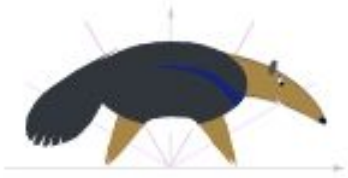


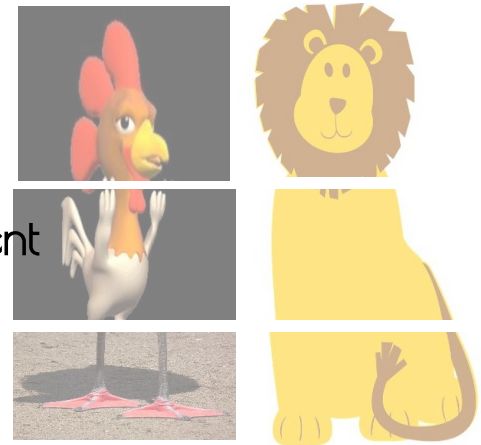
UC IRVINE MATH CEO

Community Educational Outreach

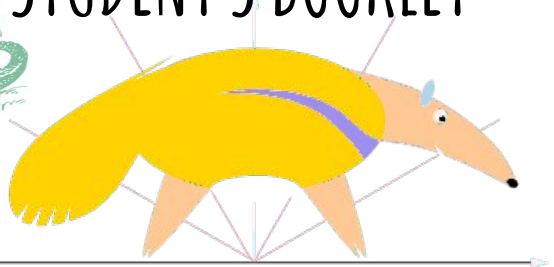
Meeting 20 Student's Booklet

Shapes, Bees and Balloons

April 27 2016 @ UCI



STUDENT'S BOOKLET



Contents

UC IRVINE MATH CEO
http://www.math.uci.edu/mathceo/

1 A Shape Experiment

Choose an integer n strictly between 9 and 12. n = ☐

In a blank page, draw n shapes, following orientations 1,2 and 3 below (read them well before you begin):

1. Make sure that most of the shapes that you draw are RED (*most* means "more than half").

2. Make sure that most of the shapes that you draw are HEXAGONS (*most* means "more than half").

3. Try to **minimize** the number of RED HEXAGONS. This means drawing as few as possible red hexagons (zero if you can, or if not, the least possible number).

After you are done drawing:

How many red hexagons did you draw? ☐

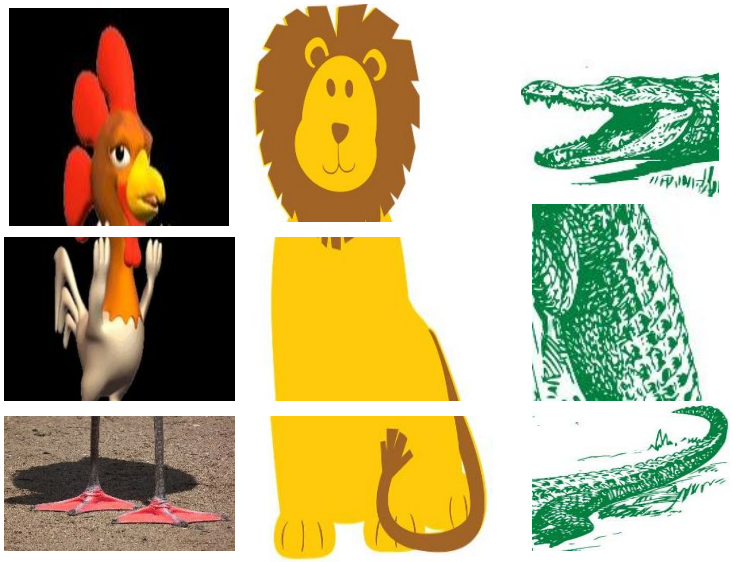
Do you think that you could have drawn less red hexagons (while still satisfying 2 and 3)?

Is is possible to draw no red hexagons at all? Explain.

Do you think that if you repeat the exercise with a different value of n, you can manage to draw no red hexagons?

