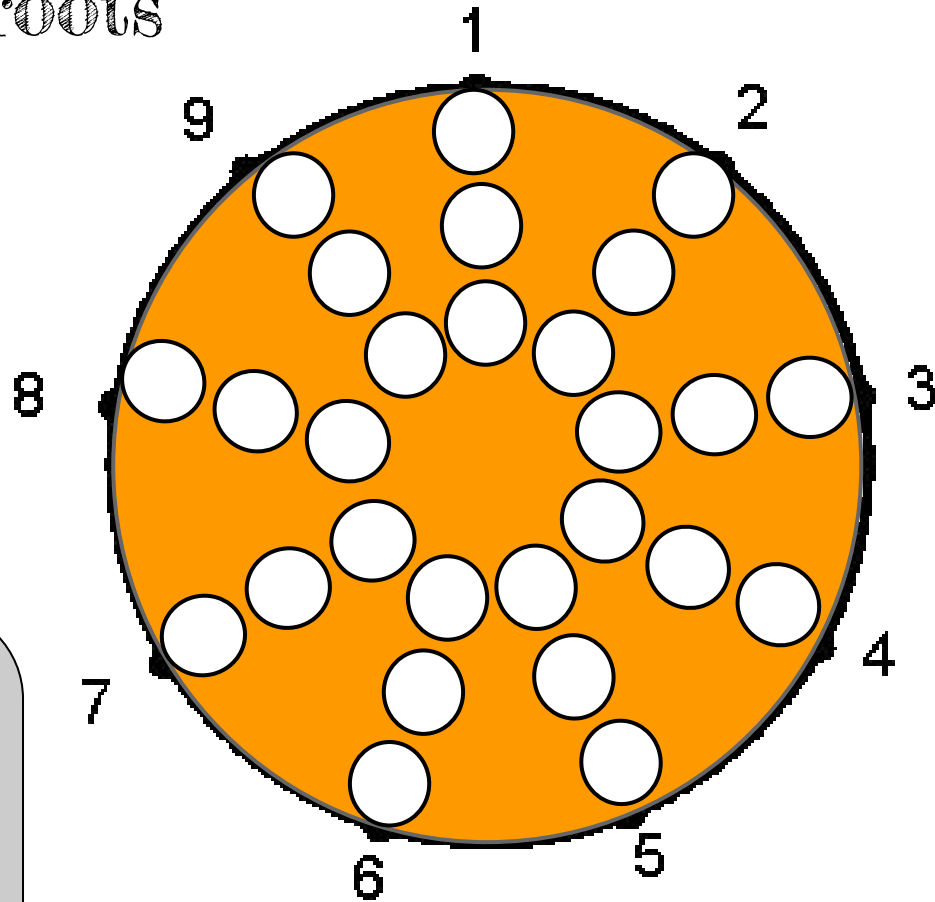
- $123456 \rightarrow$ add the digits: $1 + 2 + 3 + 4 + 5 + 6 = 21$ (*too big, add the digits again!*)
- $21 \rightarrow$ add the digits: $2 + 1 = 3$

The digital root of $N = 123456$ is equal to 3.

At your table, you will find stickers with the following numbers:

37, 23, 45, 46, 52, 53, 39, 20, 47, 36, 26, 25, 50,
40, 19, 49, 41, 43, 18, 51, 44, 21, 42, 24, 38, 22, 48.

Your table will also be given a paper plate with rays labeled 1 through 9. . As a team, your job is to compute the digital sum of the numbers above, and place the corresponding stickers on the appropriate rays of the paper plate.



GROUP ACTIVITY: once you have placed all the activities, look at the plate. What do the numbers on the same ray have in common?

Fill in the blanks with numbers between 1 and 90.

