Meeting 4  Winter 2018

Exploring Sets

January 31st

Contents

1) Heavy Sets
2) Break and Keep

www.math.uci.edu/mathceo
Meeting 4
Review Session

Tue., Jan 30th
1:30 - 2:30 PM
Rowland Hall 340P

If this time does not work, please email Lucy Dolmadjian: ldolmadj@uci.edu
About MATH CEO

UC Irvine Math CEO program is committed to offering free enrichment activities in mathematics for middle school students in underserved communities, and creating free educational material to be used in classrooms around the world. An essential goal of our program is to encourage students in low income, minority communities in Southern California to pursue a college degree in Mathematics, Science or related fields through a solid foundation and adequate planning.

Our Goals

**Inspire** and support the interest in mathematics of middle school students from nearby communities who might not otherwise have access to math enrichment activities.

**Impart** the mathematical foundations and critical thinking skills required for students to successfully pursue a college education and a career in STEM areas.

**Instill** the desire in these young achievers and their families for a college education and share college access information and financial aid opportunities.
Exploring sets
Meeting 4, Jan 31 2018

☐ Identify the Leader mentor
☐ Write names of any new mentors and students
    (find form inside folder, write new names if needed)
☐ Place check-marks in the Meeting 4 column (same form inside folder)
☐ Tell math goals to students in each activity
☐ Call students by name
☐ Keep students silence while doing the Bonus Round (Quiz)
☐ Keep your table neat and clean at all times
☐ Get help if there are behavior problems before they escalate

☐ Bonus Round: Group Quiz (pink) (end of Activity 1): 10 min
    + 5 min correction
☐ Student Survey (pink) (start survey at 3:35 PM)
☐ Fill Meeting Report (blue) (if you are the Leader)
☐ Put back into folder: Student Surveys (pink), Meeting Report (blue)
Dear Mentor

In this meeting we explore simple sets and allow students to get familiar with set writing notation, as well as how to combine sets, discovering the operations of union and intersection. We also go deeper with the “Break and Keep” activity from last meeting.

To see explanatory videos, please visit this link.

Math Goals

- Kids can write a set in bracket notation and describe it using words.

- Kids can find the union and the intersection of two sets and write it in bracket notation.

- Given a set of elements that have values associated to them, kids can find the sum of those values.

- Kids understand that when sets overlap one needs to remove the overlap, when finding numerical values.

- Kids can visually construct different fractions using bars, squares or circles.

- Kids understand the notion of a “sequence of instructions” and can produce a numerical result by following one.
### MATERIALS

<table>
<thead>
<tr>
<th>Item</th>
<th>Color</th>
<th>Quantity</th>
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<tbody>
<tr>
<td>INSTRUCTOR MANUAL</td>
<td>Green color</td>
<td>One per person</td>
</tr>
<tr>
<td>STUDENT WORKBOOK</td>
<td>White color</td>
<td>One per student</td>
</tr>
<tr>
<td>MEETING REPORT</td>
<td>Blue color</td>
<td>One per table</td>
</tr>
<tr>
<td>STUDENT SURVEYS (INCLUDES QUIZZES)</td>
<td>Pink Color</td>
<td>One per student</td>
</tr>
<tr>
<td>WHITEBOARDS</td>
<td></td>
<td>One per student</td>
</tr>
<tr>
<td>DRY ERASE MARKERS</td>
<td></td>
<td>A pouch with several</td>
</tr>
</tbody>
</table>

### AGENDA

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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<tbody>
<tr>
<td>2:10 pm</td>
<td>Introduction</td>
</tr>
<tr>
<td>2:15 pm</td>
<td>Career Invitation</td>
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<tr>
<td></td>
<td>Economist</td>
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<tr>
<td>2:25 pm</td>
<td>1) Heavy Sets</td>
</tr>
<tr>
<td></td>
<td>Exploring basic properties of sets</td>
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<tr>
<td>2:55 pm</td>
<td>Bonus Round (Quiz)</td>
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<tr>
<td>3:10 pm</td>
<td>2) Break and Keep</td>
</tr>
<tr>
<td></td>
<td>A game of proportionality and areas</td>
</tr>
<tr>
<td>3:35 pm</td>
<td>Student Survey</td>
</tr>
<tr>
<td>3:40 pm</td>
<td>End of the meeting</td>
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</tbody>
</table>
MATERIALS

INSTRUCTOR MANUAL
Green color
(No student Booklet)

MEETING REPORT
Blue color
One per table
Online meeting report

STUDENT SURVEYS
(PERQUIZES)
Pink Color
One per student

WHITEBOARDS
One per student

DRY ERASE MARKERS
A pouch with several

AGENDA

2:10 pm Introduction

2:15 pm 1) Heavy Sets
          Exploring basic properties of sets

2:45 pm Career Invitation

2:55 pm Bonus Round (Quiz)

3:10 pm 2) Break and Keep
          Making sense of fractions visually

3:35 pm Student Survey

3:40 pm End of the meeting
INDIVIDUAL ASSESSMENT

- Right after Activity 1 (Heavy Sets), there is time for an individual Quiz called **Bonus Round** (to be done as a group in this meeting). Give students **10 minutes to do both parts A and B** and have them answer (in their surveys).
  - Give first 5 minutes for individual work, and then let them cooperate for the rest of the time (5 minutes). Students may work in pairs, groups of 3 or all together. Students can change their answers during this time.
  - After collecting answers, grade them and quickly correct them with the kids (5 minutes).

