Meeting 15 Student's Booklet

ANCIENT PLACES

February 24 2016 @ UCI

Contents

UC IRVINE MATH CEO
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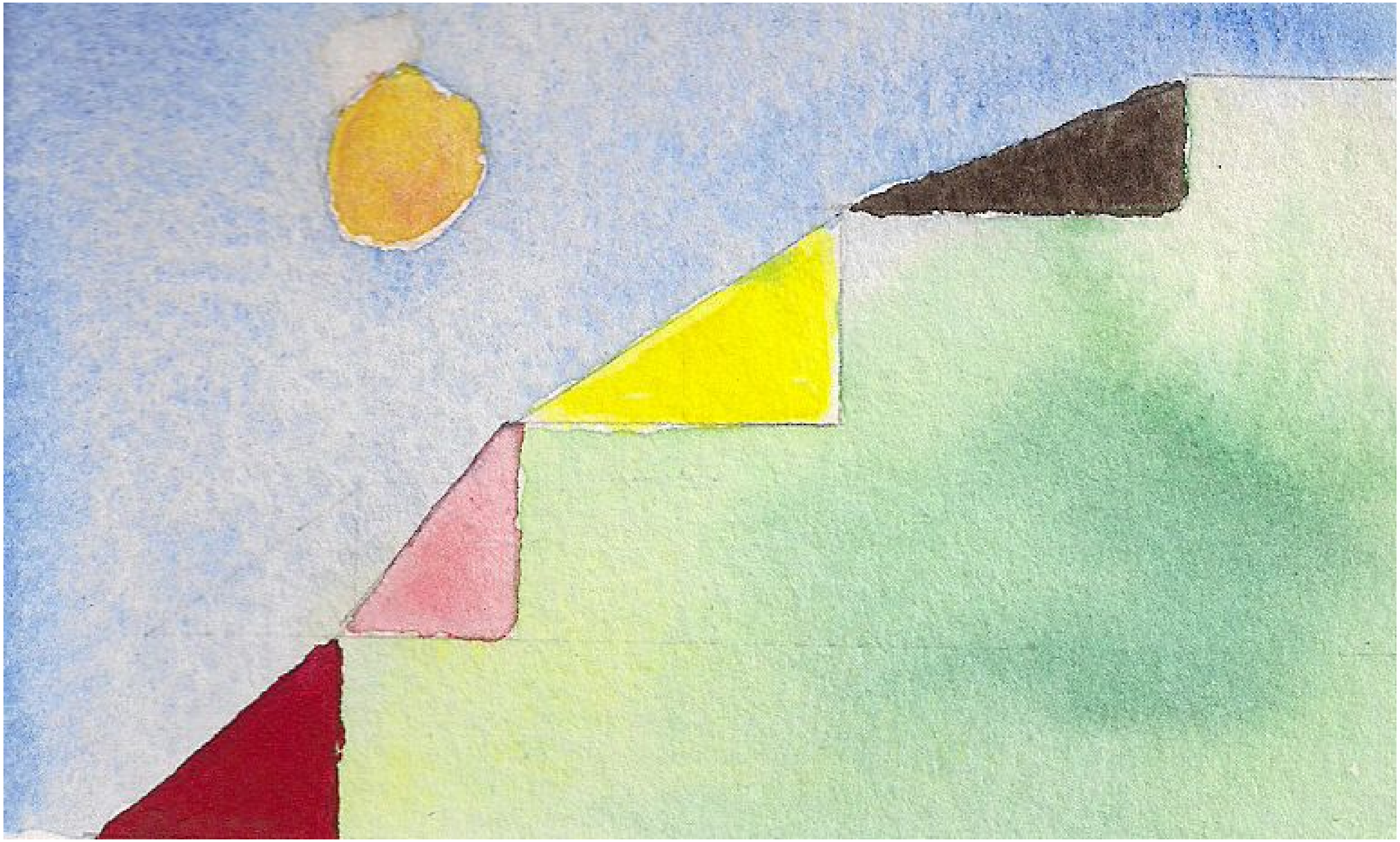
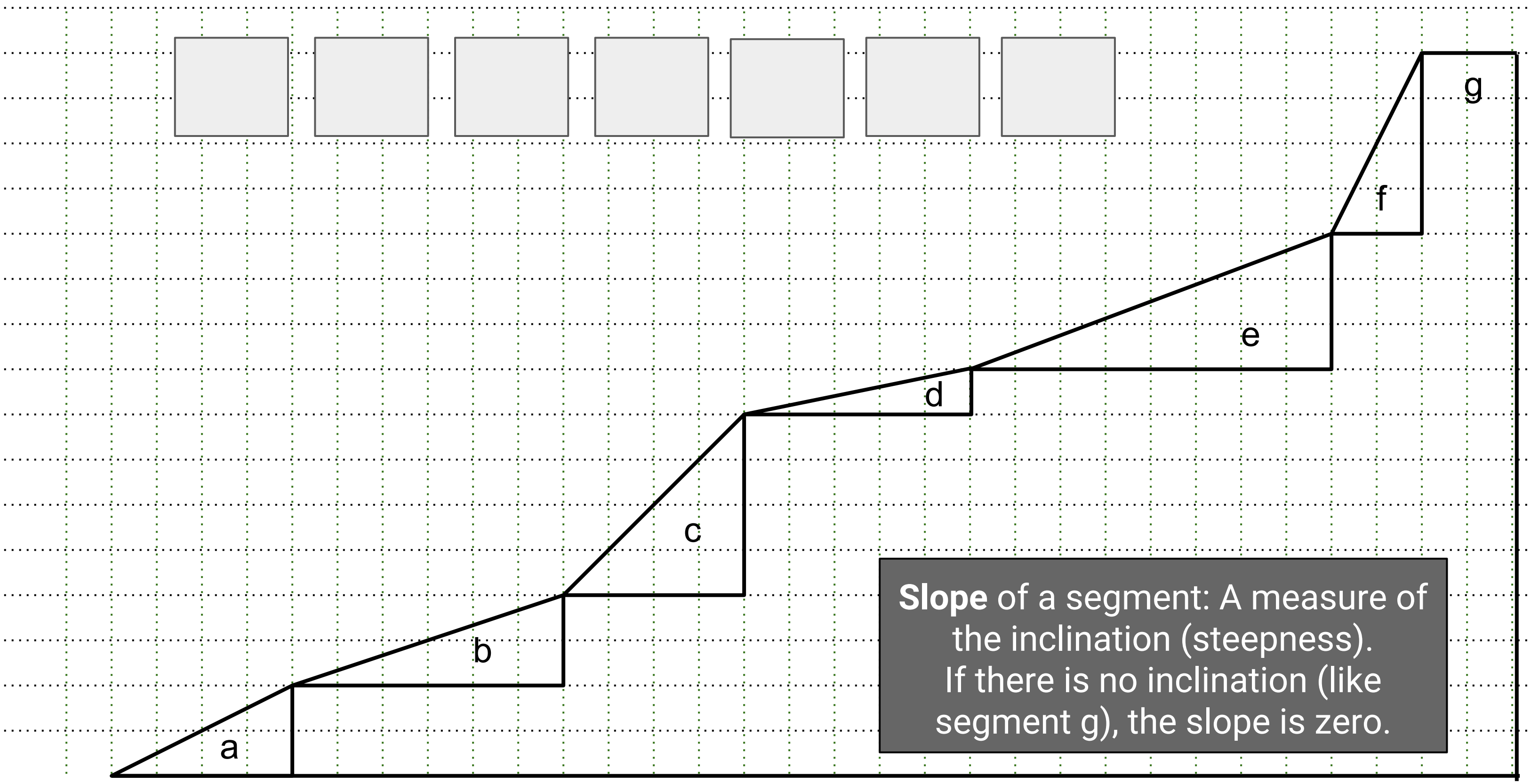


1 The Mountains

a The following mountain leading to an ancient civilization is composed by seven segments of different levels of inclination. The last segment (g) is horizontal, so it has zero inclination. The slope is a measure of the inclination. **Sort the segments in increasing order according to their inclinations.**

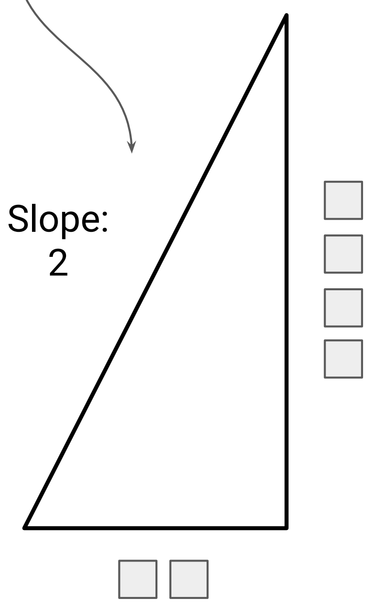


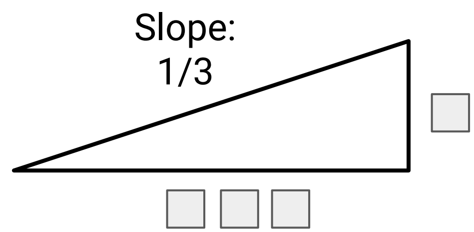
For a right triangle, we define its *slope* as:

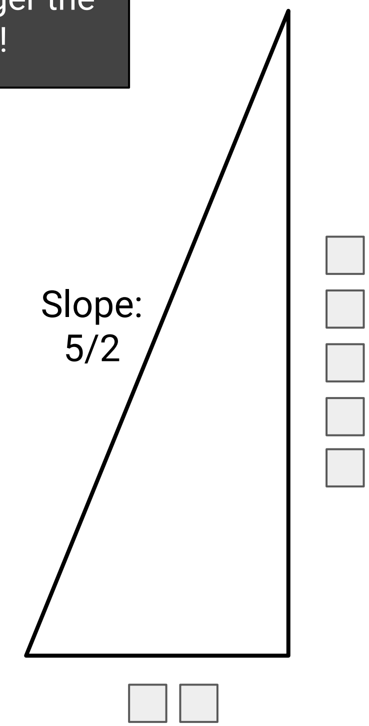
$$\textbf{Slope} = \frac{\textbf{RISE}\uparrow}{\textbf{RUN}\rightarrow}$$

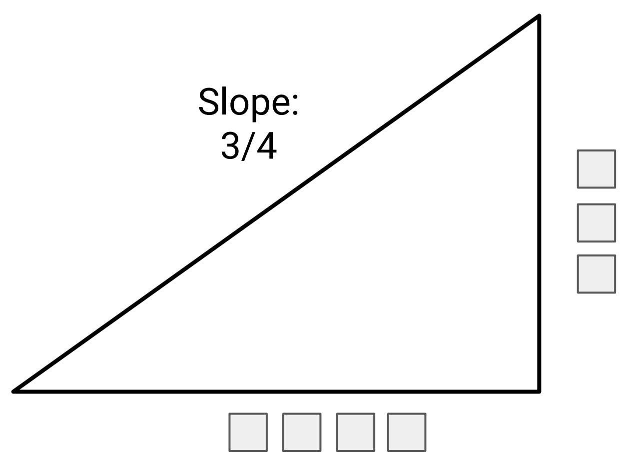
Note: the slope is a <u>number.</u> The larger the slope, the steeper the diagonal!

