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1 The Mountains

The following mountain leading to an ancient civilization is composed by seven segments of different levels of inclination. The last segment (g) is horizontal, so it has zero inclination. The slope is a measure of the inclination. **Sort the segments in increasing order according to their inclinations.**
For a right triangle, we define its \textit{slope} as:

\[
\text{Slope} = \frac{\text{RISE} \uparrow}{\text{RUN} \rightarrow}
\]

For example, in the triangle below the rise is 4 units, the run is 2 units, so the slope of the triangle is \( \frac{4}{2} = 2 \).

\begin{align*}
\text{Slope:} & \quad \frac{1}{3} \\
\text{Slope:} & \quad \frac{3}{4} \\
\text{Slope:} & \quad \frac{5}{2}
\end{align*}

Note: the slope is a number. The larger the slope, the steeper the diagonal!
Based on the pattern observed in the previous page, find the slopes of these right triangles:

b) From left to right:

1. \( \frac{3}{3} = 1 \)
2. \( \frac{3}{2} \)
3. \( \frac{5}{1} = 5 \)
4. \( \frac{1}{2} \)
5. \( \frac{1}{4} \)

To solve these problems, count the number of boxes on the side of the triangle showing the rise and the number of boxes representing the run of the triangle and divide the two numbers.

The slope of the first triangle is \( \frac{3}{3} = 1 \) because the rise is 3 and the run is 3, so \( \frac{3}{3} = 1 \).

The slope of the second triangle is \( \frac{3}{2} \) because the rise is 3 and the run is 2, so \( \frac{3}{2} \).
Recall that SLOPE = RISE/RUN. Find the missing information.

- Slope: $\frac{3}{2}$
- Slope: $\frac{1}{4}$
- Slope: $\frac{1}{2}$
d Sort the triangles in increasing order of steepness:

1) Slope: 1
2) Slope: 3/2
3) Slope: 5/2
4) Slope: 1/2
5) Slope: 1/4

Discuss: Did you solve this problem visually, numerically or both? Why?
**When the run is 1**

All these triangles have RUN = 1.

i) Compute the slope of each triangle.

ii) For each triangle, compare the rise with the slope. *What do you notice?*

Recall that $\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}}$. 

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Rise</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>$\frac{3}{6} = \frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>c</td>
<td>$\frac{4}{6} = \frac{2}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>d</td>
<td>$\frac{9}{6} = \frac{3}{2}$</td>
<td>$\frac{3}{2}$</td>
</tr>
</tbody>
</table>

The kids should see that when the base is equal to 1, the rise is equal to the slope because you end up dividing the rise by 1 and that does not affect the number.
Similar right triangles have the same slope

For each triangle:

i) Find its slope without computing run & rise.

ii) Check your answers (using the formula slope = rise/run)

iii) Compare run & rise for triangles a and d, and for triangles a and c.

What do you notice?

- Triangle b) run = 1, rise = 1/2, slope = 1/2
- Triangle c) run = 6/4 = 3/2, rise = 3/4, slope = 1/2
- Triangle d) run = 16, rise = 8, slope = 8/16 = 1/2

When you compare the slopes of triangles a and d, you see that the slopes are the same.

- From this, they should see that triangles can look different but have the same slope.

When you compare the slopes of triangles a and c, you see that the slopes are also the same.

- Continues to show that triangles can be similar even if they look different.

In all, triangles a, c, and d have the same slope.
Does the slope depend on the units?

9. Compute run, rise and slope for triangle a.

\[
\begin{align*}
\text{Run:} & \quad 1 \\
\text{Rise:} & \quad 1 \\
\text{Slope:} & \quad 1/2 
\end{align*}
\]

We now cut the units in half. Compute run, rise and slope for triangle b (which is equal to triangle a).

\[
\begin{align*}
\text{Run:} & \quad 1 \\
\text{Rise:} & \quad 1 \\
\text{Slope:} & \quad 1/2 
\end{align*}
\]

What happens to the slope if the unit is halved?
2 Sumerian Tablets

Sumerians lived about 5,000 years ago. To do math, they used wedge-shaped characters inscribed on baked clay tablets.

Let us suppose that we are Sumerians, and do additions on tablets using numbers, letters, and other symbols. Every set of three tablets connected by two converging arrows represents an addition fact.

\[
\text{input} \quad 2 \quad 3 \quad \text{output} = 2+3
\]

\[
\text{input} \quad 7 \quad 13 \quad \text{output} = 20
\]

\[
\text{input} \quad x \quad 7+x \quad \text{output} \quad \star+1
\]
Each of the following sets of Sumerian tablets represents an addition facts. Complete the missing information (using numbers, letters or symbols as appropriate).

1) \[ 68 \rightarrow 73 \]

2) \[ 13 \rightarrow 24 \]

3) \[ x + 2 \rightarrow \]

4) \[ 7 + x \rightarrow 22 \]

5) \[ 2 \rightarrow x + 3 \]

6) \[ 2 + 3x \rightarrow \]

7) \[ 7 + 2 \rightarrow \]

8) \[ + 1 \rightarrow + x + 8 \]
3 Addition Sequences

An **addition sequence** is a sequence of addition facts performed one after the other, following the order indicated by the arrows.