2 Cosplay

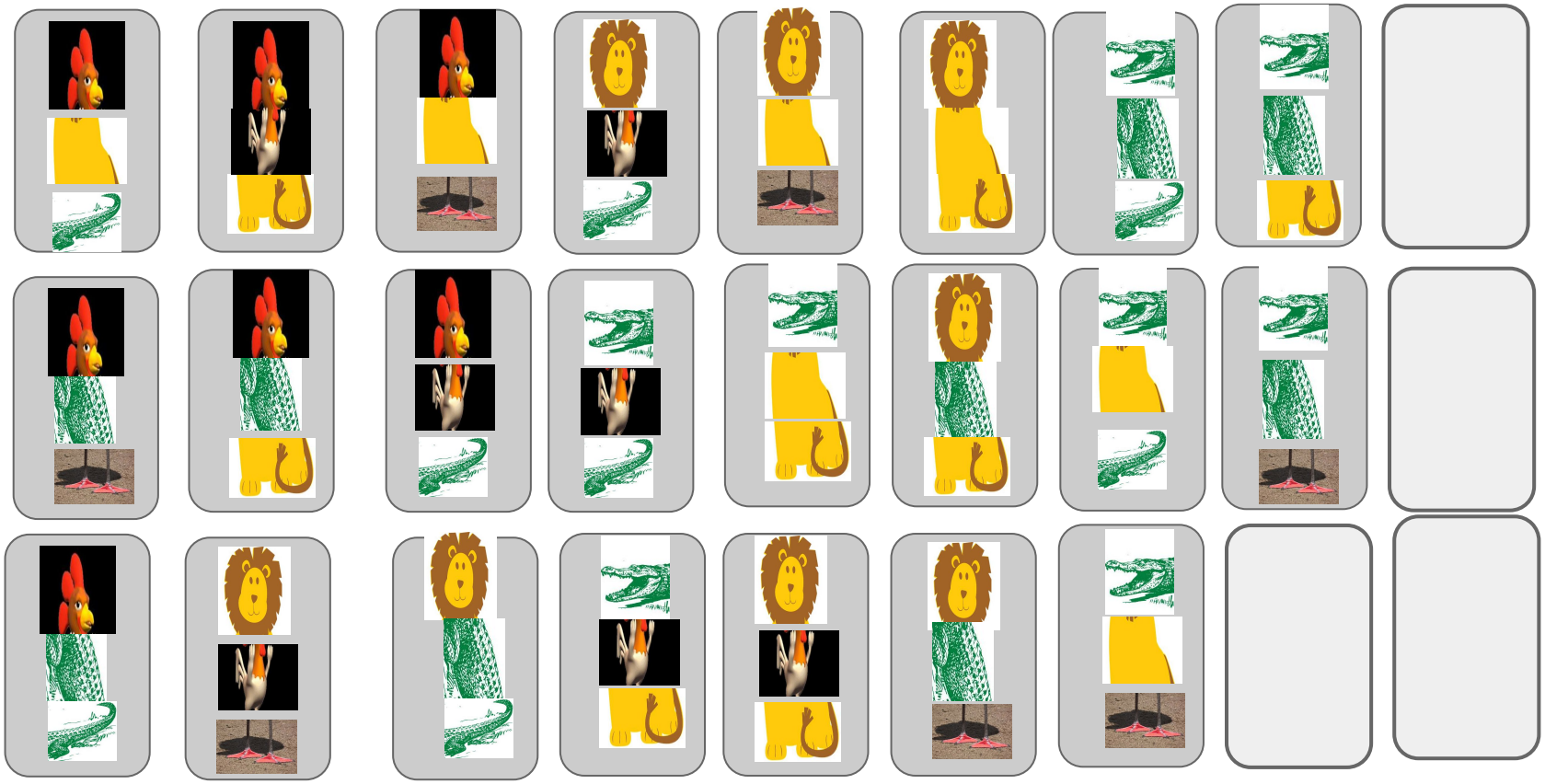
Andy is choosing his costume for a weekend-party. He loves three costumes: the alligator, the lion and the chicken. Undecided on which one to pick, he decides to cut each costume in three pieces and mix them up. He needs to wear one head, one body, and one pair of legs, but they may come from different animals.



a Draw two different costumes. Each costume must use one part of each animal.

b How many different costumes can you create using one part of a chicken and two parts of an alligator? Draw your costumes below.

c There are 27 possible ways to create a costume combining the lion, chicken and alligator pieces. You find 23 of them below: can you draw the remaining 4?