For example, in the triangle below the rise is 4 units, the run is 2 units, so the slope of the triangle is 4/2 = 2.

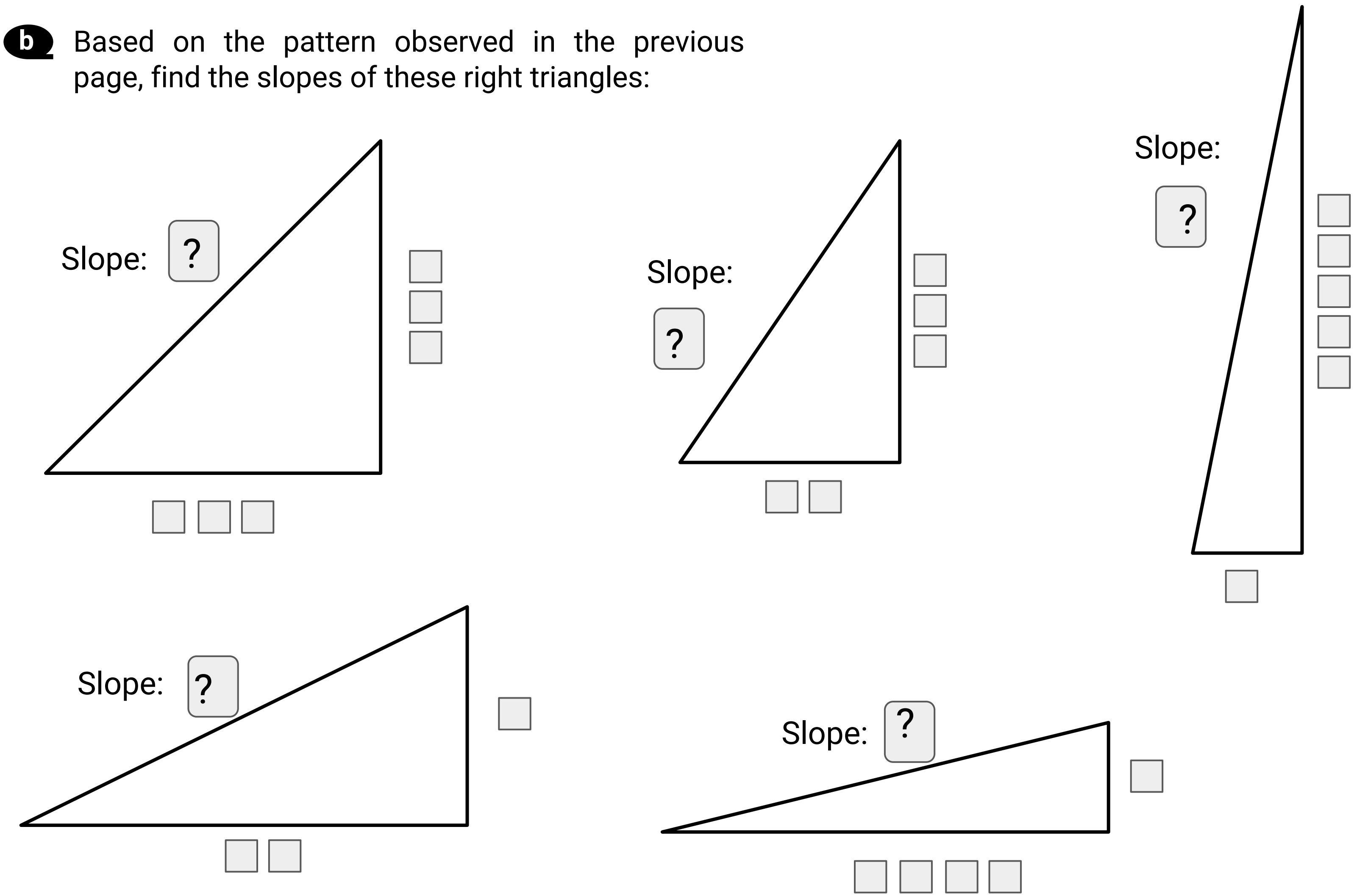




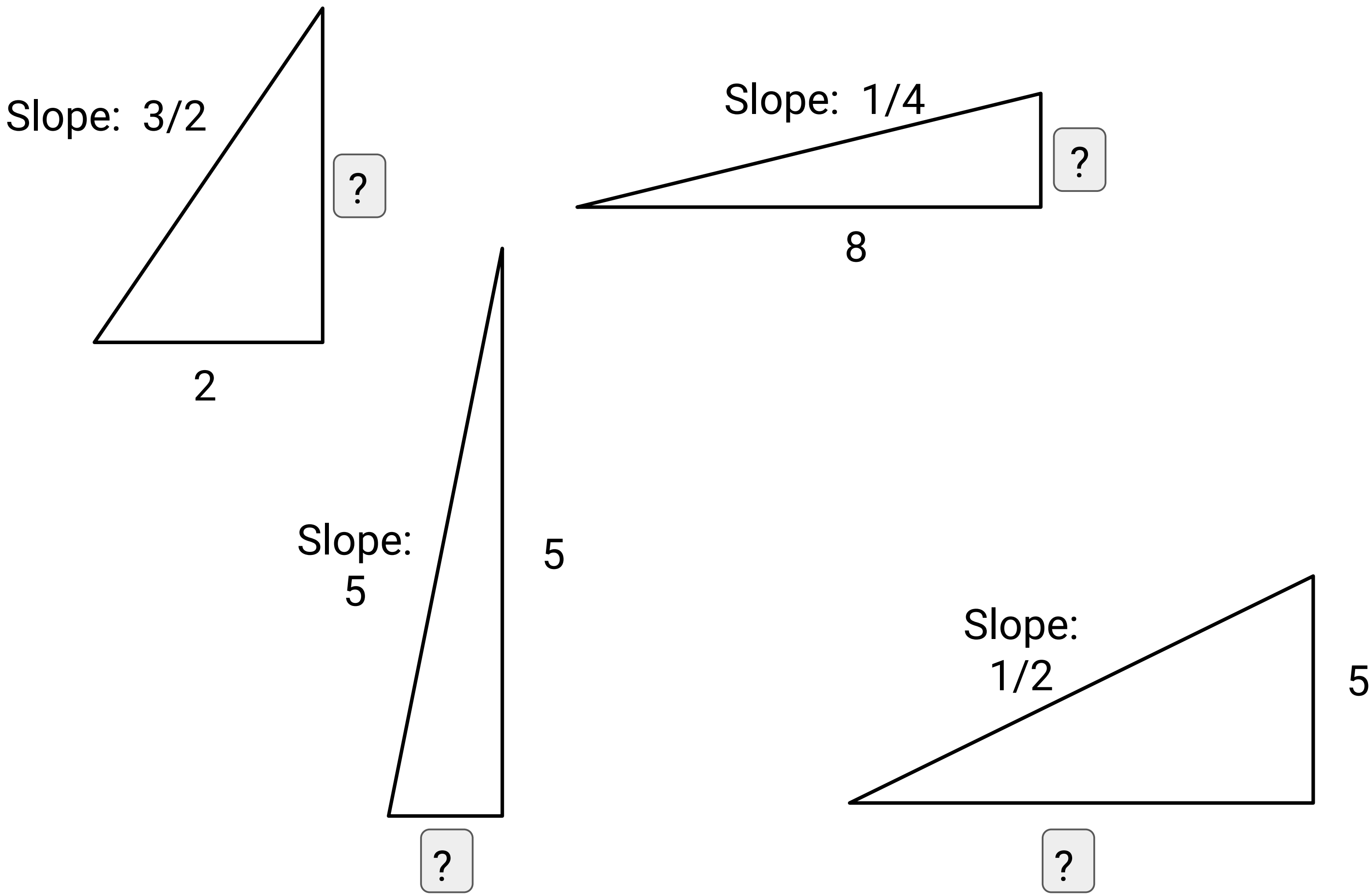




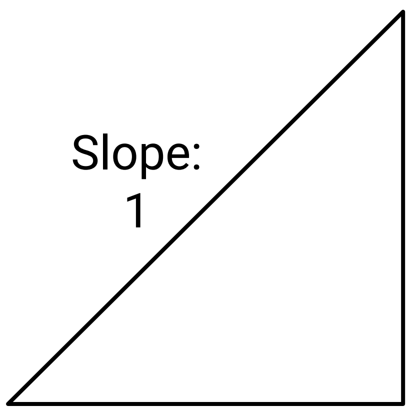
b Based on the pattern observed in the previous page, find the slopes of these right triangles:



c Recall that SLOPE = RISE/RUN. Find the missing information.



d Sort the triangles in increasing order of steepness: ☐ ☐ ☐ ☐ ☐

1)


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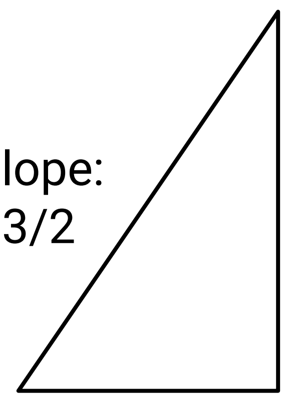

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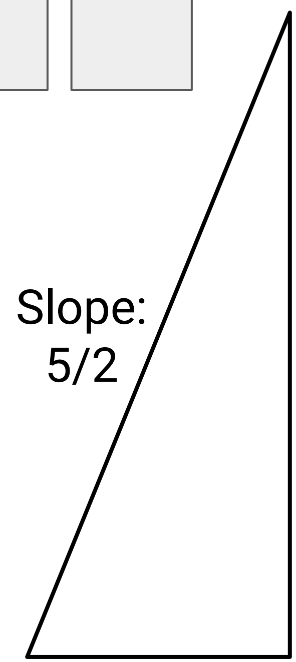

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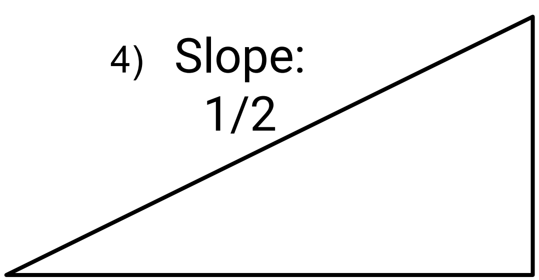

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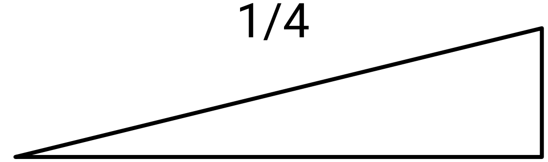


Discuss:
Did you solve this problem visually, numerically or both? Why?

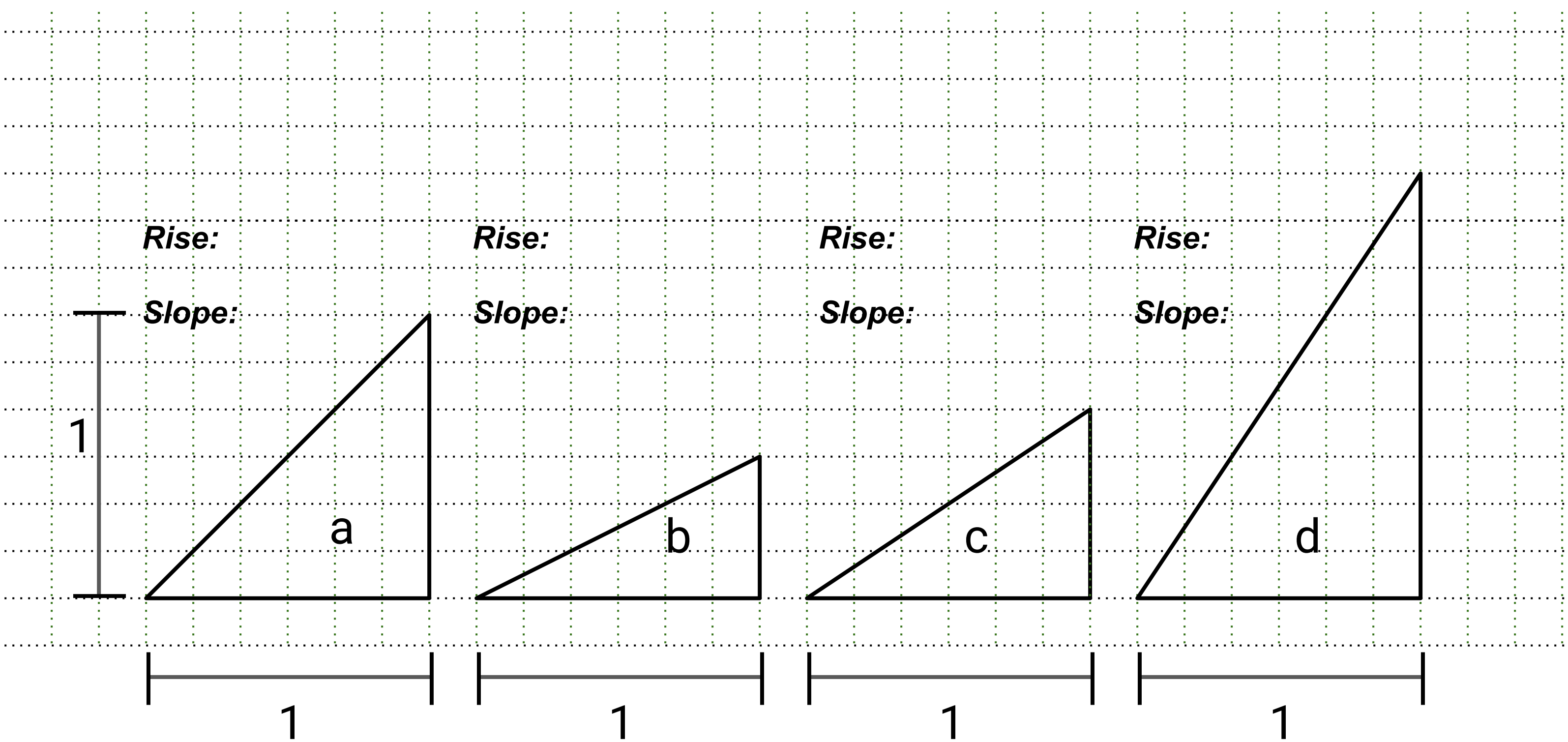
When the run is 1

Recall that SLOPE = RISE/RUN.

e All these triangles have RUN = 1.

i) Compute the slope of each triangle.

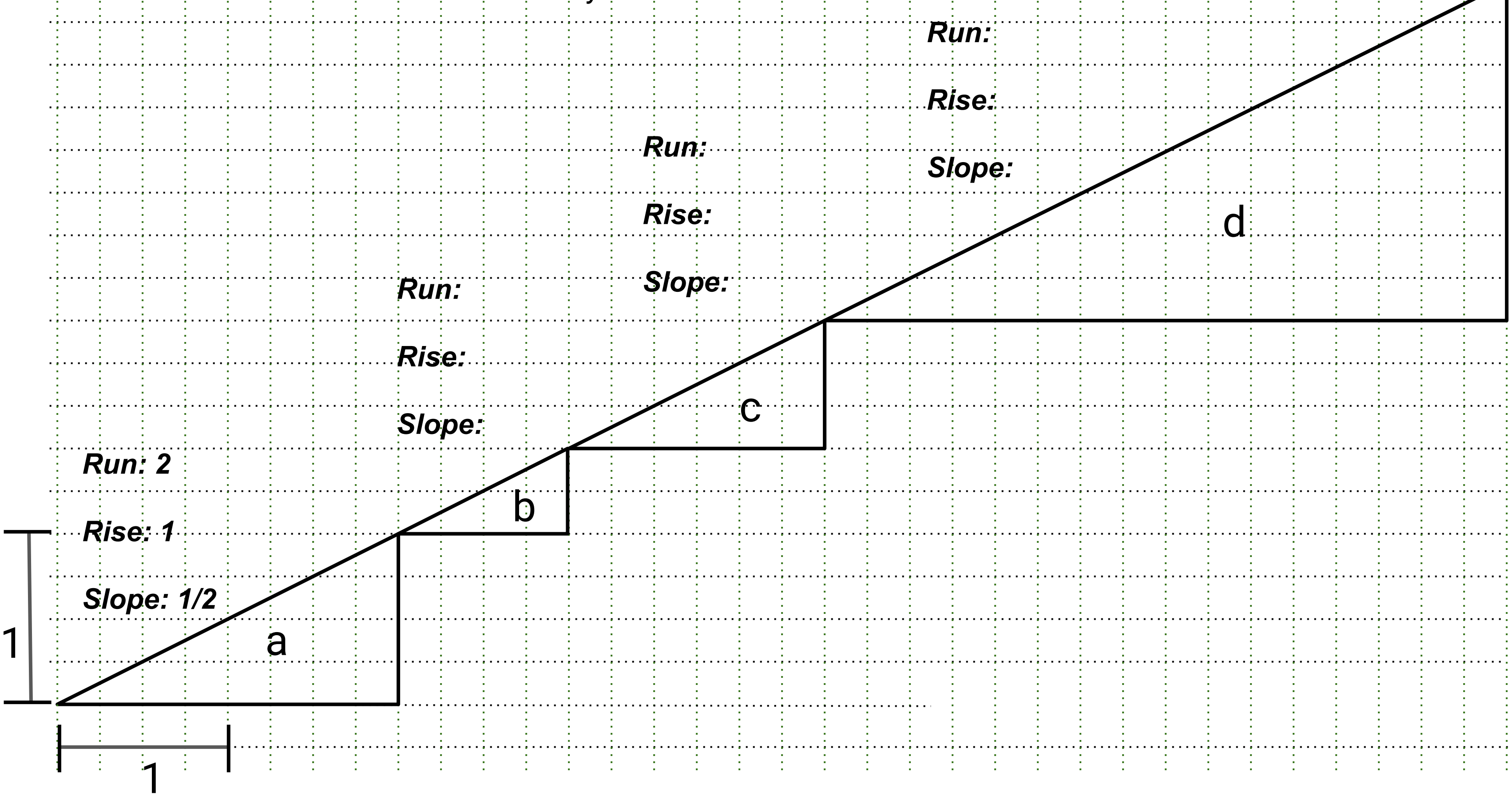
ii) For each triangle, compare the rise with the slope. *What do you notice?*



Similar right triangles have the same slope

f For each triangle:

i) Find its slope without computing run & rise.
ii) Check your answers (using the formula *slope = rise/run*)
iii) Compare run & rise for triangles a and d, and for triangles a and c. What do you notice?



Does the slope depend on the units?

g Compute run, rise and slope for triangle a.

Run:

Rise:

Slope:

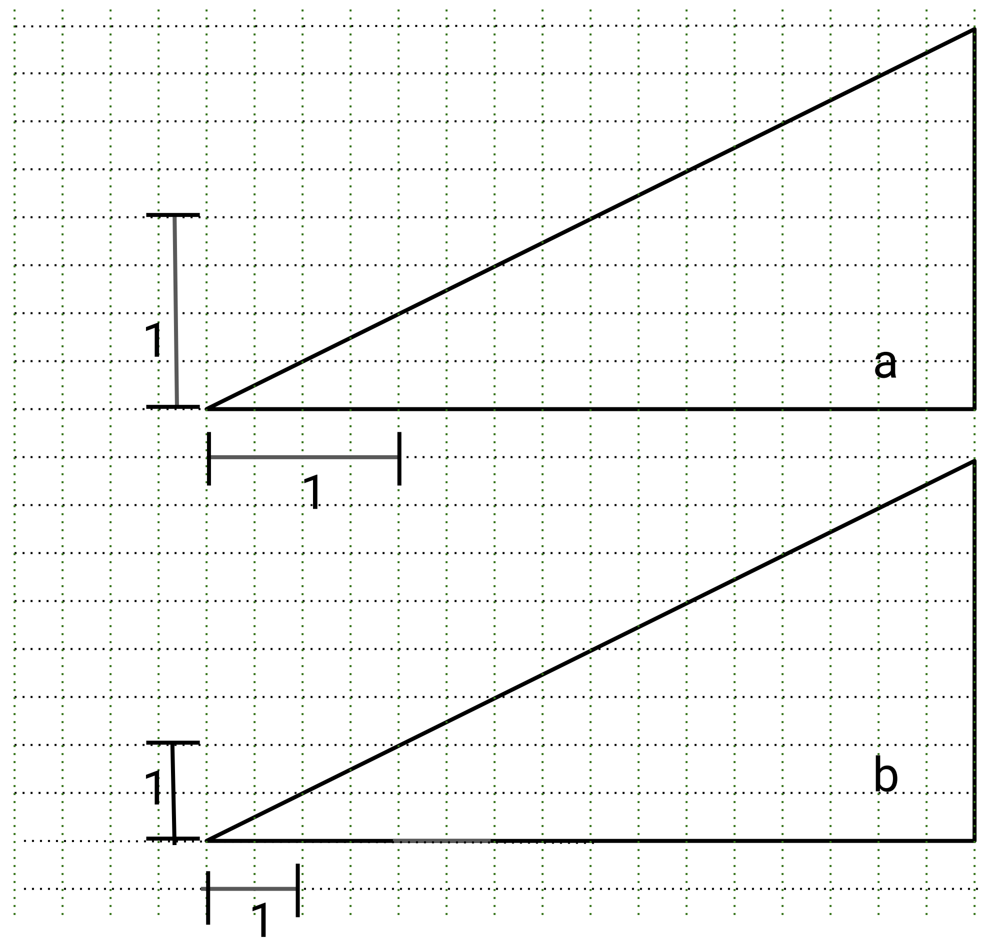
We now cut the units in half. Compute run, rise and slope for triangle b (which is equal to triangle a).

Run:

Rise:

Slope:

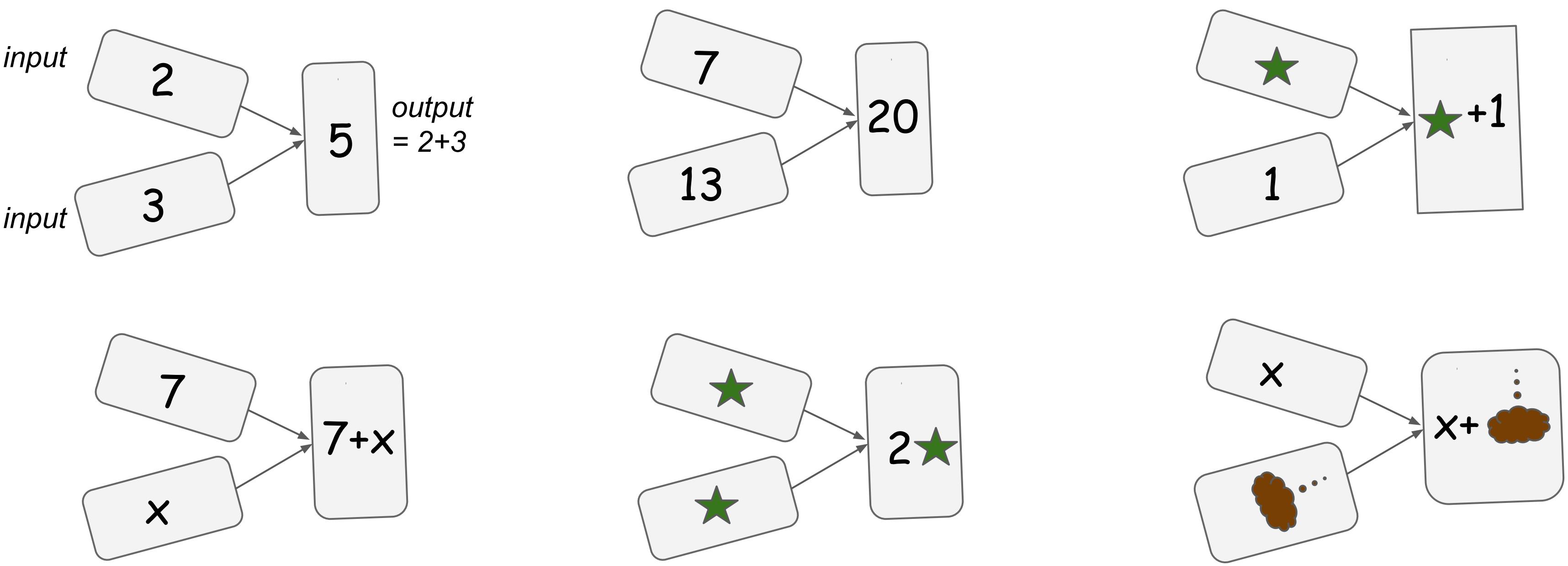
What happens to the slope if the unit is halved?



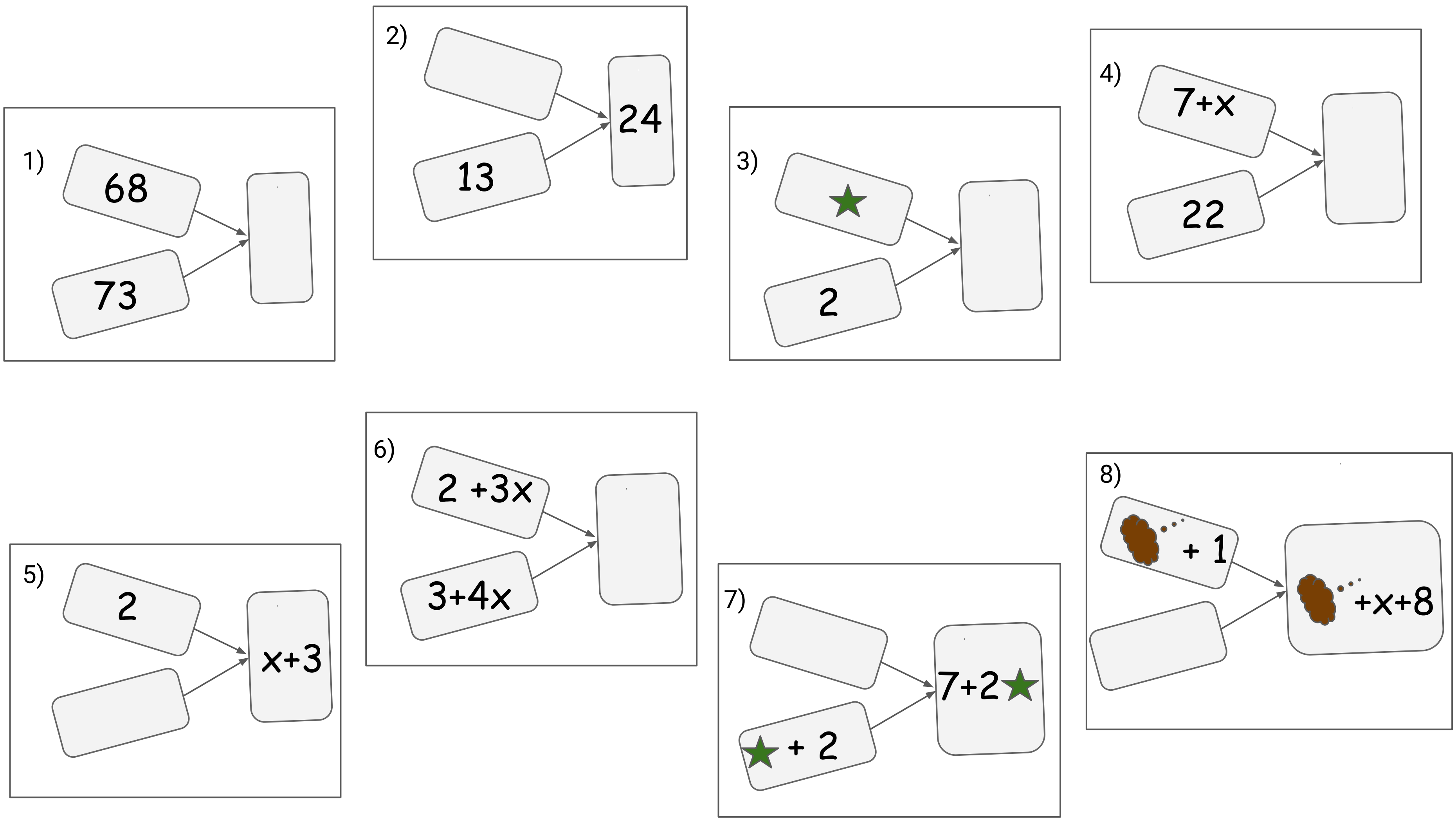
2 Sumerian Tablets

Sumerians lived about 5,000 years ago. To do math, they used wedge-shaped characters inscribed on baked clay tablets.

Let us suppose that we are Sumerians, and do additions on tablets using numbers, letters, and other symbols. Every set of three tablets connected by two converging arrows represents an **addition fact**.



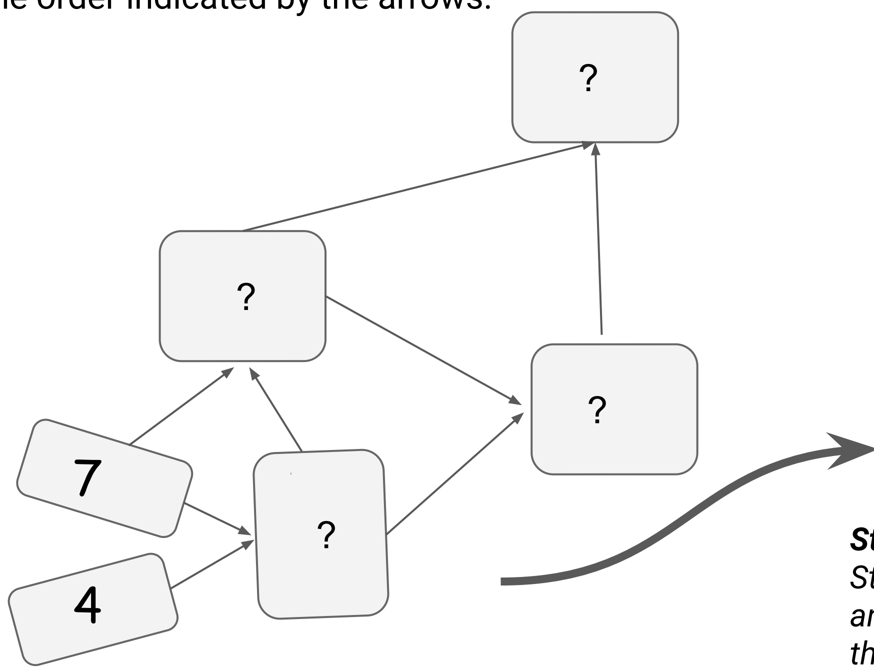
a Each of the following sets of Sumerian tablets represents an **addition facts**.
Complete the missing information (using numbers, letters or symbols as appropriate).

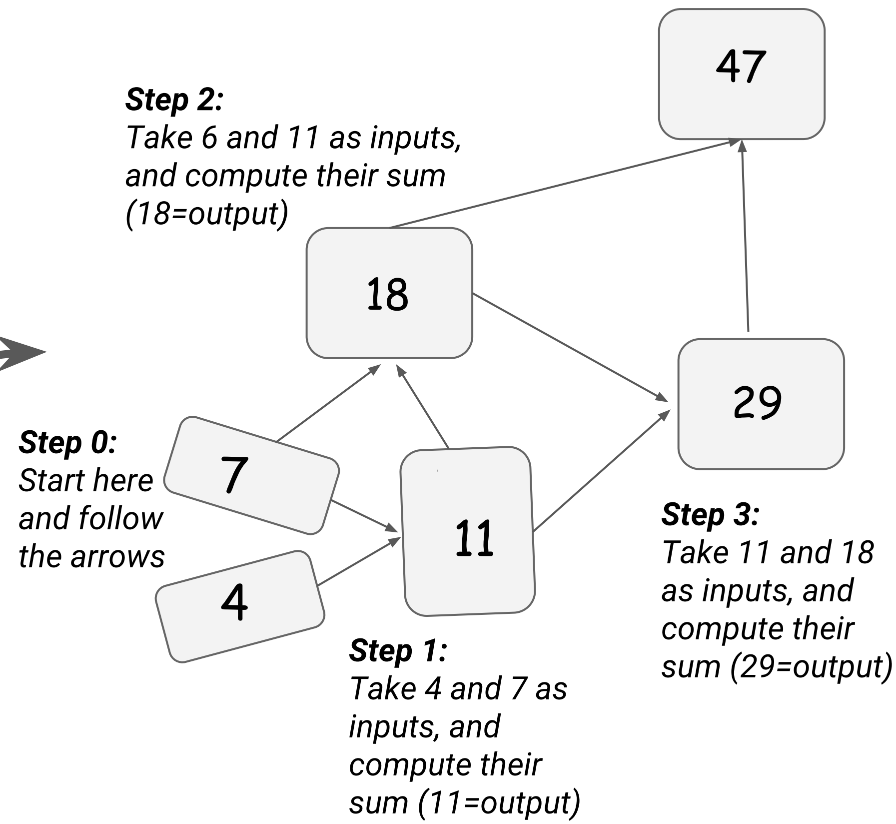


3 Addition Sequences

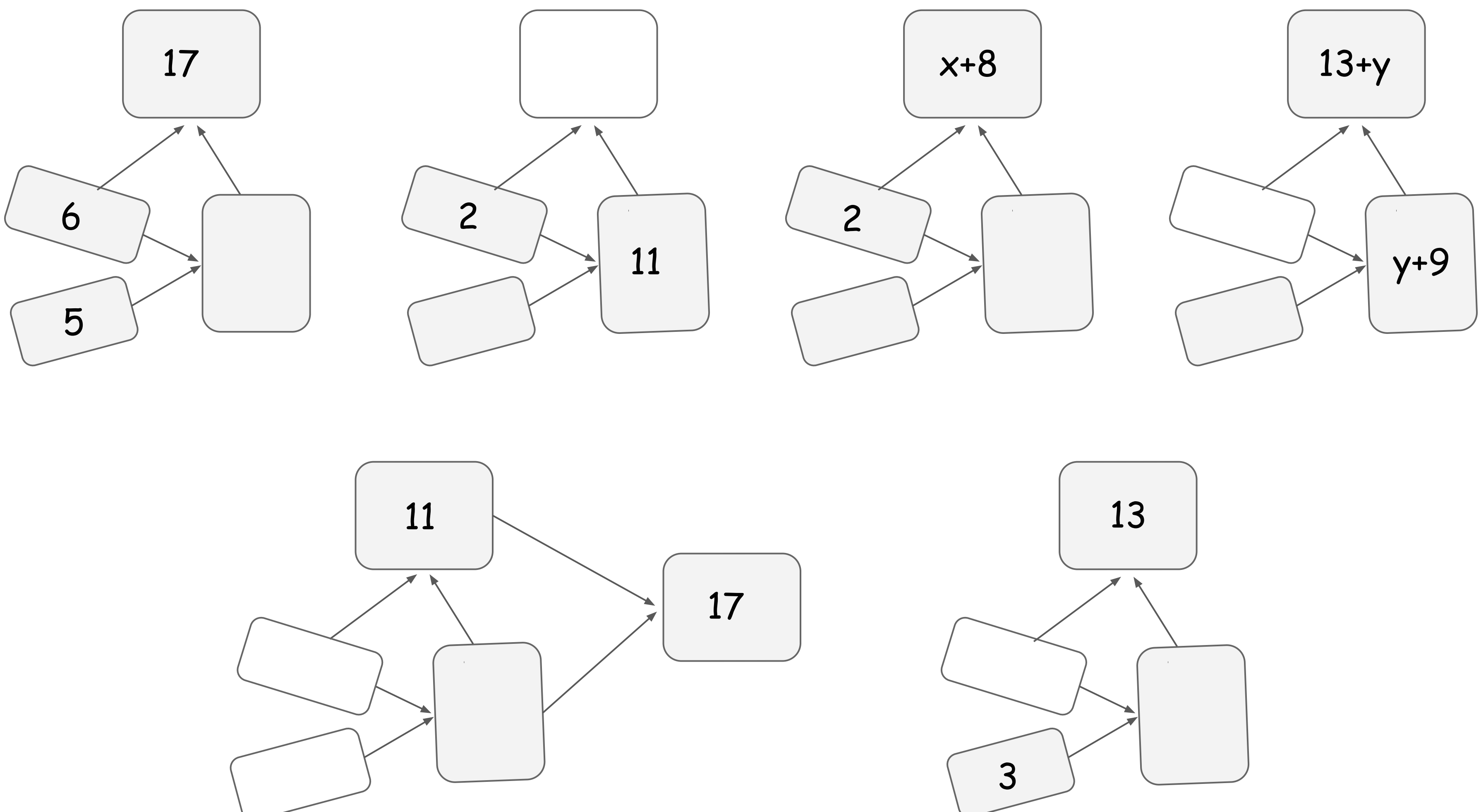
To fill out an addition sequence diagram, perform one addition fact after the other, starting with the two numbers given, and following the arrows at each step...

An **addition sequence** is a sequence of addition facts performed one after the other, following the order indicated by the arrows.

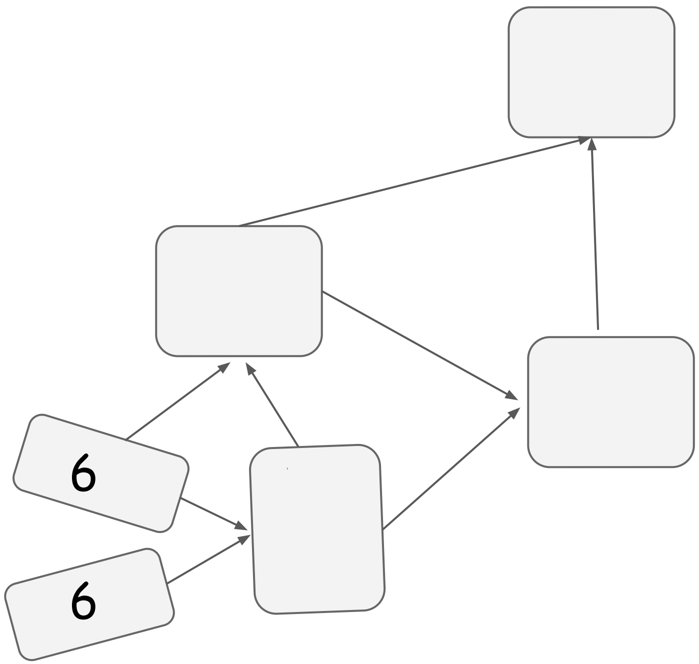




a Complete the following connected multi-diagrams:

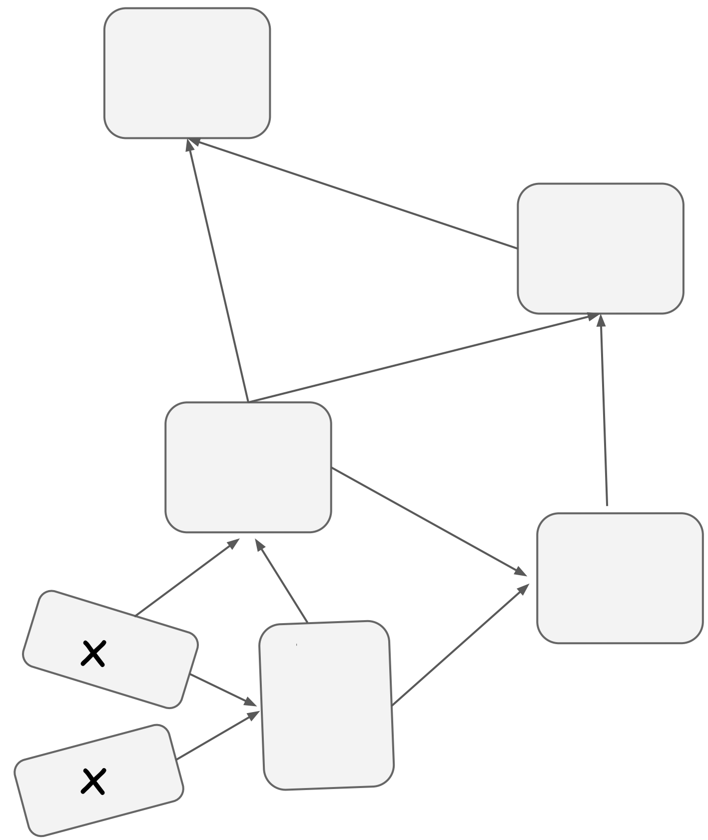


b Complete the following sequence:

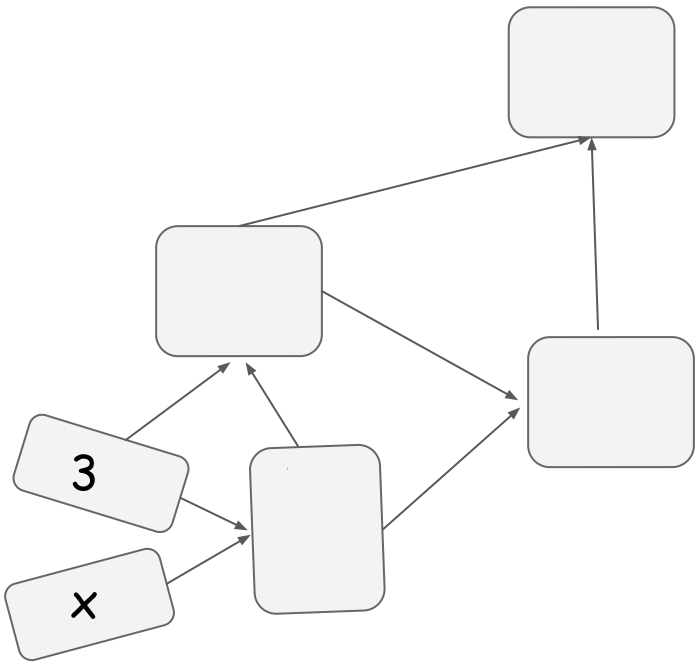


Can you find a common property of all the numbers in this sequence?

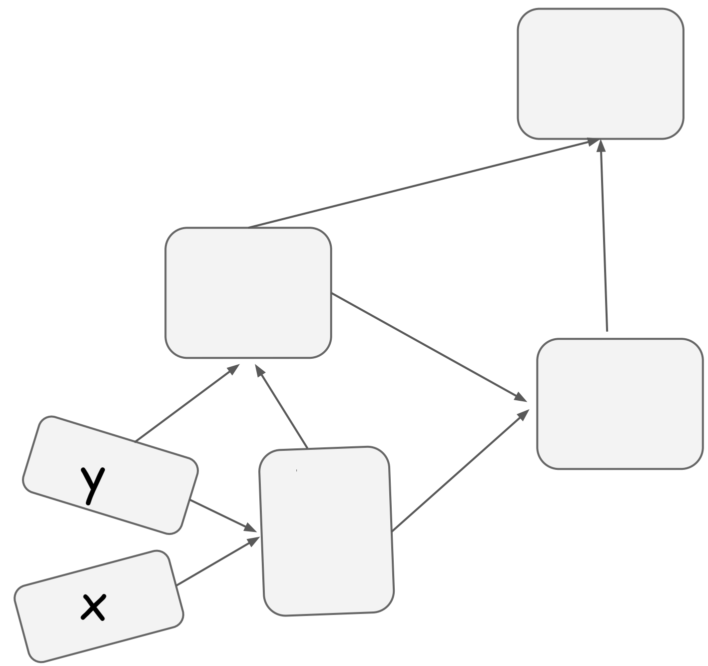
c Recall that if x is any value, then 2x is equal to x + x. Also, 3x is equal to x + x + x. And so on. Complete the following sequence:



d Complete the following sequence:



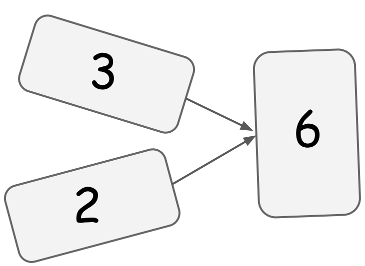
e Complete the following sequence:



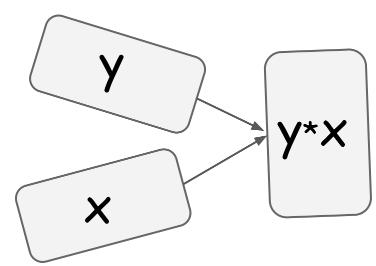
4 Multiplication Sequences

A **multiplication sequence** is a list of numbers constructed as follows: The first two numbers are given; after that, every number is the product of the previous two numbers.

For example, in the sequence 2, 3, 6 the number 6 = 2*3 is the product of the two previous numbers in the sequence. We can visualize it using the connected diagrams



We do not have to restrict to multiply only numbers, we could for example, multiply letters in the sequence: x, y, x*y where x*y is the product of the two previous elements in the sequence. We can visualize it using the connected diagrams

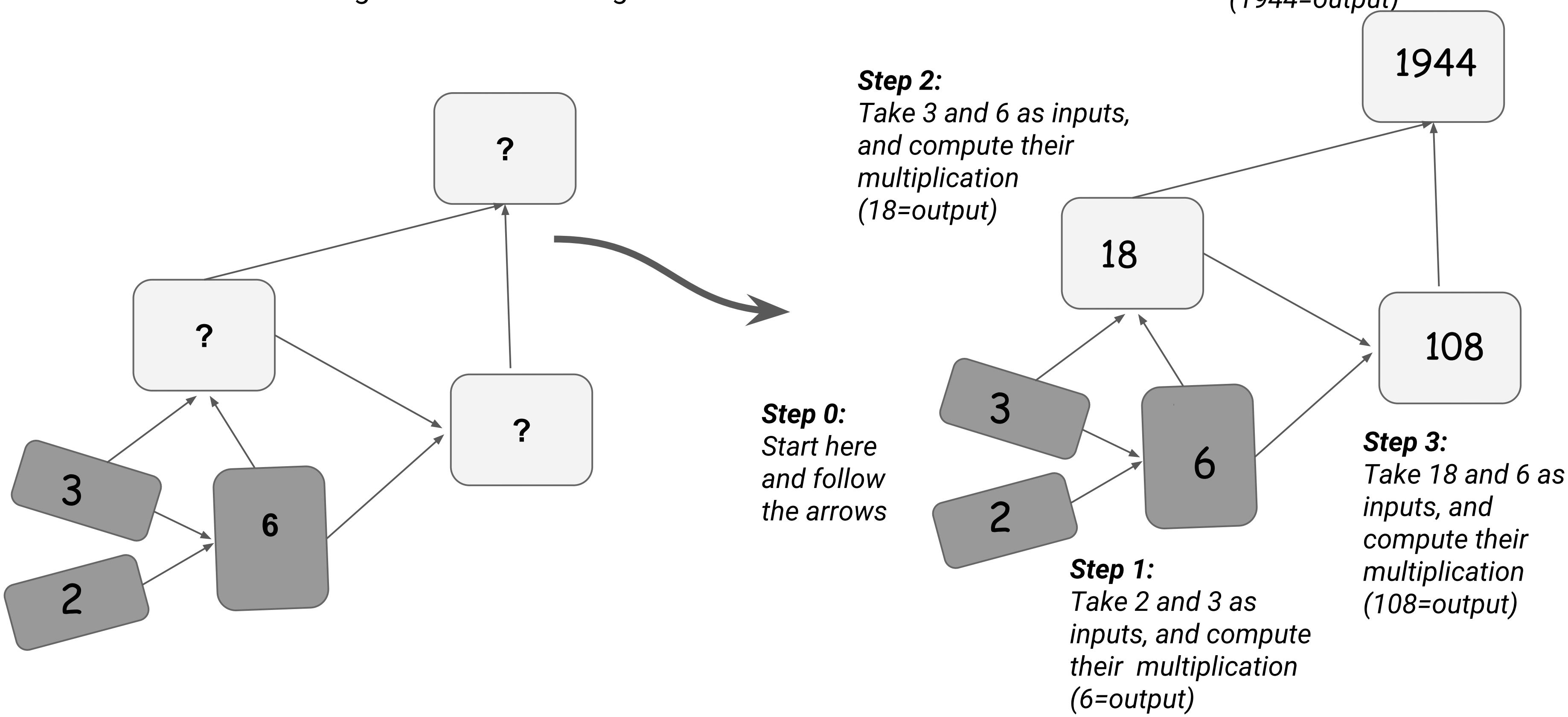


More Multiplication Sequences

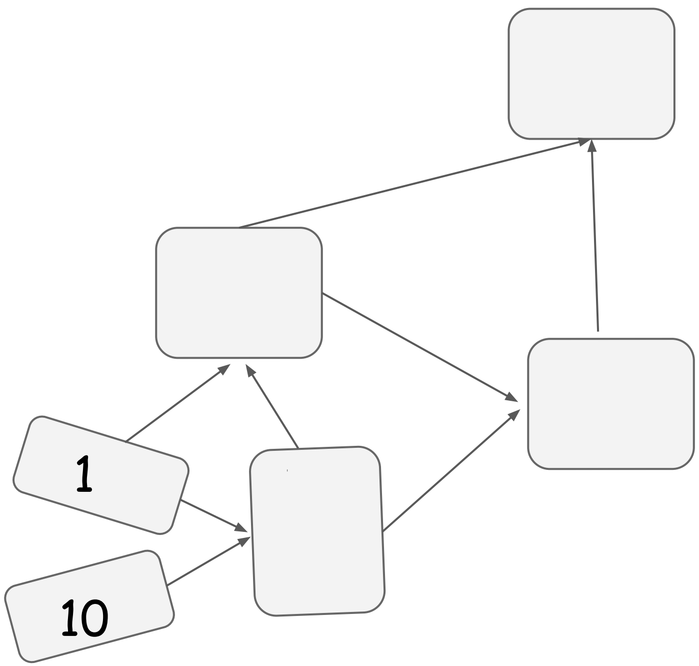
We can continue forming the first sequence:

2, 3, 6, 18, 108, 1944

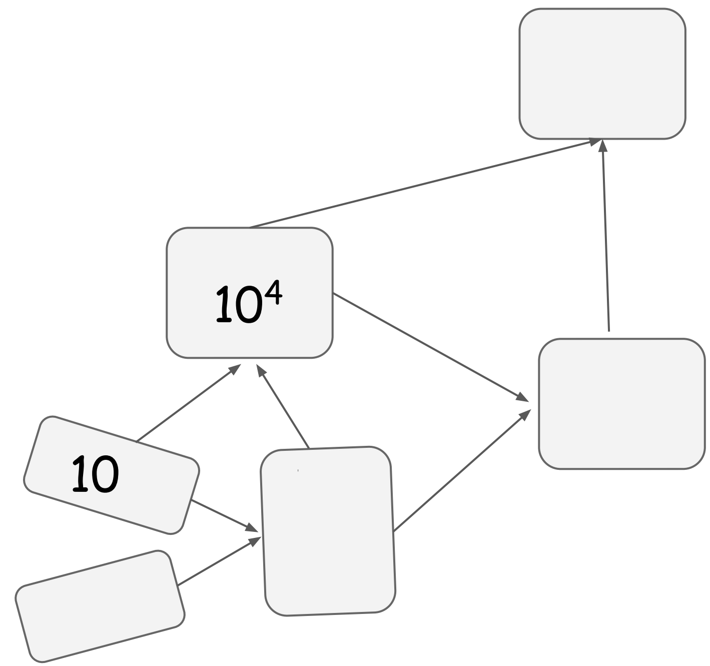
We can visualize it using the connected diagrams:



a Complete the following sequence:



b Complete the following sequence:



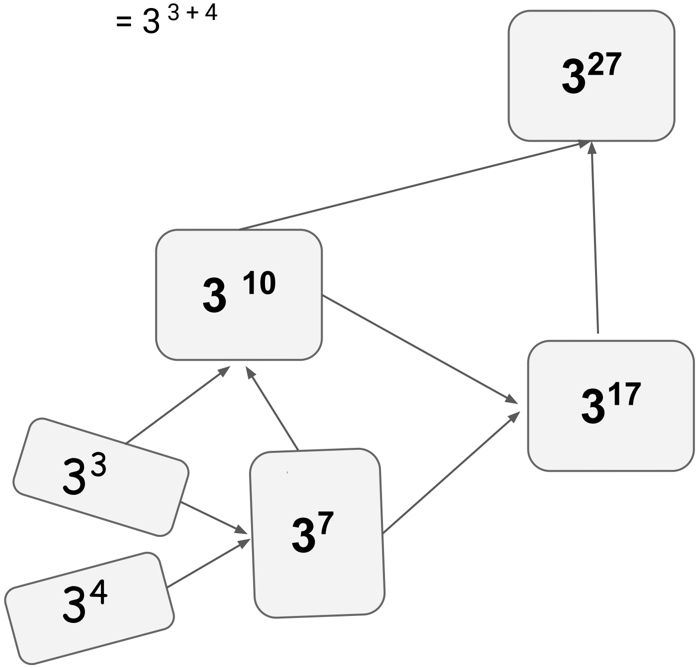
Note: Recall that when you multiply powers of 10, you can do things like:

$$10^3 \cdot 10^2 = (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) = 10^5 = 10^{3+2}$$

To multiply two powers with the same base, we only need to add the exponents: $10^n \cdot 10^m = 10^{n+m}$

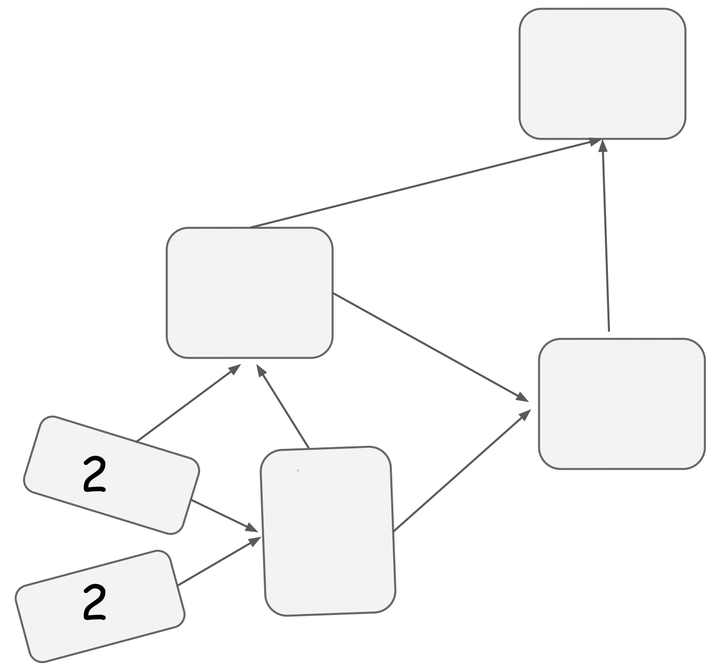
We can make powers with any number. For example:

$$
\begin{aligned}
3^3 * 3^4 &= (3 \times 3 \times 3) * (3 \times 3 \times 3 \times 3) \\
&= (3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3) \\
&= 3^7 \\
&= 3^{3+4}
\end{aligned}
$$



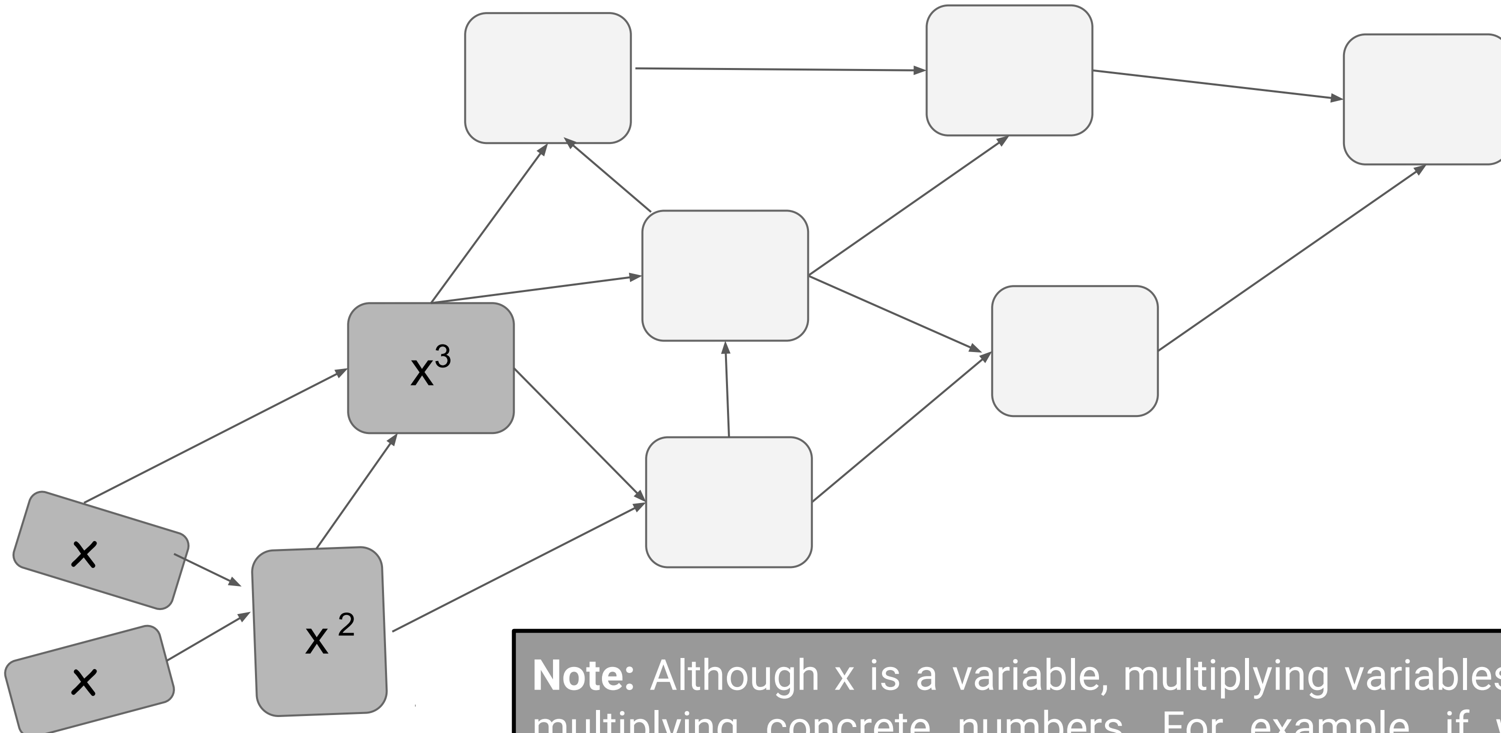
c Can you list the next 3 numbers in this sequence?

d Complete the following sequence:



e Can you list the next 3 numbers in this sequence?

f Recall that if x is any value, then x^2 is equal to x • x (read: "x times x"). Also, x^3 is equal to x • x • x, x^4 is equal to x • x • x • x, etc. Complete the following sequence:



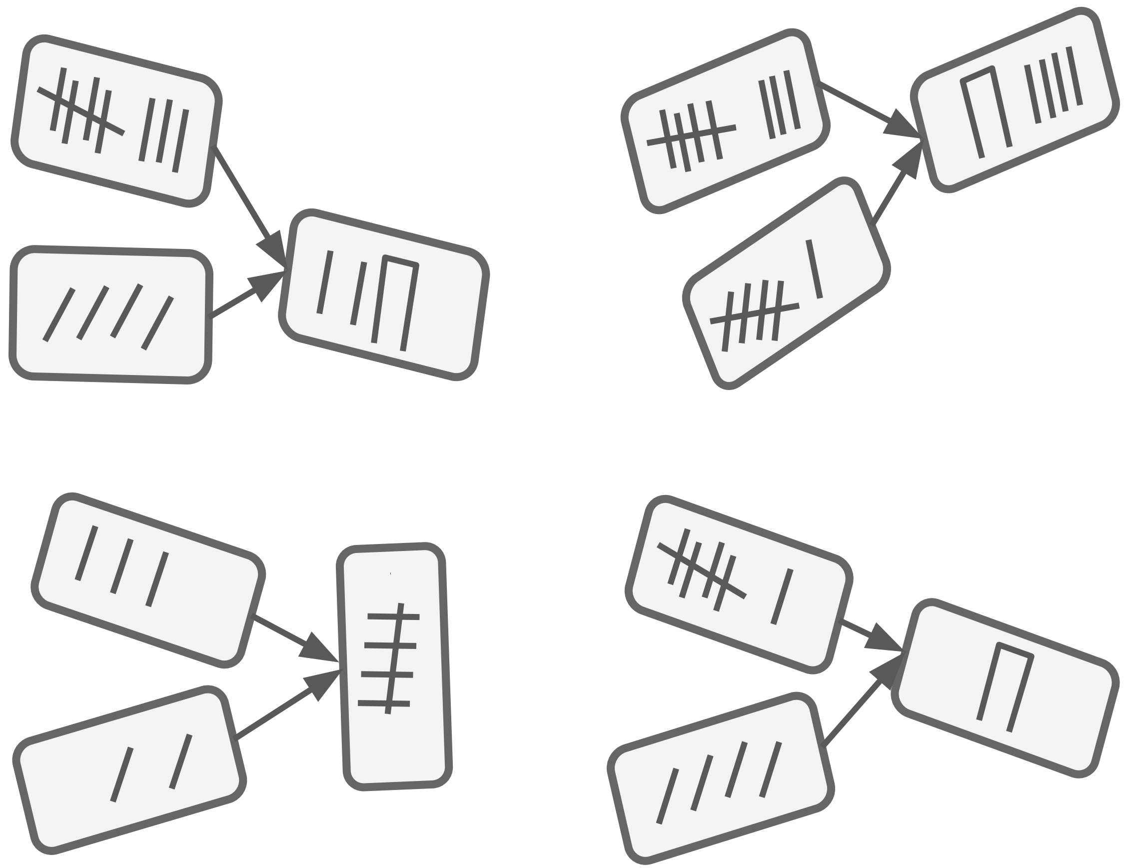
Note: Although x is a variable, multiplying variables is similar to multiplying concrete numbers. For example, if we put an x instead of the 10 in the previous note, we get:

$$x^3 \cdot x^2 = (x \cdot x \cdot x) * (x \cdot x) = (x \cdot x \cdot x \cdot x \cdot x) = x^5 = x^{3+2}$$

In general: $(x^n) \cdot (x^m) = x^{n+m}$

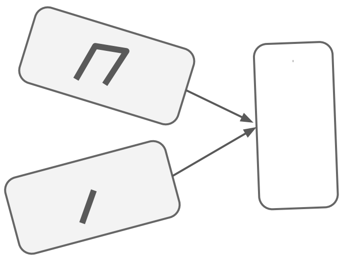
5 Ancient Tablets

Anthropologists have found several tablets from an ancient civilization which stored numeric information.

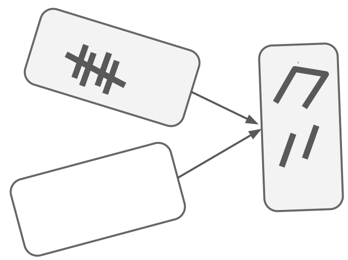


a Analyze the quantities displayed in the drawings. Can you find out which numbers and which operation are represented in these tablets?

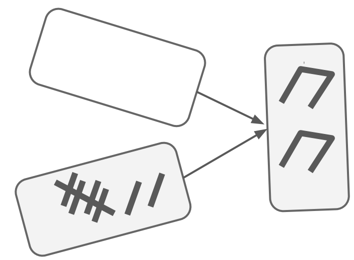
b Anthropologists also found the following set of "broken" tablets. Because the tablets are very old, some of the information was lost over time. Each diagram represents an addition fact (with symbols instead of numbers). Help the anthropologists by filling in the missing information in the broken tablets (drawn in white). **Note:** One of the broken tablets is meant to be empty.



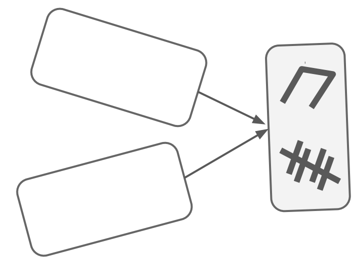
a



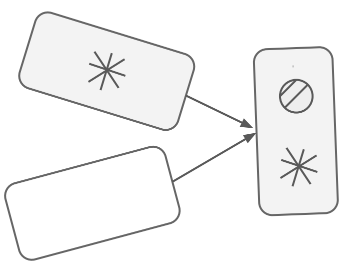
b



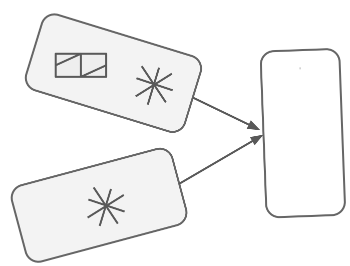
c



d



i



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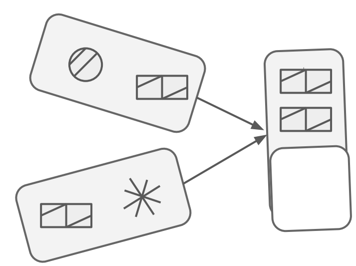
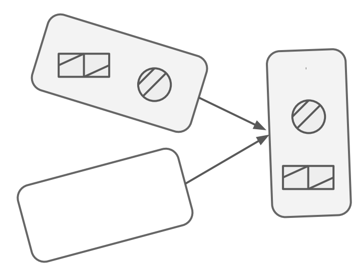
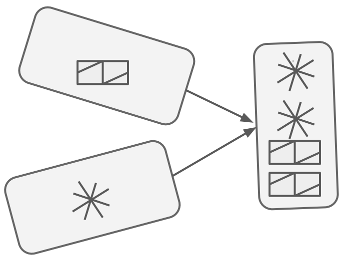


iii

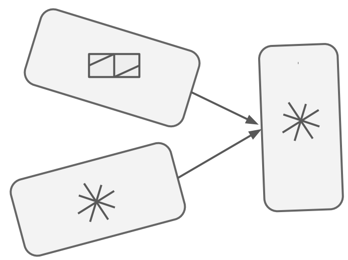


iv

c A new set of tablets was found. This time, each tablet represent a different operation (in other words, not all of them are addition). Match each trio of tablets above with one of the examples below.



