Pythagoras Trips

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1 SQUARES AND ROOTS

When we multiply an integer by itself, we obtain a **perfect square**. Compute the first perfect squares $1 \times 1 = 1$, $2 \times 2 = 4$, $3 \times 3 = 9$ ... These are the **areas** of the squares with sides $1$, $2$, $3$, $4$, ...

What is the pattern? How can you find the area of the “next” square from the previous one?

Use the transparencies to answer this question.
How does the *area* and *side* of a square change if you “multiply” or “divide” the square by 4?

- **Area:** 2
- **Side:** 2
- **Area:** 36
- **Side:** 6
- **Area:** 1
- **Side:** 1
The areas of 7 squares are given below. Find their sides.

You may use the “x4” (times four) relations between the areas of some of the squares. Note: the squares are not drawn to scale in this page.

<table>
<thead>
<tr>
<th>Area</th>
<th>1/4</th>
<th>9</th>
<th>36</th>
<th>49</th>
<th>121</th>
<th>196</th>
<th>484</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
49 \times 4 &= 196 \\
196 \times 4 &= 784 \\
484 \times 4 &= 1936 \\
121 \times 4 &= 484 \\
36 \times 4 &= 144 \\
9 \times 4 &= 36
\end{align*}
\]
B. The large square is made out of 4 equal squares. What is the side of the large square?

C. What is the side of each of the four small squares? What is the side of the large square?

You can use the transparencies to answer these questions.

Can you share the method you used? Is there another way?
Discuss: This is the graph of the function “the square of a number”
How can I use it to find squares? What is, for example, the square of 4? And the square of 4.5? How can I use it to find square roots? What is the square root of 25? And the square root 20?
Find the side of each square, using the parabola (previous page). Note: the squares are not proportional to one another in this page.

Can you estimate your answer before using the parabola?
What is the square of 5/2 (or 2.5)?

Your estimation:

By finding the area of a square (see picture)

By looking at the parabola

By squaring the numerator and the denominator
What is the square of $7/2$ (or 3.5)?

Your estimation:

- By finding the area of a square
- By looking at the parabola
- By squaring the numerator and the denominator
A frog named **Projectus**, a hockey puck named **Puckie** and a robot named **Ortho** compete to move from one location to another one in the chess-style floor of a room. Whoever does it using the shortest path in the floor (or *projected* in the floor, in case of the frog) wins. The rules for each competitor are described in the right.

In the next two challenges, find the winner and **calculate with a ruler** the distance that each competitor traveled.

**Ortho** can only move to the right/left/up/down, following the lines in the chess boards. He can **go through obstacles** (destroying them with a laser!)

**Projectus** can jump, but **only once**. He can jump over any obstacle in the room and as far as he wants, and its path for the competition is the projection of its trajectory onto the floor.

**Puckie** can **freely move** in any direction. However, he **must be always on the floor**. Puckie cannot get through obstacles.

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**Can you summarize the different options for the competitors?**

**If your finger is the frog, can you mimic the projection of a leap?**
A) FIND THE SHORTEST PATH:
B) FIND THE SHORTEST PATH:

![Diagram showing a grid with two points and a red line connecting them.]

Winner

[Box for selecting the correct answer]
Each vector (arrow) has a length. Can you guess which? Match arrows with lengths.
3 GAME OF VECTORS

Make 2 teams. Goal: Pass through all the cities in a board by traveling the least possible distance. Each team starts with two vector cards, randomly chosen. Place one on top of the other, team chooses. Put all other 4 cards on the table, face up. Each team selects its starting city, placing its token there.

Each turn:

1. Play your top card:
   - Move your token according to the vector card
   - Write the distance traveled
   - Discard the card
   - Draw a new card from the ones on the table

2. Draw a new card and place it below your other card.

Once a team completes the cities, that team stops playing. The game ends when both teams complete all cities, or after 20 rounds (in which case the team which did not complete the mission loses). Whoever traveled the least distance wins.
The Pythagorean Theorem tells the relation between the areas of 3 squares built on the sides of a right triangle.

The picture shows such three squares, with areas 4, 9 and 13.

Can you guess what is the relation between these areas?

Let us call Alan and Bob the squares built on the legs of the right triangle, and let’s Clara be the square build on the hypotenuse. Then the area of Clara is the sum of areas of Alan and Bob!

\[ A + B = C \]
A Alan and Bob are drawn. Draw the third square (Clara) and find its area.

B Draw an “Allan, Bob & Carla” configuration where Clara has an area of 20.

Are the answers that you find reasonable? Do they “agree” with the picture?
If Carla has area 52 and Allan has area 36, what is the side of Bob?

If Allan, Bob and Clara are squares with sides of length \(a\), \(b\) and \(c\) respectively, what is the relation among \(a\), \(b\) and \(c\)?
D. Find the side of Carla (the largest square), using the Pythagorean theorem.

E. In this configuration, Alan and Bob are equal. The enclosed triangle is called “right isosceles.”

Who has more area: Clara, or Alan and Bob combined?

Is the side of Clara more that 150% of the side of Alan? Why?
THE PYTHAGOREANT ANT

Now we apply our knowledge to a situation with distances:

An ant moves on a wall, 6 mm in a horizontal direction, then 8 mm in a vertical direction. Finally, it returns to its original position along a straight line.

How long is that straight line? In other words, what distance does the ant travel from Q to P?

*Hint: draw Alan, Bob and Carla!*
THE PYTHAGOREAN PROCEDURE

Suppose that you want to move from P to Q in a straight line drawn on a plane, and need to measure this distance, which we will call d.

1. Move horizontally starting from P, until you are right below Q. Let x be the distance that you moved.
2. Move vertically until you reach Q. Let y be the number of units that you moved.
3. Find “x squared” $x^2$, and also square $y^2$. Now add them. Call this value $d^2$.
4. Now take the square root of $d^2$ to find d.

Find the distance d in the diagram:

$\sqrt{d^2}$

$\text{Horizontal distance: } x = \ \square$

$\text{Vertical distance: } y = \ \square$

$d^2 = x^2 + y^2$

$\quad d^2 = \ \square$

$d = \ \square$

Can you explain this procedure?
PROVING THE PYTHAGOREAN THEOREM

Your group will be given a sequence of pictures A through G. Put them in order and make sense of the situation. Keep track of the area in each picture.