Discuss with your friends

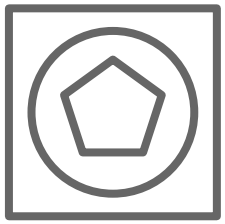
d Can you explain why there are a total 27 costumes without having to draw all of them?

e If you randomly select one of the 27 costumes from part (c), what is more likely:

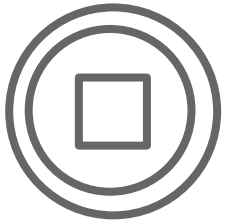
Select a costume with a part of each animal **or** *Select a costume where two parts of the same animal*

3 TRINITIES

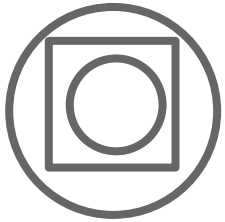
A *Trinity* is a geometrical arrangement of three shapes, not necessarily different, one trapped inside another. Here are some examples of Trinities:



In this trinity: a pentagon is the internal shape, a circle is the middle shape and a square is the external shape.



In this trinity: a square is the internal shape, a circle is the middle shape and a circle is the external shape.

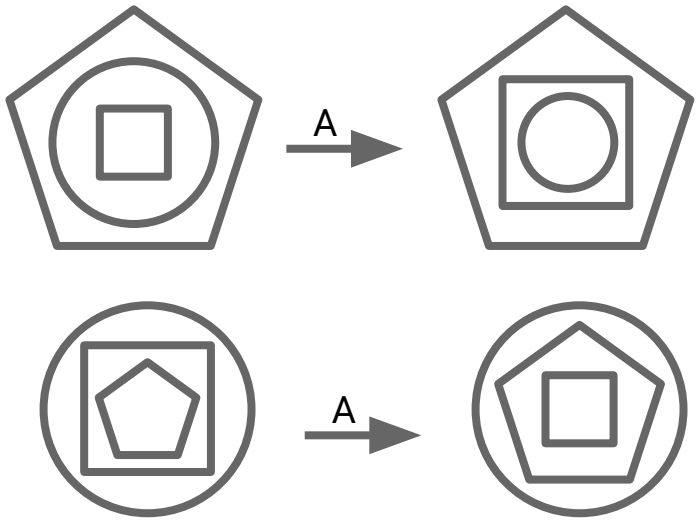


This trinity is different from the previous one, because even though it uses the same shapes (square, circle and circle), the positioning is different.

Trinity Transformations

We introduce two transformations, which we call *type A* and *type B*. These transformations turn a trinity into a new trinity, by switching the position of (some of) the shapes within the trinity.

Here are some examples of what a *type A* transformation does:

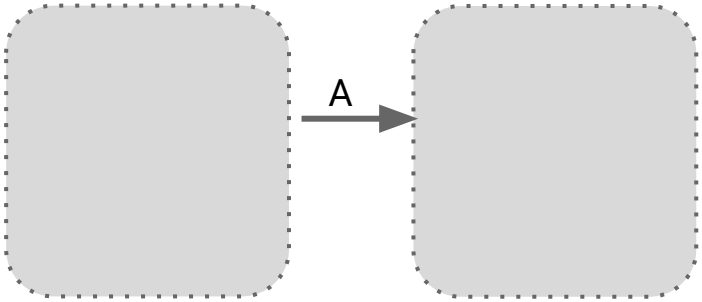


As you can see, **a *type A* transformation always keeps the outer shape fixed and switches the inner and middle shapes.**

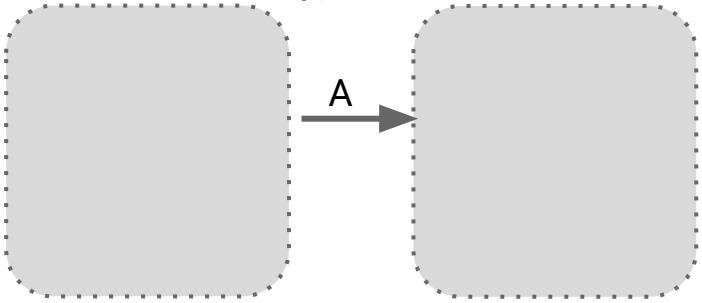
Type A Transformations

a Draw your own example of *type A* transformation.

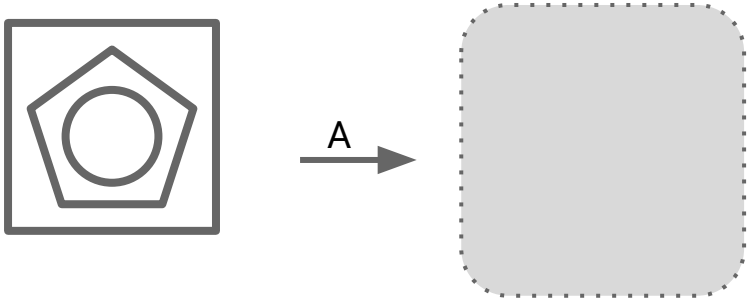
(Recall the "type A rule": keep the outer shape fixed and switches the other two shapes.)

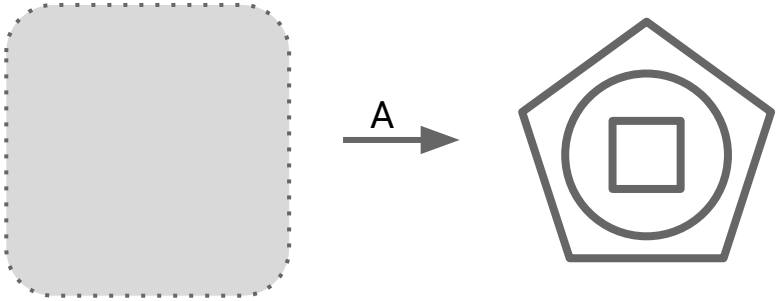


Can you draw of a trinity that does **not change (look the same)** under the transformation *type A* ?



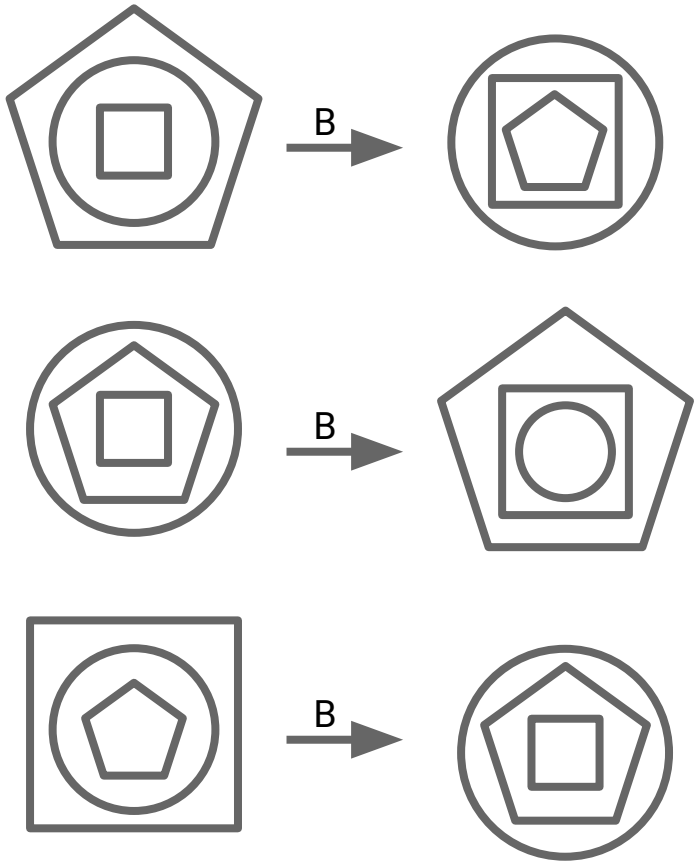
b Complete the following pictures with the corresponding trinities before or after the transformation A:



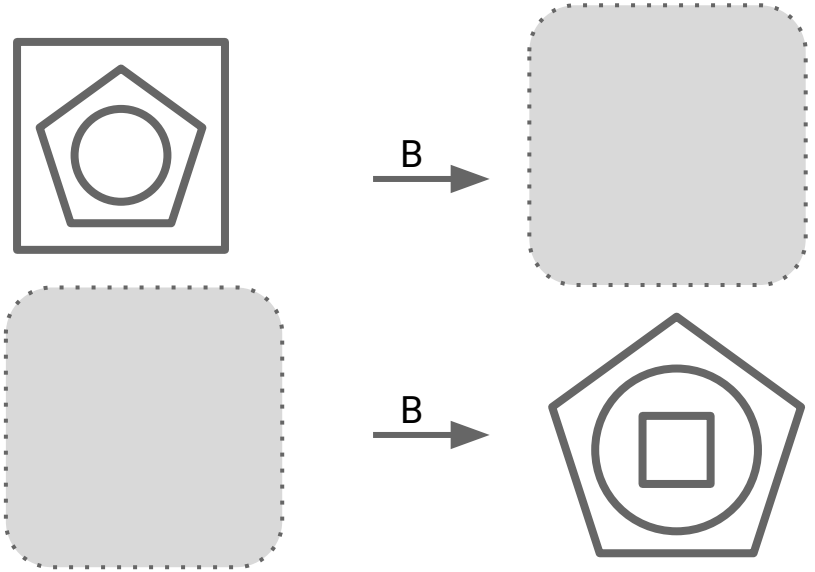


Type B Transformations

Now we introduce a *type B* transformation by a series of examples. **Your job is to figure out what a *type B* transformation does.**



c Complete the following pictures with the corresponding trinities before or after the *type B* transformation:

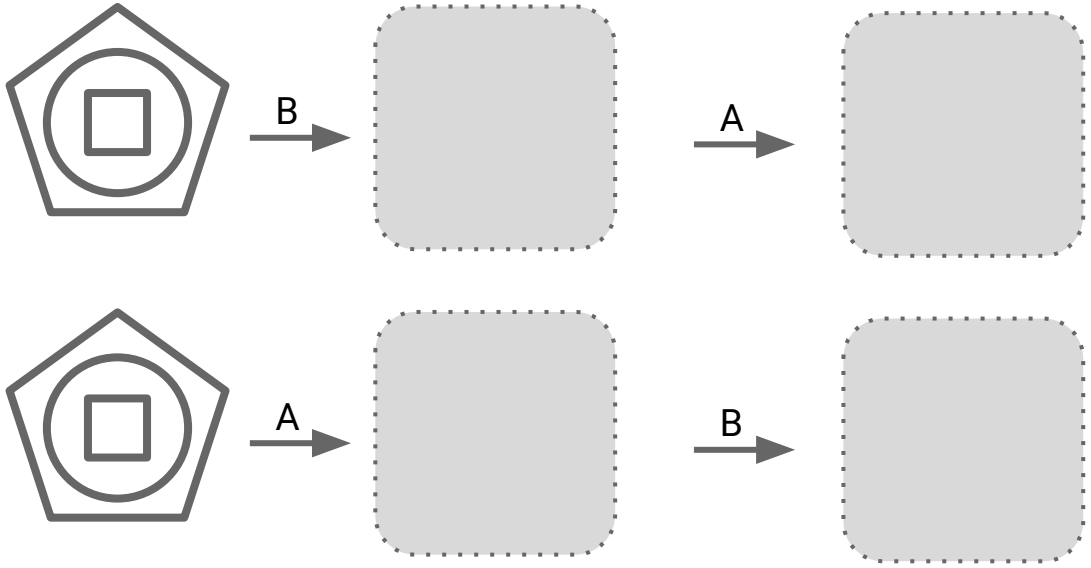


d Can you describe in few words how does a *type B* transformation work in general? What does it do?

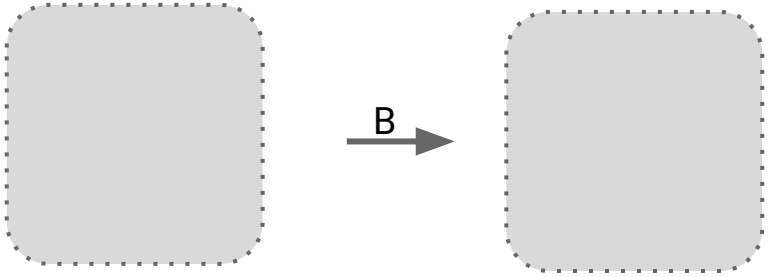
CHALLENGE (Booklet)

(i) In each of the two diagrams, apply the transformations A and B in the corresponding order (in the first picture, apply B first, then A. In the second diagram, apply A first, then B). Note that in both cases we are starting with the same trinity.

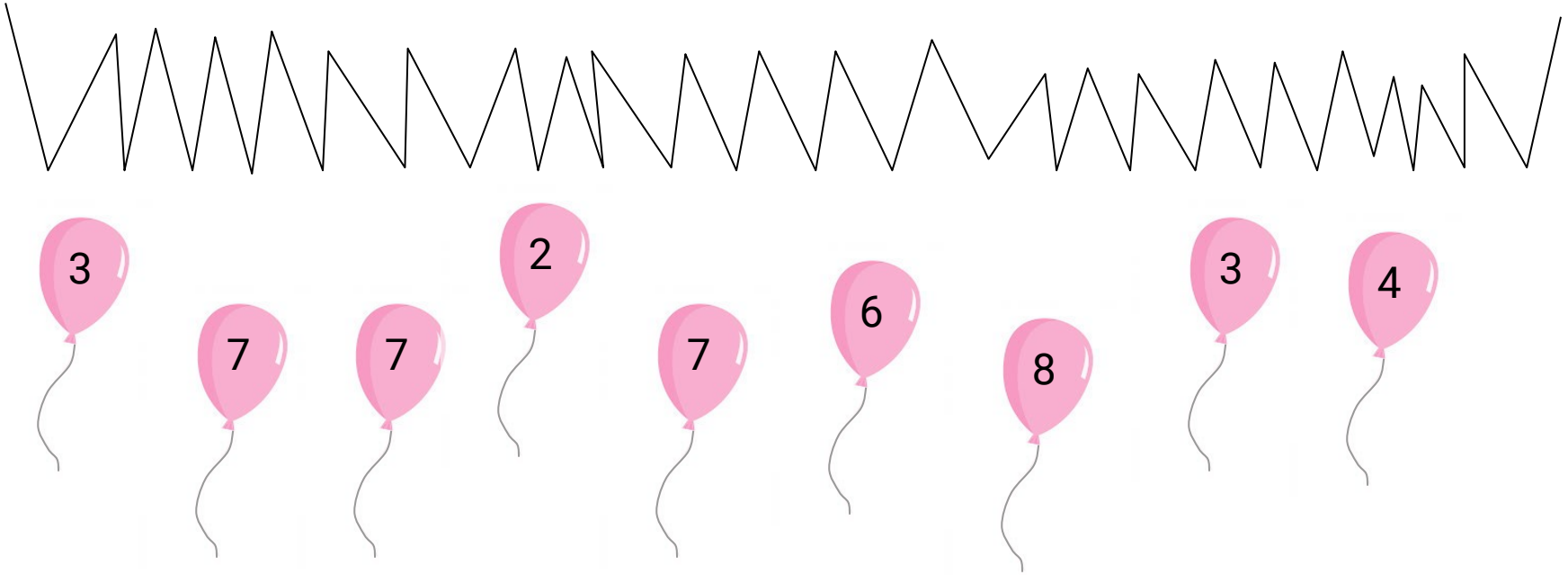
Did you arrive to the same trinity? In other words, **does the order of the transformations matter**?