1



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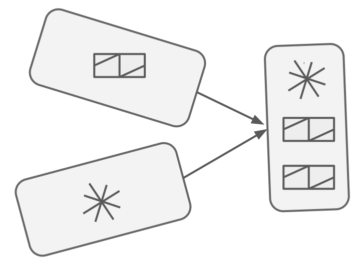

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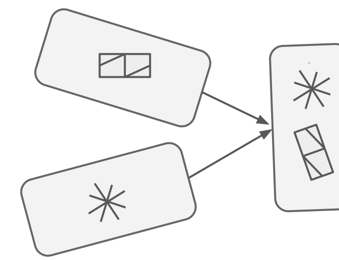

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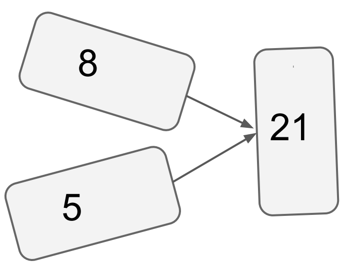
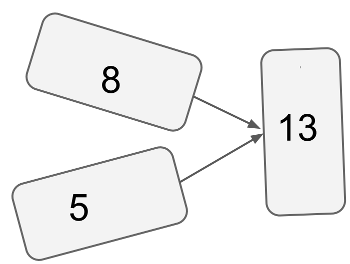
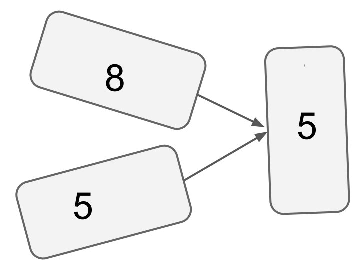


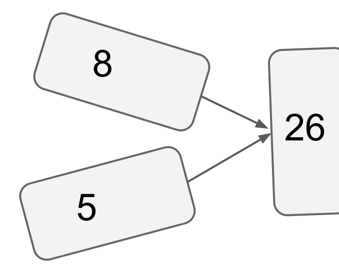
A



B

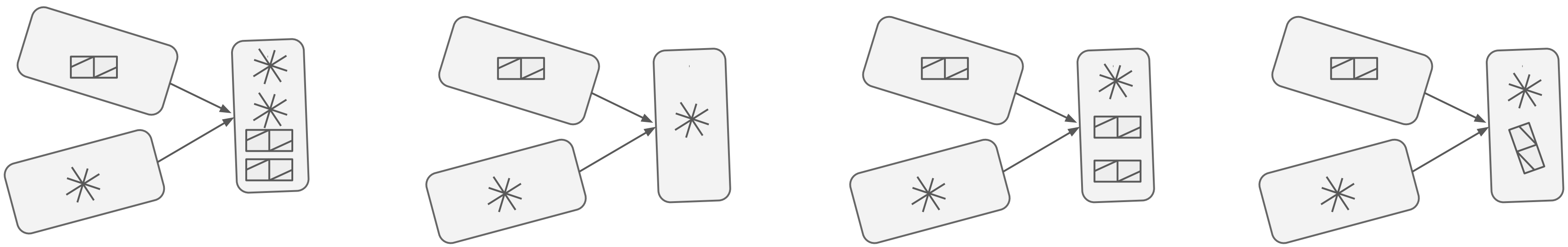


C



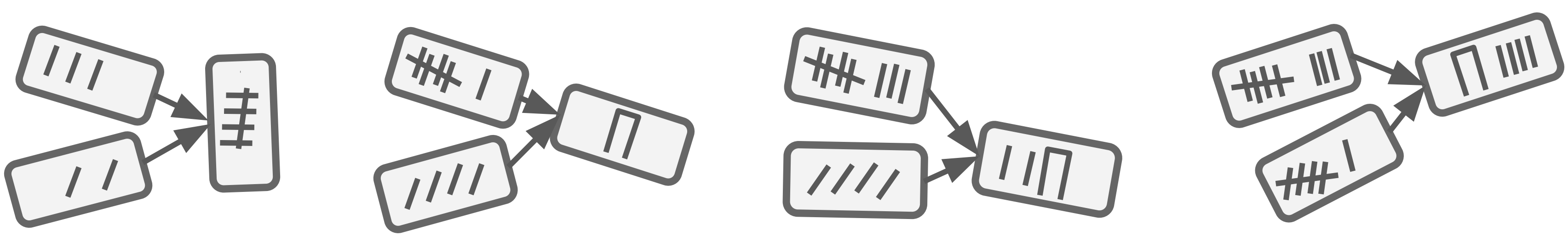
D

d The diagrams 1,2,3,4 below represent 4 different operations.



1	2	3	4

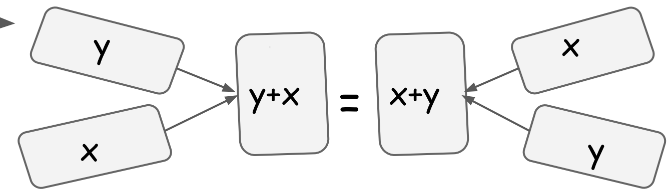
Which of the above operations was used in the ancient tablets discovered by our team of anthropologists?



CHALLENGES

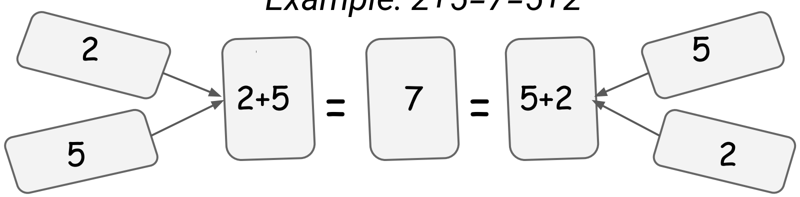
Addition of numbers has a lot of interesting properties.

Commutativity: x + y = y+ x



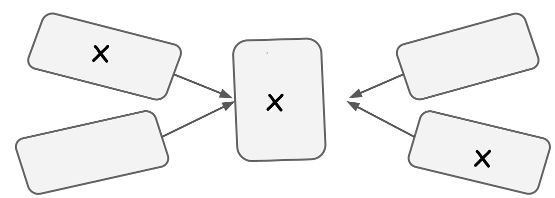
When we add two numbers, the order is not relevant.

Example: 2+5=7=5+2

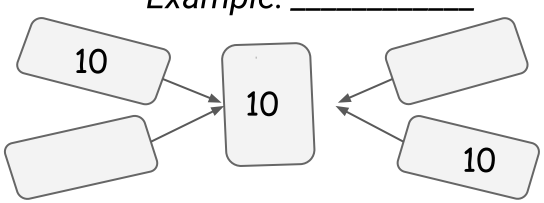


Which property of addition is represented by the diagram on the right?

____



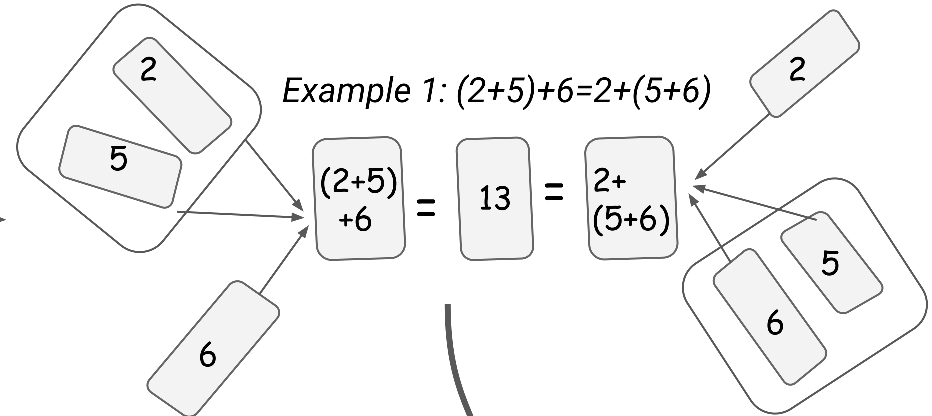
Example: ___________



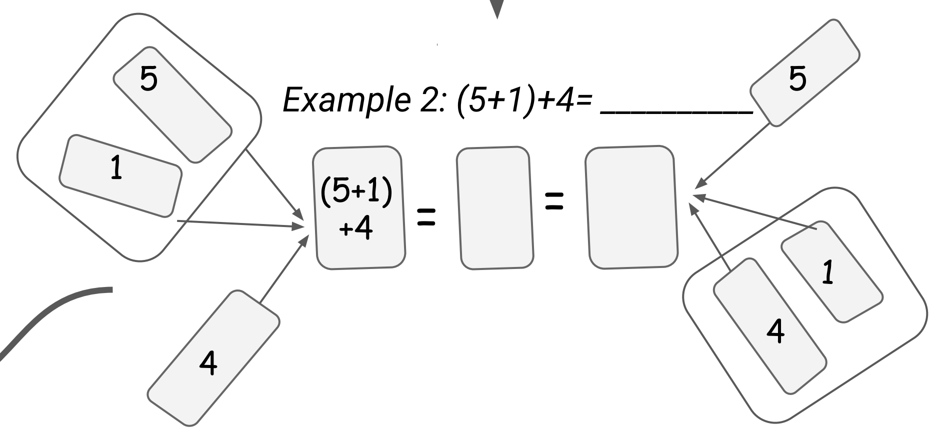
Complete the connected diagrams corresponding to:
Associativity: **(x + y) + z = x + (y + z)**

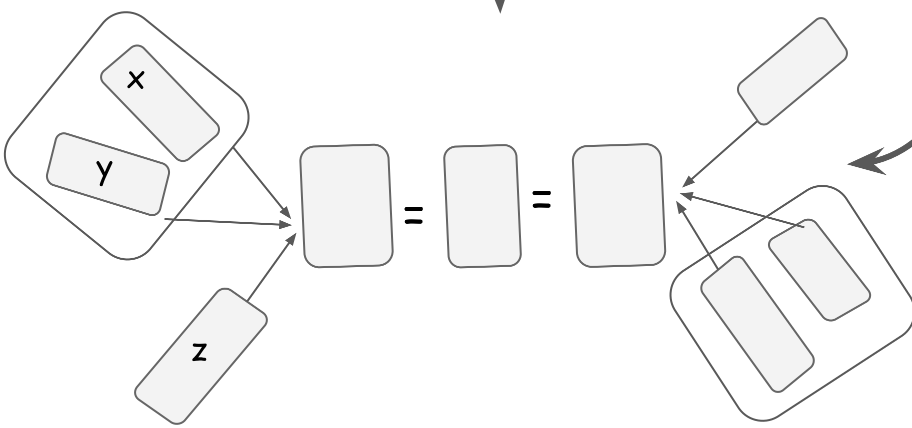
When we add three numbers together, we first have to add two of them, and then the other one. No matter which two numbers we start adding, we will obtain the same result in the end.

Example 1: (2+5)+6=2+(5+6)



Example 2: (5+1)+4= __________





CHALLENGES

Complete the following connected multi-diagrams (using addition):