**Step 0:**
Start here and follow the arrows

**Step 1:**
Take 4 and 7 as inputs, and compute their sum (11=output)

**Step 2:**
Take 6 and 11 as inputs, and compute their sum (18=output)

**Step 3:**
Take 11 and 18 as inputs, and compute their sum (29=output)

**Step 4:**
Take 18 and 29 as inputs, and compute their sum (47=output)
Complete the following connected multi-diagrams:

- 17
- 6
- 5

- 11
- 2

- x+8
- 2

- 13+y
- y+9

- 17
- 11

- 3
b. Complete the following sequence:

Can you find a common property of all the numbers in this sequence?

c. Recall that if $x$ is any value, then $2x$ is equal to $x + x$. Also, $3x$ is equal to $x + x + x$. And so on. Complete the following sequence:
d) Complete the following sequence:

3  x

Complete the following sequence:

x  y  e
d

y  x

e) Complete the following sequence:
4 Multiplication Sequences

A multiplication sequence is a list of numbers constructed as follows: The first two numbers are given; after that, every number is the product of the previous two numbers.

For example, in the sequence 2, 3, 6 the number $6 = 2 \times 3$ is the product of the two previous numbers in the sequence. We can visualize it using the connected diagrams.

We do not have to restrict to multiply only numbers; we could for example, multiply letters in the sequence: $x, y, x \times y$ where $x \times y$ is the product of the two previous elements in the sequence. We can visualize it using the connected diagrams.
More Multiplication Sequences

We can continue forming the first sequence:

\[2, 3, 6, 18, 108, 1944\]

We can visualize it using the connected diagrams:

- **Step 0:** Start here and follow the arrows.
- **Step 1:** Take 2 and 3 as inputs, and compute their multiplication (6=output).
- **Step 2:** Take 3 and 6 as inputs, and compute their multiplication (18=output).
- **Step 3:** Take 18 and 6 as inputs, and compute their multiplication (108=output).
- **Step 4:** Take 18 and 108 as inputs, and compute their multiplication (1944=output).
Complete the following sequence:

\[ \begin{align*}
&10^3 \cdot 10^2 = (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) = 10^5 = 10^{3+2} \\
\end{align*} \]

To multiply two powers with the same base, we only need to add the exponents: \[ 10^n \cdot 10^m = 10^{n+m} \]
4  Multiplication sequences

We can make powers with any number. For example:

\[ 3^3 \times 3^4 = (3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) = (3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3) = 3^{3+4} \]

\[ 3^3 \times 3^4 = 3^{17} \]

\[ 3^3 \times 3^4 = 3^{27} \]

\[ 3^3 \times 3^4 = \text{Can you list the next 3 numbers in this sequence?} \]

\[ 3^3 \times 3^4 = \text{Can you list the next 3 numbers in this sequence?} \]

\[ 3^3 \times 3^4 = \text{Complete the following sequence:} \]
Recall that if $x$ is any value, then $x^2$ is equal to $x \cdot x$ (read: “$x$ times $x$”). Also, $x^3$ is equal to $x \cdot x \cdot x$, $x^4$ is equal to $x \cdot x \cdot x \cdot x$, etc. Complete the following sequence:

$$x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) = (x \cdot x \cdot x \cdot x \cdot x) = x^5 = x^{3+2}$$

In general: 

$$(x^n) \cdot (x^m) = x^{n+m}$$
5 Ancient Tablets

Anthropologists have found several tablets from an ancient civilization which stored numeric information.

Analyze the quantities displayed in the drawings. Can you find out which numbers and which operation are represented in these tablets?
Anthropologists also found the following set of “broken” tablets. Because the tablets are very old, some of the information was lost over time. Each diagram represents an addition fact (with symbols instead of numbers). Help the anthropologists by filling in the missing information in the broken tablets (drawn in white). Note: One of the broken tablets is meant to be empty.
A new set of tablets was found. This time, each tablet represent a different operation (in other words, not all of them are addition). Match each trio of tablets above with one of the examples below.

A) As we can see from the pictures, the connected diagram represents the addition operation. So the answer to A) is Number 4.

B) In i, we need to subtract. In ii) we need to add (note that the star symbol duplicates). In part iii, we need to include the circle and the star in the answer. In part iv we do not need to do anything! It is already complete.

The following are algebraic equivalents to the visual representations:

i) \((x) + (y)\) \(\rightarrow (x + y)\).

ii) \((x + y) + (x)\) \(\rightarrow (2x + y)\).

iii) \((x + y) + (y + z)\) \(\rightarrow (2y + x + z)\).

iv) \((x + y) + (0)\) \(\rightarrow (x + y)\).
d) The diagrams 1, 2, 3, 4 below represent 4 different operations.

Which of the above operations was used in the ancient tablets discovered by our team of anthropologists?
Addition of numbers has a lot of interesting properties.

**Commutativity:** $x + y = y + x$

When we add two numbers, the order is not relevant.

Example: $2 + 5 = 7 = 5 + 2$

Which property of addition is represented by the diagram on the right?

Example: ____________
Complete the connected diagrams corresponding to: **Associativity**: 
\[(x + y) + z = x + (y + z)\]

**Example 1**: 
\[(2+5)+6=2+(5+6)\]

**Example 2**: 
\[(5+1)+4=\]
CHALLENGES

Complete the following connected multi-diagrams (using addition):

I.  
\[
\begin{array}{c}
41 \\
/ \quad / \\
9 \\
/ \quad / \\
\end{array}
\]

II.  
\[
\begin{array}{c}
\text{ } \\
/ \quad / \\
\text{ } \\
/ \quad / \\
4 \\
/ \quad / \\
\text{ } \\
/ \quad / \\
11 \\
\end{array}
\]