Digital root = 9

Digital root = 1

Digital root = 2

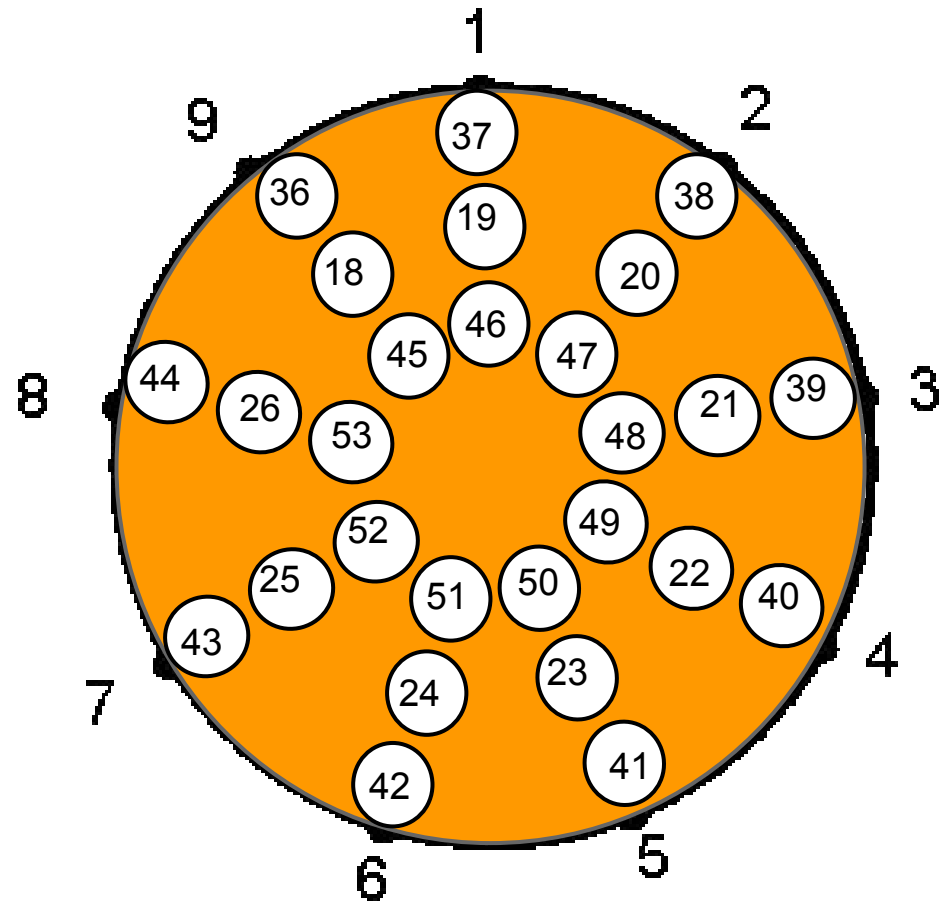
Look for patterns

*What do the numbers
in each trapezoid
have in common?*

Digital sum and divisibility by 9

A positive integer N is divisible by 9 if and only if its digital root is 9.

If N is NOT divisible by 9, then its digital root equals its remainder after division by 9.



**Pick 3 cards from a deck of cards.
Put them together to get a 3-digit number.
Is your number divisible by 9?
If not, what is the remainder?**

