Meeting 19 Student’s Booklet

Machines

April 20, 2016 @ UCI

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1. Extreme Makeover
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The Extreme Makeover Car Wash has three different stations: Soap, Wash and Dry. Once a car is done at the Soap station, it goes automatically to the Wash station. Similarly, when the car finishes at the Wash station, it is moved directly to the Dry station. After this, the car is released.

Unfortunately, at any given time only one station can be running and each station takes 5 minutes to complete its job.

The following rules also apply:

* There is no limit to the number of cars that can be at each station.
* If a car is in the Wash station, it means that it was already soaped. If a car is in the Dry station, it means that it was already soaped and washed.

The control panel below has three buttons which activate the stations.

For each of the settings presented in the next page, we will find which buttons to press so that all cars are soaped, washed and dried in the least amount of time. Thus, in this problem we seek to minimize the amount of time.

Did you know?... in mathematics, a minimization problem is a problem in which you make a quantity to be as small as possible.
Which method is faster?

There are three cars at the Extreme Makeover Car Wash, one in each station. We propose two methods to clean them.

method 1: press buttons “Soap”, “Wash”, “Dry”, in this order


Compare the times needed to clean the three cars.
**Extreme Makeover**

In which order should we press the buttons to minimize the cleaning time for any given arrangements of cars? If so, please describe your general method or strategy.

Discuss:

These pictures represent different stages of the washing process. In which order should we press the buttons (Soap, Wash, Dry) to complete the job by minimizing the cleaning time?

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<thead>
<tr>
<th>Buttons: ____________</th>
<th>Time: ___ min</th>
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<tbody>
<tr>
<td>Soap</td>
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<tr>
<td>Wash</td>
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Draw cars! Complete the pictures.

**Solutions:**

a) Buttons: P. Time: 5 min.
c) Buttons: W, P. Time: 10 min.

Discussion: The general rule is to look for the first non-empty station from left to right and press the buttons for that station and the ones to the right.
At most 3 cars at each station

We are still at the Extreme Makeover Car Wash (with its three different stations: Soap, Wash and Dry), but now we add the following limitation: **there can be at most 3 cars at each station at any given time.**

Suppose, for example, that there are 2 cars in the Soap station, 3 cars in the Wash station, and 1 car in the Dry station. You should not press the Soap Button to begin, because if you do so, then when trying to move the 2 cars to the Wash station you would not be able to, since it is already full. You should free the Wash station first.

Here is an example of what you could do:

In this example we pressed 5 buttons, so it took 25 minutes to complete the job.
At most 3 cars at each station

If there are 3 cars in the Soap station and 2 cars in the Wash station, and you press the Soap Button, then only 1 car will move from Soap to Wash and the 2 cars that remain in the Soap station will need to be moved later.

Discuss

How long did this take? Is there a better strategy (one that takes less time) to clean all these cars?
For each of the following settings, determine which buttons should be pressed and in which order, so all the cars are soaped, washed and polished in the least amount of time. Remember the new rule: **at most 3 cars in each station.**

**e** Buttons: _______________ Time: ____ min

![Soap] ![Wash] ![Dry]

**f** Buttons: _______________ Time: ____ min

![Soap] ![Wash] ![Dry]

**g** Challenge your friend! Draw two situation and give them to your friends to solve.

Buttons: ____________ Time: ______ min

![Soap] ![Wash] ![Dry]

Buttons: ____________ Time: ______ min

![Soap] ![Wash] ![Dry]
Challenges

For each of the following settings, determine which buttons should be pressed and in which order, so all the cars are soaped, washed and polished in the least amount of time. Remember the new rule: at most 3 cars in each station.

### h

Buttons: ___________  Time: ________ min

### i

Buttons: ___________  Time: ________ min
You start with 1 sloth.

You want to end up with as many sloths as possible! So you go to a sloth machine, which will help you increase your collection of sloths (for a charge).

There are four types of sloth-growing machines:

- ** duplicator (cost: $10): duplicate your sloth collection**
- ** triplicator (cost: $20): triplicate your sloth collection**
- ** +2-ator (cost: $3): add 2 new sloths to your collection**
- ** +5-ator (cost: $10): add 5 sloths to your collection**

Solution:

The first observation is that buying the triplicator is not worth it, since we can buy two duplicators for the price of one triplicator, and two duplicators are equivalent to what would be a fourplicator, which is better than a triplicator.

The next observation is that it is better to spend $9 and buy three +2-ators (equivalent to what would be a +6-ator) than spending $10 to buy a +5-ator.