TIPS

- Build the habit of having students work during the first 5 minutes in complete silence and without any help of peers of mentors.

BEHAVIOR EXPECTATIONS

If a kid is behaving improperly or disrupting students, or does not follow directions at all, talk to them. If problem persists or is really serious, please let Brandi, Alessandra, Li-Sheng or an Assistant know immediately.
**UCI MATH CEO MEETINGS: BASIC GUIDELINES FOR VOLUNTEERS**

**1. KNOW YOUR STUDENTS**
Call students by their names most of the time: make sure they know your name, talk briefly about their day before you start the math activities.

**2. ASK FOR EXPLANATIONS**
Ask students how they got their answers. Say things like “How do you know?” “Why?” “Draw a picture” “Convince me!” “Can you explain to Juan?” etc.

**3. MOVE & MONITOR**
Move around your table; monitor all students; use an adequate tone of voice; encourage kids to work in teams.

**4. CHECK WORK**
Verify that the students write the answers to the problems and that they are correct and complete.

**5. AT THE END**
Ask students to fill out the survey individually (no help), and to help pick up trash from the table and floor.

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**TEACHING TIPS**
This icon refers to specific tips which you will find embedded in the booklet activities: procedures, questions to ask to the students, recommended methodologies, and so on.

**Example:** After you introduce a new concept, it is a good idea to ask students to rephrase the concept, explain it in their own words. You can choose particular students, for example those who are disengaged.

**Example:** It is convenient to ask one student to read out instructions for a problem or definitions of a concept. This keeps your group focused on the task and improves their reading skills if you give feedback on reading.

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This icon means that the students should work individually in the corresponding problem, before discussing. Be flexible and adapt to your situation.

Activity to be done in pairs

Note: if not specified, the booklet problem can be done as a group activity involving a discussion.
What is a set? Does the order of the elements matter in a set?

How do we write sets using brackets? Can you give some examples? How do you say them in words?

Consider the sets { 1, 2 } and { 2, 3 }. Can you think of a set that is larger than both? Can you think of another one?
1) HEAVY SETS

Materials

- Student’s Workbook (pages 2 to 6)

In this exploratory activity students will learn to write sets using bracket notation, understand some basic properties of sets and manipulate them. We will also explore the relation between a set of numbers and the sum of its elements, developing ideas of conservation of quantities that will be useful for topics in probability, as we promote number and combinatorial sense.

Finally, students will discover, through problem solving and conjectures, the operations of union and intersection of sets.

Advice to mentors

Start with the 3 introductory questions from page 10 to know your students previous knowledge about the topic. Then do a mini-lecture on sets (see 1.1). After, you can do 1.2 (matching) and 1.3 (heavy sets). You may do 1.2 as individual work and then compare. For 1.3 students may work in pairs or you can do it also as a group discussion.

Depending on the pace of your students and how much you want to spend in the first activities, you may only do up to 1.3; if you have time, 1.4 (and even 1.5), which are a bit more challenging.

Can someone explain what a set is and give examples?

Before we start, i would like you to ask some questions to see how much we already understand.

Activities 1.4 and 1.5 from pages 17 to 21 are optional challenges. Only do them if there is still time in your table.
1. Sets
A set is a collection of elements. In a set, the order of the elements does not matter.

Example: If \( a = \) “apple” and \( b = \) “boat”, then we can create the set \( \{ a, b \} \), which is the same as \( \{ b, a \} \). This set has two elements: the apple \( a \) and the boat \( b \). The order in which we list them does not matter.

\[
\{\text{apple}, \text{boat}\}
\]

The set \( \{ p, q, r \} \) is read: “the set whose elements are \( p, q \) and \( r \)”. Here \( p, q \) and \( r \) are three objects, which can be whatever we want!

2. Matching activity
Match each description on the left with the set on the right. Each description corresponds with a unique set.

A
The set has 4 elements

\[
\{\text{apple}, \text{boat}, \text{football}\}
\]

B
The set does not have any spherical elements

\[
\{\text{boat}, \text{football}, \pi, \text{moon}\}
\]

C
The set has only spherical elements

\[
\{\text{ball}, \pi, \text{shovel}, \text{bug}\}
\]

D
The set has a bug

\[
\{\text{hammer}, \text{pyramid}\}
\]
1 HEAVY SETS

1.1 Sets
A set is a collection of elements. In a set, the order of the elements does not matter.

Example: If \( a = \text{“apple”} \) and \( b = \text{“boat”} \), then we can create the set \( \{ a, b \} \), which is the same as \( \{ b, a \} \). This set has two elements: the apple \( a \) and the boat \( b \). The order in which we list them does not matter.

\[
\{ a, b \}
\]

The set \( \{ p, q, r \} \) is read: “the set whose elements are \( p, q \) and \( r \)”. Here \( p, q \) and \( r \) are three objects, which can be whatever we want!

1.2 Matching activity
Match each description on the left with the set on the right. Each description corresponds with a unique set.

A
The set has 4 elements

B
The set does not have any spherical elements

C
The set has only spherical elements

D
The set has a bug
1.3 Sets and weights

Now suppose that the elements are given some weights in Kg, and that to find the weight of a set we need to add the weight of its elements. Remember that in sets elements cannot be repeated, so you only add the weight once!

Suppose that we have elements labeled by the letters a through z, and each one has a given weight in kg. We know some of the weights (for elements a to j), but we ignore others (elements k to z, which are not pictured):

- a = 12 kg
- b = 10 kg
- c = 2 kg
- d = 22 kg
- e = 14 kg
- f = 8 kg
- g = 24 kg
- h = 20 kg
- i = 7 kg
- j = 7 kg
1) HEAVY SETS

Find the weight of the set \( \{a, b\} \).

If we know that the weight of the set \( \{a, b, x, y\} \) is 35 kg, what is the weight of the set \( \{a, x, y\} \)?

Build a set that weighs **exactly 44 kg**. How many such sets can you construct? Make sure to write them using brackets.

- a = 12 kg  
- b = 10 kg  
- c = 2 kg  
- d = 22 kg  
- e = 14 kg

- f = 8 kg  
- g = 24 kg  
- h = 20 kg  
- i = 7 kg  
- j = 7 kg
Find the weight of the set \{a, b\}.

\(a\) weighs 12 kg.
\(b\) weighs 10 kg.
If we add 12 + 10 we obtain 22.

So \{a, b\} weighs 22 kg.

If we know that the weight of the set \{a, b, x, y\} is 35 kg, what is the weight of the set \{a, x, y\}?

There are several ways to solve this problem. One such way:
We can break \{a, b, x, y\} into two sets that do not share any element: \{a, b\} and \{x, y\}. Since \{a, b\} weighs 22 kg and \{a, b, x, y\} weighs 35 kg, we must have that \{x, y\} weighs 35 - 22 = 13 kg. Since \(a\) weighs 12 kg, then \{a, x, y\} weighs 25 kg.

Another way: because \{a, b, x, y\} weighs 35 kg and \(b\) weighs 10 kg, we conclude that \{a, x, y\} weighs 25 kg.

Build a set that weighs exactly 44 kg. How many can you construct? Make sure to write them using brackets.

There are several possible combinations. It is important to note that since repeating elements in a set does not count, the set \{d, d\} is not a solution, because the set \{d, d\} is really just \{d\}, which weighs 22 kg.

Also, the answer \{a, b, d\} is really the same as \{b, d, a\} or \{d, a, b\}, etc, since order of elements does not matter in sets.

Here are some combinations. You may find other ones:

- \{a, b, d\} (12 + 10 + 22 = 44)
- \{c, d, h\} (2 + 22 + 20 = 44)
- \{d, e, f\} (22 + 14 + 8 = 44)
- \{g, h\} (24 + 20 = 44)
- \{b, h, i, j\} (10 + 20 + 7 + 7 = 44)
- ...
Suppose that we are already given the weights of the following four sets (which you can check):

- \{ a, b, c, d, e \} weighs 60 kg.
- \{ a, c, e, f, g \} weighs 60 kg.
- \{ a, c, e \} weighs 28 kg.

What is the weight of the following set?
\{ a, b, c, d, e, f, g \}

Can you find this weight using only the information in the three bullets above? If so, how? Explain.

Did you find a general rule in this exercise that can help you find the weight of a set that is created by joining two smaller sets? Describe the rule.
Suppose that we are already given the weights of the following four sets (which you can check):

- \{ a, b, c, d, e \} weighs 60 kg.
- \{ a, c, e, f, g \} weighs 60 kg.
- \{ a, c, e \} weighs 28 kg.

What is the weight of the following set?
\{ a, b, c, d, e, f, g \}

Can you find this weight using only the information in the three bullets above? If so, how? Explain.

Did you find a general rule in this exercise that can help you *find the weight of a set that is created by joining two smaller sets*? Describe the rule.

One way to solve the problem is to find the weight of the set \{a,b,c,d,e,f,g\} by using the information in page 14, by adding the item’s weights: $12+10+2+22+14+8+24 = 92$.

However, if students decide to use this method (which is perfectly fine), you can encourage them to use only the information in the bullets, which we can model with either a set diagram (see top left diagram), or for example, the following table that represents the first two bullets:

<table>
<thead>
<tr>
<th></th>
<th>60</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>d</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>e</td>
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<tr>
<td></td>
<td>e</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>f</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g</td>
<td></td>
</tr>
</tbody>
</table>

The table shows that by adding $60+60$ we obtain the sum of the weights of all items in the middle column, but a, c and e are being repeated (counted twice) so we must subtract the combined weight. Bullet 4 gives this combined weight, which is 28. So the answer is $60+60-28 = 120 - 28 = 92$.

General Rule: *If we have Set 1 and Set 2 and we join them to create Set 3, then we find its weight as follows: Add the weights of Sets 1 and 2, and then remove the weight of the set of the common elements of Sets 1 and 2.*
Assume that we are given the following information:

- \{ k, l, m, q, r, s \} weighs 80 kg.
- \{ a, k, n, p, q, s \} weighs 70 kg.
- \{ k, q \} weighs 20 kg.
- \{ s, q \} weighs 24 kg.

Also, we are given that q weighs 6 kg.

What is the weight of the following set?
\{ a, k, l, m, n, p, q, r, s \}

Assume that we have the sets Set 1, Set 2, Set 3 and Set 4, such that:

- Set 1: \{ t, u, w, x \} weighs 46 kg.
- Set 2: \{ w, x, y, z \} weighs 36 kg.
- Set 3: \{ t, u \} weighs 30 kg.
- Set 4: \{ y, z \} weighs 20 kg.

Suppose that Set 5 is formed with the elements that belong to both Set 1 and Set 2. What is the weight of Set 5? Find more than one way to solve this problem if you can.
1) HEAVY SETS

Assume that we are given the following information:

- \( \{ k, l, m, q, r, s \} \) weighs 80 kg.
- \( \{ a, k, n, p, q, s \} \) weighs 70 kg.
- \( \{ k, q \} \) weighs 20 kg.
- \( \{ s, q \} \) weight 24 kg.

Also, we are given that \( q \) weighs 6 kg.

What is the weight of the following set?
\( \{ a, k, l, m, n, p, q, r, s \} \)

One solution: If we add 80+70 we obtain 150, which is the weight of the following objects (where those underlined were counted twice, as they are repeated in the count: \( a, k, l, m, n, p, q, r, s \)).

Thus, if we find the weight of \( k, q \) and \( s \) combined, we can subtract this to find the answer. So we need the weight of the set \( \{ k, q, s \} \). We already know that \( \{ k, q \} \) weighs 20 kg, and \( \{ s, q \} \) weighs 24, and so the objects \( k, q, s \) (with \( q \) counted twice) weighs 44. Since \( q \) weighs 6 kg, the set \( \{ k, q, s \} \) really weighs 44-6=38.

Therefore \( \{ a, k, l, m, n, p, q, r, s \} \) weighs 150 - 38 = 112 kg.
See a picture in the next page.

Assume that we have the sets Set 1, Set 2, Set 3 and Set 4, such that:

- Set 1: \( \{ t, u, w, x \} \) weighs 46 kg.
- Set 2: \( \{ w, x, y, z \} \) weighs 36 kg.
- Set 3: \( \{ t, u \} \) weighs 30 kg.
- Set 4: \( \{ y, z \} \) weighs 20 kg.

Suppose that Set 5 is formed with the elements that belong to both Set 1 and Set 2. What is the weight of Set 5? Find more than one way to solve this problem if you can.
Assume that we are given the following information:

- \{k, l, m, q, r, s\} weighs 80 kg.
- \{a, k, n, p, q, s\} weighs 70 kg.
- \{k, q\} weighs 20 kg.
- \{s, q\} weighs 24 kg.

Also, we are given that q weighs 6 kg.

What is the weight of the following set?

\{a, k, l, m, n, p, q, r, s\}

Assume that we have the sets Set 1, Set 2, Set 3 and Set 4, such that:

- Set 1: \{t, u, w, x\} weighs 46 kg.
- Set 2: \{w, x, y, z\} weighs 36 kg.
- Set 3: \{t, u\} weighs 30 kg.
- Set 4: \{y, z\} weighs 20 kg.

Suppose that Set 5 is formed with the elements that belong to both Set 1 and Set 2. What is the weight of Set 5? Find more than one way to solve this problem if you can.
1) Heavy Sets

Teaching Tips

- Make sure that students become familiar with expressing and writing sets using brackets. Also make sure that they do the following:
  - Use commas to separate the objects
  - Don’t repeat the elements of the set (insist in the concept that in sets repetition is not counted)
  - Know how to read a set in words.

- Students may translate statements about the weight of a set into equations of the form $a+b+... = N$, and this is fine, as one of the underlying goals of this activity is to promote algebraic thinking with equations that arise naturally from a problematic situation (in the context of sets). However, this is not required as a first approach and students may still reason with words or pictures.

- Make sure to use set diagrams and other visual aids (tables may also work) to convey the information of the different problems. This will help develop intuition.

- Use activity 1.2 to reiterate that an element may appear in several sets. This will be useful for later, when we consider intersections of sets.

- You can use activity 1.3a) as an opportunity to develop mental math and number sense in your students. In other words, you can stimulate students to do the additions mentally for some computations.

- A good problem solving strategy that you can teach students is to “slightly modify” one set that is solution to generate another one. This is an important skill. For example, if you have a solution including the 14 kg element (and not including any of the 7 kg elements), you may now replace the 14 kg element with both 7 kg elements and generate another solution, because $7+7=14$. This line of reasoning will transfer to algebra, in which one modifies only a part of an expression with an equivalent one, without altering the overall value.
If students get stuck working with many sets, or sets with many elements, it is helpful to revisit simpler examples and have the students explain those solutions. Here are examples of how you may lead these discussions:

- “What if I know a set which contains a panda and a raccoon weighs 90 kg, while a set containing only a panda weight 70 kg. How much does a set with only a raccoon weigh?”
- “How did you find the answer? Can you explain it using your own words?”
- “Let’s try a similar strategy for this problem. Do you remember what we did last time?”

You can also lead students to a solution when they are stuck by asking them conversationally to compare the sets:

- “What does this pair of sets have in common? What about this pair?”
- “What would need to be added or removed from this set to make it the same as the other one?”

For students who are already very comfortable working with set diagrams and pictures, you could also ask them to practice writing equations to represent the information for each problem, but still solve the problems with pictures. This is a nice connection to algebraic thinking (setting up equations based on word problems).

Ask students to draw the set diagrams, placing elements in the appropriate places. Use colors to mark the weights of each “piece” of the diagram, as in example 1.3(e), for extra visualization of the information.

Don’t forget to encourage students to discuss the problems and solutions! It is ok to encourage mental math, but be sure to ask the students questions like, “can you explain how you got that number?”
Dear mentors:

Dedicate 15 minutes for the Bonus Round Group Quiz, both parts A and B.

- Give the first 5 minutes for individual work,
- then give 5 minutes for group work (they solve the quiz together).

(Pink papers in your Mentor's folder).

Then grade, spend 5 minutes for formative assessment (giving feedback so that kids learn from mistakes right away).

Still collect the individual responses.

(A) A bucket had 2 liters of water. Peter removed ½ liters from it, and then Amy added 1/3 of a liter to the remaining water in the tank after that. How much water was in the tank after that? Draw a picture.

Justify your choices:
Write your work:
(A) A bucket had 2 liters of water. Peter removed \( \frac{1}{2} \) liters from it, and then Amy added \( \frac{1}{3} \) of a liter to it. How much water was in the bucket after that? Draw a picture.

Justify your choices. Write your work:

- 2 liters
- minus \( \frac{1}{2} \)
- plus \( \frac{1}{3} \)

We have 11/6 of liter, or 1+\% of liter.

(B) Let Set 1 be the set \{ 2, 4, 6, 8, 9, 0 \} and Set 2 be the set \{ 1, 2, 3, 4, 5, 8, 0, 10, 11 \}. Let Set 3 be the set of the elements that are in both Set 1 and Set 2. If we add all elements in Set 3, which number do we obtain?

Solution:
Common elements: 2, 4, 8, 0.
Sum: \( 2 + 8 + 0 = 14 \).

Answer: 14.
When we represent fractions with pictures, why do we care to break into equal shapes? Explain.

If I break a shape in 6 equal units and keep 2 of them, what fraction is being represented? Can you draw a picture?

If I break a square in 2 equal shapes, I break each of these in 2 equal new shapes, and I select 3 of these new shapes, what fraction I am representing?
2) BREAK AND KEEP

Material
Brown package (Unit Templates)

Introduction
This is a follow up to the activity done in Meeting 3. You may review examples 1 and 2 if needed.

In this activity, students use different sequences of instructions that generate different fractions. The types of instructions are restricted and must be given by students in a very precise fashion, as if they were writing a computer program in which a defined language needs to be followed. Also, the order in which the instructions are given will matter.
The goals of this activity are that students understand the visual representations of fractions and how they can relate to each other. Another goal is that kids understand at a very general level the notion of a sequence of instructions (similar to computer programs).

Start by explaining that in this activity we will start with a “unit” shape and start breaking them and keeping some of the pieces, doing this several times and in a certain order. We are simulating giving instructions to a robot, so we need to be very precise. also, the robot will execute the instructions in the order that we choose, so we want to pick the order right.

The two possible instructions are (1) Break and (2) Keep. Break is used to break the unit (or each piece we have) into several equal new pieces, while Keep selects certain number of pieces and discards the rest (think of trashing them for good). At every moment we should know how many pieces we have, and how big they are.