**MAGIC
RULE:**

**A positive integer N is divisible by 9
if and only if its digital root is 9.**

*Interesting
puzzles!*



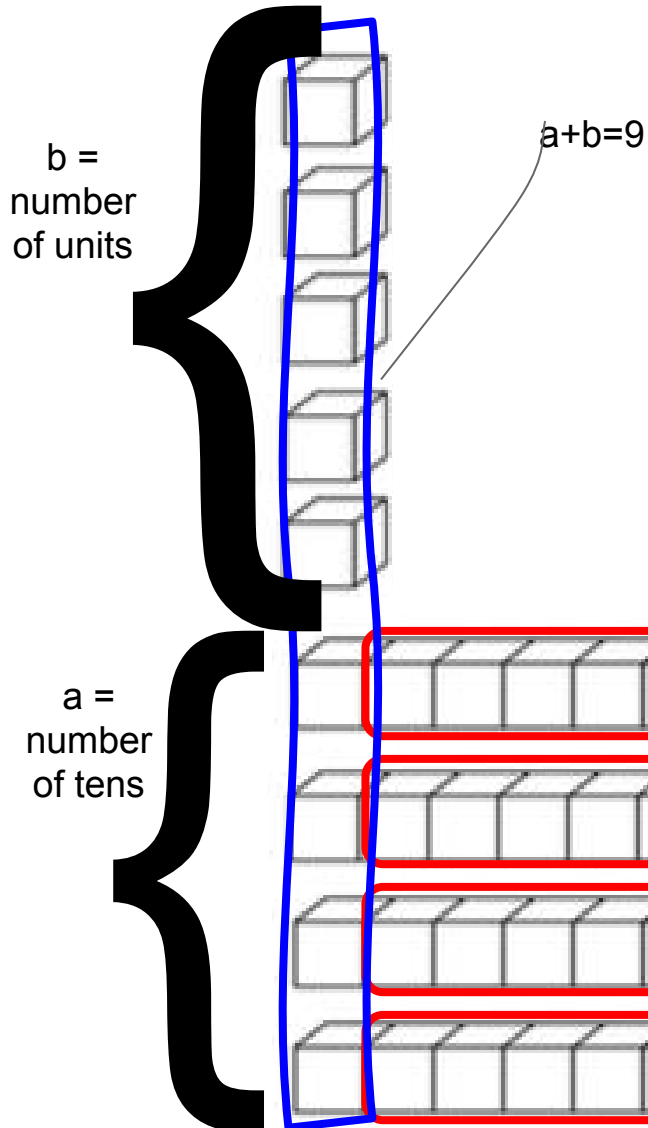
Find the missing digit
if the number
36_3452
is divisible by 9

Find the missing digit
if the number
41_23_1
is divisible by 9
(there is more than 1 solution)

Find the missing digit
if the number
6782_12
has a remainder of 2
after division by 9

Each student should make a puzzle for the volunteer at the table, but should have a solution first!

3 Back to buns and sausages...



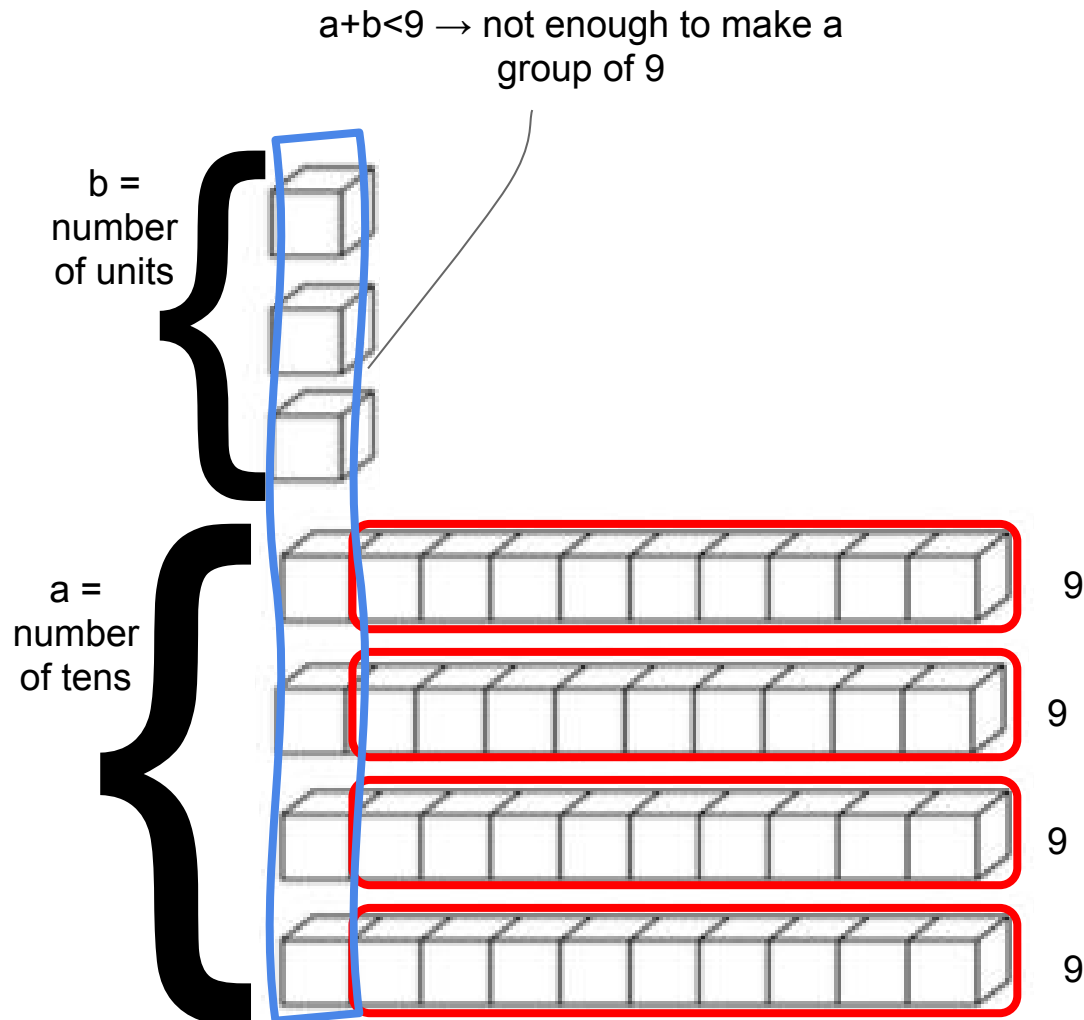
We discovered that
“an integer is divisible by 9 if and only if the digital sum is 9”
 and
“if an integer is NOT divisible by 9 then the digital sum is equal to the remainder”.

Why is this true? *Let's look at some pictures...*

case 1: $a+b=9$

If $a+b=9$, then the number ab is a multiple of 9.

(After you take out groups of 9, there is nothing left....)

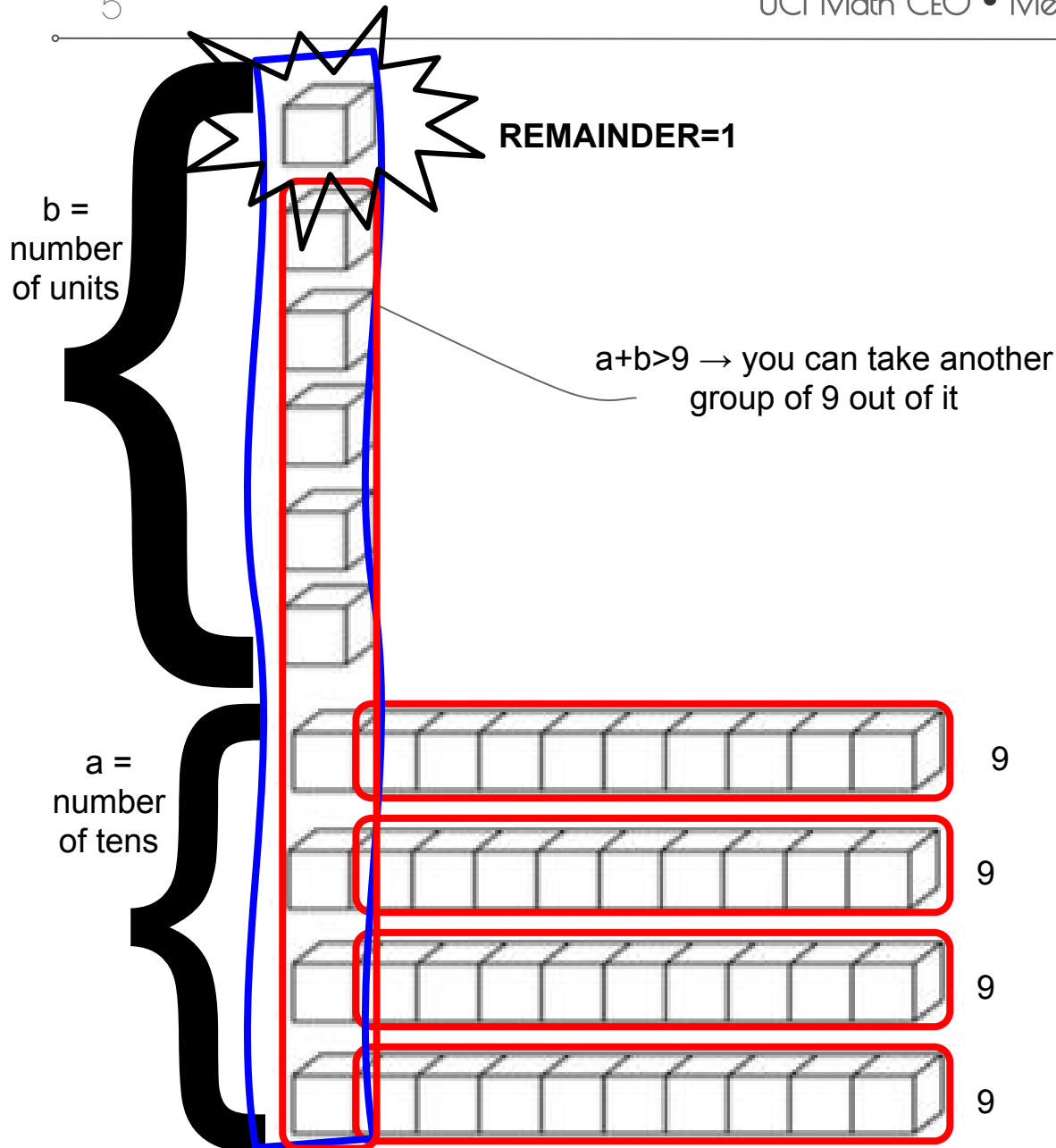
case 2: $a+b < 9$ 

If $a+b < 9$, then the number ab is not a multiple of 9.

(After you take out groups of 9, there is a remainder equal to $a+b$.)



5



case 3: $a+b > 9$

If $a+b > 9$, then the number ab is not a multiple of 9.

(After you take out a group of 9 from each of the tens, you are left with $a+b$ which contains one more group of 9 plus the remainder....)

Let's check that these rule applies when $N = 63$ and when $N = 85$.

A positive integer N is divisible by 9 if and only if its digital root is 9.

If N is NOT divisible by 9, then its digital root equals its remainder after division by 9.

$$N = 63 = 60 + 3 = 6 \cdot 10 + 3 =$$

$$= 6 \cdot (9 + 1) + 3$$

$$= 6 \cdot 9 + 6 \cdot 1 + 3 = 6 \cdot 9 + 9$$

digital root

Remainder: 0. So it's divisible by 9.

$$N = 85 = 80 + 5 = 8 \cdot 10 + 5 =$$

$$= 8 \cdot (9 + 1) + 5$$

$$= 8 \cdot 9 + 8 + 5 = 8 \cdot 9 + 13 = 8 \cdot 9 + 10 + 3 =$$

$$= 8 \cdot 9 + 1 \cdot (9 + 1) + 3 = 8 \cdot 9 + 1 \cdot 9 + 4$$

digital root

Remainder: 4. So it's not divisible by 9.

It also works for three digit numbers (and more): $231 = 200 + 30 + 1 = 2 \cdot 100 + 3 \cdot 10 + 1 =$
 $= 2 \cdot (99 + 1) + 3 \cdot (9 + 1) + 1 = 2 \cdot 99 + 2 + 3 \cdot 9 + 3 + 1 = (\text{a multiple of } 9) + (2 + 3 + 1)$

digital root

4 Feels like magic

We want to find the digital root of $(641 + 23)$.