So the question is how to distribute $70 to buy duplicators and +2-ators. The extreme cases would be:

(a) Buy only duplicators: we would buy seven, and so we would duplicate our collection seven times: 1 > 2 > 4 > 8 > 16 > 32 > 64 > 128.
For example, if you have 1 sloth and use - *in this order* - a +2-ator, a duplicator, a triplicator, a +5-ator and and then again a triplicator, you will now have 69 sloths:

![Diagram showing the process](chart.png)

This will cost you a total of ($3 + $10 + $20 + $10 + $20) = $63.

**How many sloths for that money?**

Start with 1 sloth and use the following sloth machines. How many sloths do you end up with? And how much do you spend?

![Diagram with question mark](chart.png)
$23 dollars

You start with 1 sloth and want to get as many sloths as possible. **You can spend exactly $23.**

For example, you may choose to use, *in order*, a triplicator and a +5-ator, to obtain a total of 8 sloths.

Can you end up with 9 sloths by paying exactly $23? (any number of machines is allowed)

The 3D computer animated film of Walt Disney Zootopia, in which a sloth interprets a police DMV officer, was premiered on February 17, 2016.

**Costs**
- duplicator ........... $10
- triplicator .......... $20
- “ + 2-ator” .......... $3
- “ + 5-ator” .......... $10
**Officer Special requirements**

The officer special requirements are the following: you start with 1 sloth, you must spend **exactly $23** and you must use a duplicator machine exactly once.

*For example*, you may choose to use, *in order*, a duplicator, a +5-ator and a +2-ator, to obtain a total of 9 sloths.

[Diagram of sloth machine usage]

Here is a **non-example**:

[Another diagram of sloth machine usage]

What’s wrong with this procedure?
The officer special requirements:

- you must spend **exactly $23**
- you **must use a duplicator machine exactly once**.

Decide whether the following procedure satisfy the officer’s requirements. If not, explain why not...

- **yes**
- **no, because ________________________**

- **yes**
- **no, because ________________________**

- **yes**
- **no, because ________________________**
The officer special requirements:
- you must spend **exactly** $23
- you **must use a duplicator machine exactly once**.

How many combinations of sloth machines can you use to satisfy the officer requirements? (Draw as many combinations as you can.)

**Costs**
- duplicator........... $10
- triplicator ........... $20
- “+ 2-ator”............. $3
- “+ 5-ator”............. $10

Which combination of sloth machines produces the most sloths?
**Challenge: no more than $70 dollars**

You start with 1 sloth and want to get **as many sloths as possible**.

You must spend no more than $70.

Spend your money wisely. Fill out the table on the side to indicate which machines you choose to use.

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**Costs**

- duplicator .......... $10
- triplicator .......... $20
- " + 2-ator" .......... $3
- " + 5-ator" .......... $10

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1st machine:

2nd machine:

3rd machine:

4th machine

5th machine (if any):

6th machine (if any):

7th machine (if any):

8th machine (if any):

9th machine (if any):

# SLOTHS IN THE END: 8
3 STACKING CUBES

We show a sequence of figures composed of cubes in which the number of cubes grows following a regular pattern.

In the example on this page, we build bigger and bigger rectangular figures by adding one row of cubes to the top and two columns of cubes to the right.

**Labeling our figures**

The first stage shown is just a picture (two stacked cubes), the last data is only numerical (6 x 9). The intermediate pictures display values indicating the number of cubes. The shaded cubes indicate the previous stage. The arrows are tagged with the change in number of cubes from one stage to another.

We use a **DO NOT COMPUTE** approach for indicating the number of cubes: for example, in the last stage we write 6 x 9 instead of 54, to count the number of cubes. This helps us find patterns.
Fun facts about cubes

There are many ways to draw a cube, you can practice with the following constructions:

- Draw a square
- Draw another square
- Join the corners

Another way:

- Draw an hexagon
- Use two sides to draw a diamond
- Add one more line to generate the cube (and two more diamonds)
a. Complete all the missing information:
   - Label the pictures and the arrows, as appropriate.
   - Draw the missing picture.

Describe the pattern:

(number of cubes)...

(3x2) + 1x ___

number of cubes...

(3x2) + 1x ___

(3x2) + 1x ___

(3x2) + 1x10
Complete all the missing information, and color each picture (by shading a copy of the previous figure).

Describe the pattern:

number of cubes...

number of cubes...

number of cubes...

number of cubes...

number of cubes...

number of cubes...
Challenge your friend!

Make your own design with cubes and ask your neighbor to find the pattern.

Describe the pattern:
Complete all the missing information, and color each picture (by shading a copy of the previous figure).

Describe the pattern:
These “nails” are built by adding cubes in a regular pattern...

**Challenge**

- **Label the pictures** with a tag indicating the number of cubes.
- **Label the arrows** with a tag indicating the change in number of cubes.

Describe the pattern:
Express the number of cubes, in terms of $N$: