Meeting 7 Student’s Booklet

How Should We Vote?

November 9, 2016 @ UCI

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How Best to Make a Group Decision?

Four friends Andy, Bianca, Cesia and Daniel are trying to decide where to go for a day of fun for the upcoming Veteran’s Day holiday.

Andy loves the beach but strongly dislikes the mountains. He would enjoy the museum but not as much as the beach.

Bianca loves the mountain but strongly dislikes the beach. She would enjoy the museum but not as much as the mountain.

Cesia loves the mountain but can’t stand the beach. She also would enjoy the museum although not as much as the mountain.

Daniel loves the museum. He just can’t stand the mountain. Also he likes the beach but not as much as the museum.

If everyone just vote for their favorites, then a day at the mountain is the clear winner? But is this the best decision? Andy and Daniel both really dislikes the mountain. Is there a better choice? Is having the most first place votes really the best choice? Is there a better way to vote? We will consider this question today.
1 Do You Approve?

Let’s us return to our four friends, Andy, Bianca, Cesia and Daniel, who are trying to decide where to go for the upcoming Veteran’s Day holiday. Their preferences is given in the below table. Recall that the third and last preference in the table is something they strongly disliked.

<table>
<thead>
<tr>
<th></th>
<th>Andy</th>
<th>Bianca</th>
<th>Cesia</th>
<th>Daniel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Beach</td>
<td>Mountain</td>
<td>Mountain</td>
<td>Museum</td>
</tr>
<tr>
<td>2nd</td>
<td>Museum</td>
<td>Museum</td>
<td>Museum</td>
<td>Beach</td>
</tr>
<tr>
<td>3rd</td>
<td>Mountain</td>
<td>Beach</td>
<td>Beach</td>
<td>Mountain</td>
</tr>
</tbody>
</table>

Given that each of our four friends has a very strongly feeling against one of the three options, let us consider another voting method called Approval Voting. In this voting system, each friend can vote positively for as many options as they wish. The option with the most positive votes wins.

Based on the description given earlier, determine how the 4 friends will vote. Write Yes or No in each box of the table...

Determine how many yes votes do Beach, Mountain, Museum each received?

Which option received the most “yes” votes and hence is the winner?
Let us also apply the points system voting to our four friends’ preferences. Using the points system:

1\textsuperscript{st} Preference = 3 points
2\textsuperscript{nd} Preference = 2 points
3\textsuperscript{rd} Preference = 1 point

Determine the total points total for each of the three options: Beach, Mountain, Museum?

Which option receives the most points and is the winner?

Every time that the beach is in 1\textsuperscript{st} place, it gets 3 points.
Every time it’s in 2\textsuperscript{nd} place, it gets 2 points.
Every time it’s in 3\textsuperscript{rd} place, it gets 1 point.

How many total points does the beach get?

Do the same, for the other two vacation spots to determine the winner.

Total number of points for beach: 3+2+1+ 1 = 7
2 The Best Sports Team

Angels, Ducks, Lakers and Rams are in the running for the best professional sports team in Southern California. A poll is taken of professional sports coaches to rank these four teams. In tabulating the ballot, we will use a points-based voting system, where each first place vote will get 4 points, each second place vote will get 3 points, each third place vote will get 2 points, and the last place vote will get 1 point. The team with the highest points total wins.

250 coaches casted their ballots.

Number of Voters

<table>
<thead>
<tr>
<th>Place</th>
<th>75</th>
<th>65</th>
<th>60</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Angels</td>
<td>Rams</td>
<td>Lakers</td>
<td>Lakers</td>
</tr>
<tr>
<td>2nd</td>
<td>Ducks</td>
<td>Ducks</td>
<td>Ducks</td>
<td>Ducks</td>
</tr>
<tr>
<td>3rd</td>
<td>Lakers</td>
<td>Lakers</td>
<td>Angels</td>
<td>Rams</td>
</tr>
<tr>
<td>4th</td>
<td>Rams</td>
<td>Angels</td>
<td>Rams</td>
<td>Angels</td>
</tr>
</tbody>
</table>

Which team received the most first place votes? Most last place votes?

Which team received the least number of first place votes?

Without doing any calculation, can you guess who won the poll, with the most number of total points?
Using the point system, where each

1\text{st} \text{ Place Vote} = 4 \text{ points} \\
2\text{nd} \text{ Place Vote} = 3 \text{ points} \\
3\text{rd} \text{ Place Vote} = 2 \text{ points} \\
4\text{th} \text{ Place Vote} = 1 \text{ point}

Which team won?

\textbf{HINT:} For the Angels, the total number of votes received can be found by adding the following four numbers:

(Number 1\text{st} \text{ Place Votes Angels Received}) \times 4 = \\
(Number 2\text{nd} \text{ Place Vote Angels Received}) \times 3 = \\
(Number 3\text{rd} \text{ Place Vote Angels Received}) \times 2 = \\
(Number 4\text{th} \text{ Place Vote Angels Received}) \times 1 = \\

Do the same, for the other three teams to determine the winner.

How can we characterize the winner of this points based voting system? Is it in some sense the consensus candidate, that is, it’s neither most liked or most disliked?
### Number of Voters

<table>
<thead>
<tr>
<th>Place</th>
<th>75</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Angels</td>
<td>Rams</td>
<td>Lakers</td>
<td>Lakers</td>
</tr>
<tr>
<td>2nd</td>
<td>Ducks</td>
<td>Ducks</td>
<td>Ducks</td>
<td>Ducks</td>
</tr>
<tr>
<td>3rd</td>
<td>Lakers</td>
<td>Lakers</td>
<td>Angels</td>
<td>Rams</td>
</tr>
<tr>
<td>4th</td>
<td>Rams</td>
<td>Angels</td>
<td>Rams</td>
<td>Angels</td>
</tr>
</tbody>
</table>

What if we change the point system, where instead each

- 1st Place Vote = 10 points
- 2nd Place Vote = 6 points
- 3rd Place Vote = 3 points
- 4th Place Vote = 1 point

Now which team received the most points?