Let's take the digital root of the two numbers:

- $641 \rightarrow 6 + 4 + 1 = 11 \rightarrow 1 + 1 = 2$
- $23 \rightarrow 2 + 3 = 5$

and add those up

- $2 + 5 = 7$.

Is 7 = the digital root of the answer?

- $641 + 23 = 664 \rightarrow 6 + 6 + 4 = 16 \rightarrow 1 + 6 = 7$

It feels like magic!

Check yourself! Find the digital root of $(31 + 23)$.

- $31 \rightarrow \underline{\quad}$
- $23 \rightarrow \underline{\quad}$

and add those up

- $\underline{\quad} + \underline{\quad} = \underline{\quad}$.

Also:

- $31 + 23 = \underline{\quad} \rightarrow \underline{\quad}$

Cool, eh?

The digital root of a sum of 2 numbers
= the (digital root of the) sum of the 2 digital roots

*Brainstorm with your group.
Why does this trick work?*

Does it work for the product as well?

Say, we want to find the digital root of $(41 * 11)$.

Let's take the digital root of the two numbers:

- $41 \rightarrow 4 + 1 = 5$
- $11 \rightarrow 1 + 1 = 2$

and add those up

- $5 * 2 = 10 \rightarrow 1 + 0 = 1$

Is 5 = the digital root of the answer?

- $41 * 11 = 451 \rightarrow 4 + 5 + 1 = 10 \rightarrow 1 + 0 = 1$

It feels like magic!

*Check yourself! Find the digital root of $(12 * 23)$.*

- $12 \rightarrow \underline{\quad}$
- $23 \rightarrow \underline{\quad}$

multiply those up

- $\underline{\quad} * \underline{\quad} = \underline{\quad}$.

Also:

- $12 * 23 = 276 \rightarrow \underline{\quad}$

Cool, eh?

The digital root of a product of 2 numbers = the (digital root of the) product of the 2 digital roots

***Brainstorm with your group.
Why does this trick work?***

We can use digital roots to check that our computations are (*most likely*) correct.

<u>First Number</u>	324	→	9
<u>Operation</u>	+		+
<u>Second Number</u>	226	→	1
<u>Solution</u>	550	→	1

$$9 + 1 = 10 \rightarrow 1 + 0 = 1 \quad \text{OK!!}$$

GROUP ACTIVITY: Write down a calculation using +, - or x and ask another friend at your table to check it.

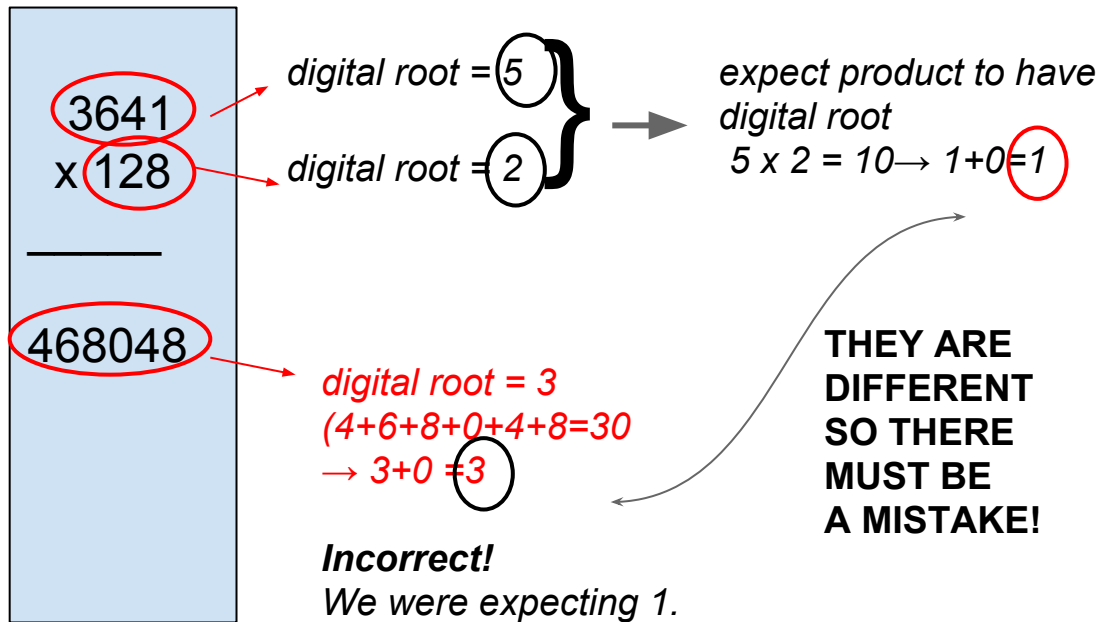
<u>First Number</u>	324	→	9
<u>Operation</u>	-		-
<u>Second Number</u>	226	→	1
<u>Solution</u>	98	→	8

$$9 - 1 = 8 \quad \text{OK!!}$$

<u>First Number</u>	324	→	9
<u>Operation</u>	*		*
<u>Second Number</u>	226	→	1
<u>Solution</u>	73,224	→	9

$$9 * 1 = 9 \quad \text{OK!!}$$

We can also use digital roots to find the error in our computations:



3641
 $\times 128$

 468048

digital root = 5
 digital root = 2

expect product to have digital root
 $5 \times 2 = 10 \rightarrow 1+0=1$

digital root = 3
 $(4+6+8+0+4+8=30 \rightarrow 3+0=3)$

Incorrect!
 We were expecting 1.

THEY ARE DIFFERENT SO THERE MUST BE A MISTAKE!

Some of the following computations are wrong. Use digital roots to spot the mistakes...

- ★ $12 \times 8 = 96$
- ★ $345 + 789 = 1234$
- ★ $324 + 227 = 540$
- ★ $324 - 226 = 98$
- ★ $325 \times 226 = 73550$

5 THE MATHEMAGICIAN



The mathemagician insisted on doing his next trick. He pointed to a child in the audience.

“Pick a number”, the said.

- “Double it.
- Add 7.
- Multiply by 5,
- Subtract the number you started with.
- Remove any non-zero digit from the answer.
- Now tell me the remaining digits in any order.”

The child said: “6 and 8”.

“Then the digit you removed is a 3,” announced the mathemagician.

He was correct. How did he know?

From “Problem solving through recreational mathematics.”

THE MATHEMAGICIAN



“Pick a number” ---> N

- “Double it. ---> $2N$
- Add 7. ---> $2N + 7$
- Multiply by 5. ---> $10N + 35$
- Subtract the number you started with. ---> $(10N + 35) - N = 9N + 35$
digital root: $3 + 5 = 8$
- Remove any non-zero digit from the answer.
(the digital root does not change!)
- Now tell me the remaining digits in any order.”

The child said: “6 and 8”. ---> $6 + 8 + \text{missing digit} = 8 (+ \text{multiple of } 9)$
---> $6 + \text{missing digit} = \text{a multiple of } 9$
--> missing digit = 3 :)

In general, the two digits they give you, plus the missing digit, must be equal to $8 + \text{a multiple of } 9$.