To begin, go over the following two examples so that students understand the general idea of the activity:
Example #1: Creating 1/4

Tell your students that you will show them how to create the fraction one fourth, 1/4, by giving instructions. You can ask all students to pretend that they are the robots and will follow your instructions.

First instruction: draw the unit, as a circle.
Students will draw a big circle in their whiteboards or paper. Stress that the circle is the unit.

Second instruction: “Break in 2”. Explain that you want them to break the unit into two equal pieces.
Students will break the circle in 2 equal semicircles.

Third instruction: “Keep 1”. Explain that of all pieces (which are 2), you want them to keep only 1, and eliminate the other one, by marking it with a red X.
Note that we now have only 1 piece (we eliminated the other one).

Fourth instruction: “Break in 2”. Explain that each piece (except those eliminated) should be broken into 2 equal new pieces.

Fifth instruction: “Keep 1”. That is, from the two pieces that we have, keep one and trash the other.

Sixth instruction: “END”. Tell students to color or shade all the pieces that were not eliminated.
Tell students that the fraction represented is 1/4 and ask them why.
Example #2:

Tell your students that you will give them some instructions like in the previous example, and they will need to tell you which fraction was created.

1st instruction: draw the unit, as a square.
Students will draw a big square in their whiteboards or paper. Stress that the square is the unit.

2nd instruction: “Break in 3”. Explain that you want them to break the unit into 3 equal pieces.

3rd instruction: “Keep 2”. Explain that of all pieces (which are 3), you want them to keep only 2, and eliminate the other one, by marking it with a red X. We now have only 2 pieces.

4th instruction: “Break in 2”. Explain that each piece (except those eliminated) should be broken into 2 equal new pieces. There are now 4 equal new pieces.

5th instruction: “Keep 1”. That is, from the 4 pieces that we have, keep one and trash the other 3.

6th instruction: “END”. Tell students to color or shade all the pieces that were not eliminated. Ask students which fraction they think was created.

The answer is: 1/6 (which can be seen by splitting the right column in two, to obtain 1 out of 6 equal pieces). Help students reason this out if they seem to be struggling.
Activity

Tell students that they will be cooperating to create several fractions that you will give them. Introduce the following rule:

Rule 1: You can only break in 2, in 3, 5 or 7 (the first 4 prime numbers).

Explain that, for example, to generate the fraction 1/4, the sequence “Start, Break in 4, Keep 1, End” is invalid, because we are only allowed to break in 2, 3 or 5. Ask one student how he thinks we could generate ¼ (Solution, for example: “Start, Break in 2, Keep 1, Break in 2, Keep 1”, or “Start, Break in 2, Break in 2, Keep 1”).

Fractions to give to your students: 5/6, 12/20, 19/20, 5/1, 1/50, 4/50, 15/14.

You may choose which, skipping some of them; depending on the time remaining, manage the time designated to each fraction).

Example:

To produce ⅝, one option is: “Start with a bar, break in 2, break in 3, keep 5, end”.

Example: To produce 12/20, one option is to simplify first (6/10, 3/5) and then do: “Start with a bar, break in 5, keep 3, end”. Or, without simplifying: “Start with a bar, break in 2, break in 2, break in 5, keep 12, end”

Challenge

If you have time and you feel that your students have the appropriate developmental level, do the following: Start with a fraction and put another fraction as a goal, and students need to give a sequence of instructions to go from the first fraction to the “goal fraction”.

- Example: Start with 2/4 (this is the new “unit”, which has 1 piece since it is a unit) and arrive to 3/2: “keep 3”. (since 2/4 times 3 is 3/2).
- Example: Start with 3/4 and arrive to ½: “Break in 3, keep 2”.
- Example: Start with 2/3 and arrive to 5/6: Break in 2, Keep 1, Break in 2, Keep 5.
- Example: Start with 2/5 and arrive to 2: Keep 5.
- Example: Start with 2/5 and arrive to 1: Break in 2, Keep 5.
- Example: Start with 2/5 and arrive to 2/3: Keep 5, Break in 6, Keep 2.
### Example:
To produce 5/1: “Start, Keep 5, End”

<table>
<thead>
<tr>
<th>Start</th>
<th>Break in 3</th>
<th>Break in 2</th>
<th>Keep 2</th>
<th>Keep 30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>![X]</td>
<td>![X]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>![X]</td>
<td>![X]</td>
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<tr>
<td></td>
<td>![X]</td>
<td>![X]</td>
<td>![X]</td>
<td>![X]</td>
</tr>
</tbody>
</table>

Note: to challenge students you may tell them, for example: “How can you get 5, by starting with: “Start, Break in 3, keep 2?””. (One answer: “continue with “break in 2, keep 30, end”.}
2) Break and Keep  
Teaching Tips

- An important goal of this activity is that students can explain fractions using verbs and phrases involving actions (break, keep, discard, etc), instead of just saying what the numerator or denominators are. Encourage students to describe the fractions using this relational language, in which they describe the picture and the different relations between all parts.