**Hint:** The required calculation is similar. But now we must add the following four numbers:

- (Number 1st Place Votes Received) \( \times \) 10 =
- (Number 2nd Place Vote Received) \( \times \) 6 =
- (Number 3rd Place Vote Received) \( \times \) 3 =
- (Number 4th Place Vote Received) \( \times \) 1 =

Such points-based voting system is commonly used in sports for instance to determine the Most Valuable Player in Major League Baseball, the Heisman Trophy Winner in College Football, and the Ranking of NCAA College Teams.
Halloween just passed by recently and everyone has a lot of leftover chocolates. We would like to give them out, but not everyone likes the same type of chocolates. Some people prefer one type of chocolate over another, but everyone also has their 2nd and 3rd favorites too.

David, Anthony, Crystal, and Maria have listed their preferences for three chocolate bars from most favorite to least favorite. If we asked all of them to vote for their favorite, they would pick their #1 choice. If that choice was not available, they would pick their next favorite.

<table>
<thead>
<tr>
<th></th>
<th>David</th>
<th>Anthony</th>
<th>Crystal</th>
<th>Maria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>Twix</td>
<td>Snickers</td>
<td>Snickers</td>
<td>Hershey</td>
</tr>
<tr>
<td>2nd</td>
<td>Snickers</td>
<td>Twix</td>
<td>Twix</td>
<td>Twix</td>
</tr>
<tr>
<td>3rd</td>
<td>Hershey</td>
<td>Hershey</td>
<td>Hershey</td>
<td>Snickers</td>
</tr>
</tbody>
</table>

Which chocolate wins if everyone vote for their favorite?

What if Hershey chocolate is not available? Is there a clear winner, or does it become a tie?

This kind of voting is called a preference ballot, and it tries to select the winner by taking into account each voter’s preferences.

Trying to know each person’s preferences may be too complicated a task if we have many, many voters. Suppose we have a class of 50 students voting, do you think such preference voting can still be feasible?
Chocolate Preferences

If we look at the preferences of a class of 50 students, then we know a lot of people will have the similar preferences for chocolate bars. Let us list the number of people who have the same preferences instead of just their names.

<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twix</td>
<td>Snickers</td>
<td>Hershey</td>
</tr>
<tr>
<td>Hershey</td>
<td>Twix</td>
<td>Snickers</td>
</tr>
<tr>
<td>Snickers</td>
<td>Hershey</td>
<td>Twix</td>
</tr>
</tbody>
</table>

Let us just assume everyone has one vote and therefore only vote for their first preference.

Which chocolate wins?

How many first place votes are needed to reach a majority (more than half)? Did the winner get a majority?

What percentage of students disliked the winning choice?

What fraction (percentage if 7th or 8th graders) of students chose Twix as their second favorite?

You may have noticed that there was no majority winner, but one chocolate did get more votes than the other choices. This is called a plurality winner.
Chocolate Preferences continued…

Another way people can vote with their preferences clearly ordered is by means of single-elimination. This voting system may require going through multiple rounds and after each round getting rid of the lowest-scoring option.

After a round where an option is eliminated, the rankings of the remaining options will likely shift. For the voting preference on the right, Twix is eliminated in the first round since it had the lowest amount of votes. Everything that was below Twix in the “14 students” column then moves up in the ranking.

Which chocolate becomes the winner in Round 2?
Challenge Problem: Breakfast, Lunch or Dinner?

Everyone in class wants to vote for their favorite meal of the day, but we know beforehand that there is no winner in this election (majority nor plurality). We have 30 students who voted for their favorite meal. Can you distribute those students’ votes so that there is no winner at all when we do single elimination?

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lunch</td>
<td>Breakfast</td>
<td>Dinner</td>
<td>Dinner</td>
</tr>
<tr>
<td></td>
<td>Breakfast</td>
<td>Dinner</td>
<td>Dessert</td>
<td>Lunch</td>
</tr>
<tr>
<td></td>
<td>Dinner</td>
<td>Lunch</td>
<td>Breakfast</td>
<td>Dessert</td>
</tr>
<tr>
<td></td>
<td>Dessert</td>
<td>Dessert</td>
<td>Lunch</td>
<td>Breakfast</td>
</tr>
</tbody>
</table>
3:30 PM Game Time!

It’s time to vote in our own Math CEO student president! This is for the whole room. A volunteer moderator selects 3 students who want to be president. They come up to the front and give a quick speech (max of 30 seconds) about why they should be president.

Moderator proceeds to draw a preference ballot on the board.

<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
</table>

To make it easier, only have three combinations of the way the votes can be spread. Ideally, have each of the 3 candidates maintain at least one 1st place spot. Ask students to participate in making the preference ballot and voting. To vote, moderator will ask students to raise hands and tally the total for each column.

Moderator will go over all 4 voting systems discussed in booklet: Most Popular (counting only 1st choice votes), Single-Elimination (lowest vote getter in each round gets eliminated), Approval (Yes or No for each option), Points-Based Voting. Moderator will use each method to decide the winner of the election.