6 Mathematics on a wheel

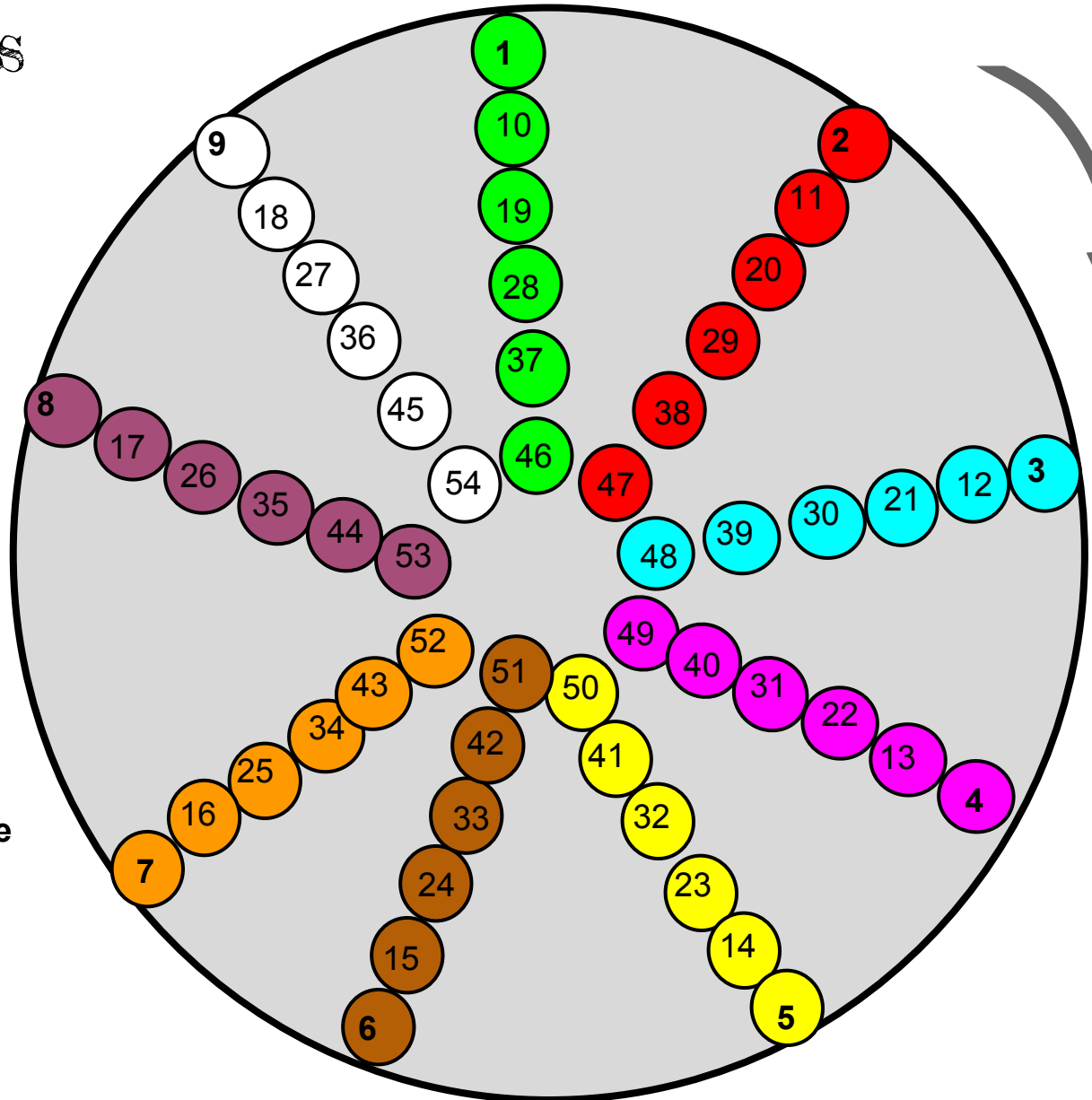
Color the numbers on the wheel:

- green, if the digital sum=1
- red, if the digital sum=2
- blue, if the digital sum=3
- pink, if the digital sum=4
- yellow, if the digital sum=5
- brown, if the digital sum=6
- orange, if the digital sum=7
- purple, if the digital sum=8
- white, if the digital sum=9

What is Blue + 4?

Start from a blue value and move +4. Where do you get? The answer should be a color.

Talk with your group. Does the answer depend on what blue color you chose at the beginning?



Compute the following operations. Remember, your answers should be colors.

- Pink + 5 = _____
- Red - 1 = _____
- Orange + 6 = _____
- Yellow * 2 = _____
- Blue * 3 = _____

Talk with your group. Why is the answer independent of what value you pick within the original color?