- Students may start discovering some regularities the more they get in touch with the sequence of instructions. For example, when we do two consecutive “Break” instructions, the order does not matter since it will break into the same number of pieces which is the total. This is a basic fact for fraction multiplication and which is important to understand. Encourage students to talk about these discoveries, and even write them down as rules for the group to use.

- It is perfectly fine in this game to keep more pieces than you have, and that is why we can actually create fractions which are greater than the unit (that is, improper fractions). It is a bit weird here using the language of keep since in real life you cannot really keep more than what you have, but you can tell students that math is kind of magical and whatever that we can do using our imagination and following rules, its perfectly OK. For example, to generate the fraction 3/2, you would do: “Start with a unit”, then “Break in 2”, then “Keep 3”, then “End”.

- It is very important to always start with the instruction that sets up the unit. This rule can help students keep in mind that the unit must always be identified, as fractions can be relative and depend on which unit you begin with.
Exploring Sets
January 31st

Contents

1) Heavy Sets
2) Break and Keep

www.math.uci.edu/mathceo
What is a set? Does the order of the elements matter in a set?

How do we write sets using brackets? Can you give some examples? How do you say them in words?

Consider the sets \{1, 2\} and \{2, 3\}. Can you think of a set that is larger than both? Can you think of another one?
1. Sets
A set is a collection of elements. In a set, the order of the elements does not matter.

Example: If \( a = \) “apple” and \( b = \) “boat”, then we can create the set \( \{a, b\} \). This set has two elements: the apple \( a \) and the boat \( b \). The order does not matter, so we could also write \( \{b, a\} \) and it is the same set.

\[
\{\text{apple}, \text{boat}\}
\]

The set \( \{p, q, r\} \) is read: “the set whose elements are \( p, q \) and \( r \)”. Here \( p, q \) and \( r \) are three objects, which can be whatever we want!

2. Matching activity
Match each description on the left with the set on the right. Each description corresponds with a unique set.

<table>
<thead>
<tr>
<th>Description</th>
<th>Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>A The set has 4 elements</td>
<td>{ball, circle, square, triangle}</td>
</tr>
<tr>
<td>B The set does not have any spherical elements</td>
<td>{tennis ball, hat, ( \pi ), apple}</td>
</tr>
<tr>
<td>C The set has only spherical elements</td>
<td>{tennis ball, ( \pi ), hammer, bug}</td>
</tr>
<tr>
<td>D The set has a bug</td>
<td>{tennis ball, hammer}</td>
</tr>
</tbody>
</table>
3. Sets and weights

Now suppose that the elements are given some weights in Kg, and that to find the weight of a set we need to add the weight of its elements. Remember that in sets elements cannot be repeated, so you only add the weight once!

Suppose that we have elements labeled by the letters a through z, and each one has a given weight in kg. We know some of the weights (for elements a to j), but we ignore others (elements k to z, which are not pictured):

<table>
<thead>
<tr>
<th>Letter</th>
<th>Image</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td><img src="image" alt="a" /></td>
<td>12 kg</td>
</tr>
<tr>
<td>b</td>
<td><img src="image" alt="b" /></td>
<td>10 kg</td>
</tr>
<tr>
<td>c</td>
<td><img src="image" alt="c" /></td>
<td>2 kg</td>
</tr>
<tr>
<td>d</td>
<td><img src="image" alt="d" /></td>
<td>22 kg</td>
</tr>
<tr>
<td>e</td>
<td><img src="image" alt="e" /></td>
<td>14 kg</td>
</tr>
<tr>
<td>f</td>
<td><img src="image" alt="f" /></td>
<td>8 kg</td>
</tr>
<tr>
<td>g</td>
<td><img src="image" alt="g" /></td>
<td>24 kg</td>
</tr>
<tr>
<td>h</td>
<td><img src="image" alt="h" /></td>
<td>20 kg</td>
</tr>
<tr>
<td>i</td>
<td><img src="image" alt="i" /></td>
<td>7 kg</td>
</tr>
<tr>
<td>j</td>
<td><img src="image" alt="j" /></td>
<td>7 kg</td>
</tr>
</tbody>
</table>
1) HEAVY SETS

Find the weight of the set \{a, b\}.

If we know that the weight of the set \{a, b, x, y\} is 35 kg, what is the weight of the set \{a, x, y\}?

Build a set that weighs exactly 44 kg. How many can you construct? Make sure to write them using brackets.
Suppose that we are already given the weights of the following four sets (which you can check):

- \{ a, b, c, d, e \} weighs 60 kg.
- \{ a, c, e, f, g \} weighs 60 kg.
- \{ b, d \} weighs 32 kg.
- \{ a, c, e \} weighs 28 kg.

What is the weight of the following set?
\{ a, b, c, d, e, f, g \}

Can you find this weight using only the information in this page (some of the four bullets above)? If so, how? Explain.

Did you find a general rule in this exercise that can help you find the weight of a set that is created by joining two smaller sets? Describe the rule.
Assume that we are given the following information:

- \{ k, l, m, q, r, s \} weighs 80 kg.
- \{ a, k, n, p, q, s \} weighs 70 kg.
- \{ k, q \} weighs 20 kg.
- \{ s, q \} weighs 24 kg.

Also, we are given that q weighs 6 kg.

What is the weight of the following set?
\{ a, k, l, m, n, p, q, r, s \}

Assume that we have the sets Set 1, Set 2, Set 3 and Set 4, such that:

- Set 1: \{ t, u, w, x \} weighs 46 kg.
- Set 2: \{ w, x, y, z \} weighs 36 kg.
- Set 3: \{ t, u \} weighs 30 kg.
- Set 4: \{ y, z \} weighs 20 kg.

Suppose that Set 5 is formed with the elements that belong to both Set 1 and Set 2. What is the weight of Set 5? Find more than one way to solve this problem if you can.
M4  Additional Resources

(we will have them at the meeting in your table)

- Pages 1-33: Instructor Manual
  45 copies, Green (2 per table + extras)

- Pages 34-40: Student Booklet
  120 copies, White, (6-8 per table + extras)

- Page 42: Cover, Activity 2
  18 copies, Brown (1 for each table)
  *Fold and place contents from P41-42 inside*

- Pages 43-44: Units Templates
  60 copies, Brown, one sided
  3 per table + extras for both rooms

- Pages 46-47: SS
  120 copies, 1-sided, PINK, 8 per table

- Pages 42-43: MR
  20 copies, 2-sided, BLUE, (1 per table + 2 extra)
Fold paper

Brown Package

Activity 2
BREAK AND KEEP

Manipulatives for
Units Templates I

Brown Paper
Bonus Round (Quiz) + Student Survey (Pink paper) 120 copies 1-sided
BONUS ROUND

(A) Every time I eat 6 grapes, I eat 4 pears: If I ate 10 pears, draw how many grapes I ate.

(B) I have 200 bats in total. Some are wooden, some metallic. If the ratio WOODEN : METALLIC is of 3 : 2, how many bats are wooden?

Answer:

(B) Let Set 1 be the set \{ 2, 4, 6, 8, 9, 0 \} and let Set 2 be the set \{ 1, 2, 3, 4, 5, 8, 0, 10, 11 \}.

Let Set 3 be the set of the elements that are in both Set 1 and Set 2. If we add all elements in Set 3, which number do we obtain?

Justify your choices. Write your work:

Answer

Justify your choices. Write your work:

Answer
UCI MATH CEO STUDENT SURVEY
MEETING 4, JANUARY 31, 2018

First AND Last Name: ________________________ _______________________________ Table Number: _____ Lathrop ( ) Villa ( )

Questions:
1) How boring were today’s activities?  1= not at all boring  2  3= somewhat boring  4  5= very boring
2) How satisfied are you with how you on today’s tasks?  1= not satisfied at all  2  3= somewhat satisfied  4  5= very satisfied
3) How did you feel while solving today’s activities?  1= not nervous at all  2  3= somewhat nervous  4  5= very nervous
4) How much energy did you put into today’s activities?  1= no energy at all  2  3= some energy  4  5= lots of energy
5) How close do you feel to your mentor at Math CEO?  1= not close at all  2  3= somewhat close  4  5= very close
6) How close do you feel to your peers at Math CEO?  1= not close at all  2  3= somewhat close  4  5= very close

Feedback for your mentor: ________________________

3 words to describe Math CEO: ________________________________

______________________________________________________

2 THINGS WHICH I LEARNED TODAY

2 THINGS THAT I FOUND INTERESTING

I QUESTION THAT I STILL HAVE

Clean your table when you finish, return the dry-erase markers, pick up your trash and take your belongings. Thank your mentor!

Thanks for your responses!
Meetings Report (Blue paper)
Dear leader mentor,

Please complete this survey about each of the students at your table. Circle the number that better reflects how you feel. We really value your input. THANK YOU for your thoughtful answers, and for your amazing contribution to Math CEO.

<table>
<thead>
<tr>
<th>STUDENT’S FIRST NAME: ___________________</th>
<th>LAST NAME: ______________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compared to his/her peers, how good was this student at solving today’s math activities? 1 (worse) 2 3 (average) 4 5 (a lot better)</td>
<td></td>
</tr>
<tr>
<td>How much innate ability or talent in math did this student show today? 1 (not at all) 2 3 (a little) 4 5 (very much)</td>
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</tr>
<tr>
<td>How much effort did this student put in today’s math activities? 1 (not at all) 2 3 (a little) 4 5 (very much)</td>
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<td>How much did this student participate in today’s math activities? 1 (not at all) 2 3 (a little) 4 5 (very much)</td>
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