(ii) Can you draw a trinity which is **left unchanged** by a *type B* transformation? This means that the trinities before and after the transformation are equal.

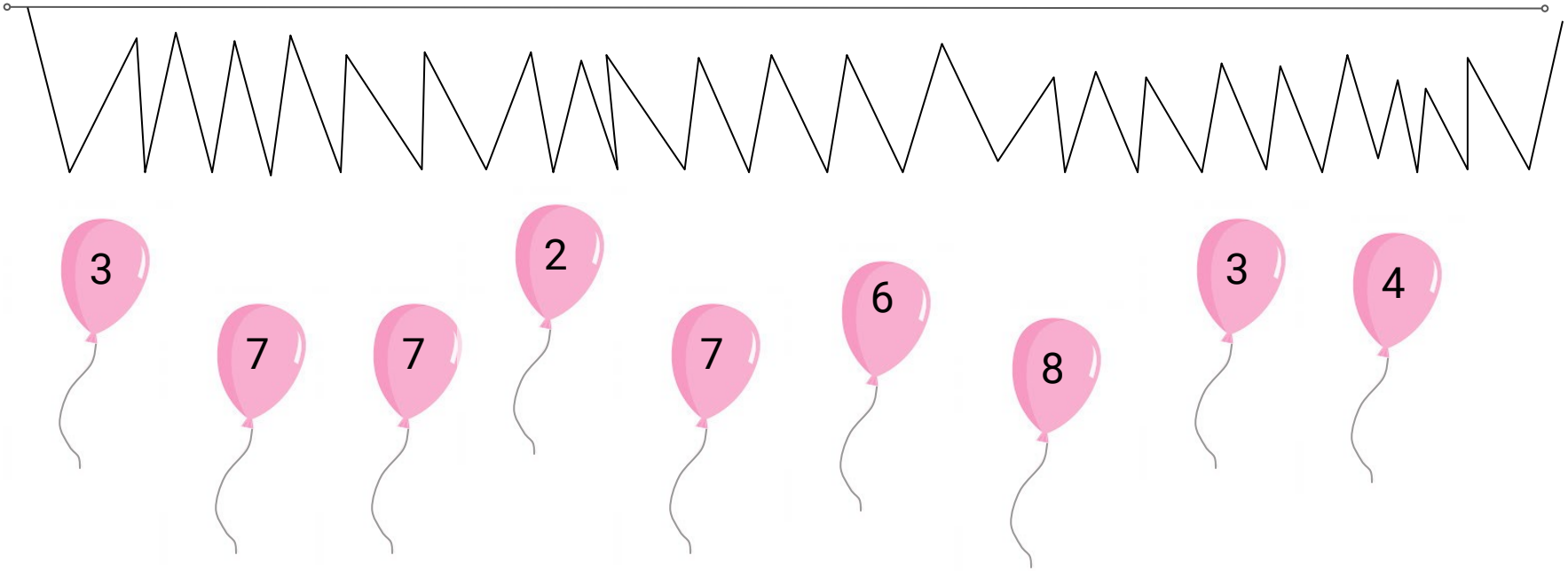


4 BALLOONS



The picture shows **nine balloons** which are flying up and are about to hit a rough ceiling, and pop. Each balloon has a number that indicates **in how many seconds the balloon will pop**. For example, the balloon on the left will pop exactly 3 seconds after the picture was taken. The fourth balloon from the left is higher, so it will pop sooner (in 2 seconds).

We would like to **summarize the events** and answer some questions about the balloons. Namely: (a) How many seconds will pass between the first and last pop? (b) What will be the “loudest” moment? (c) What will be the first moment when more than half of the balloons have popped?



a We define the **range** to be the number of seconds between the first and the last pop.

What is the range? ____

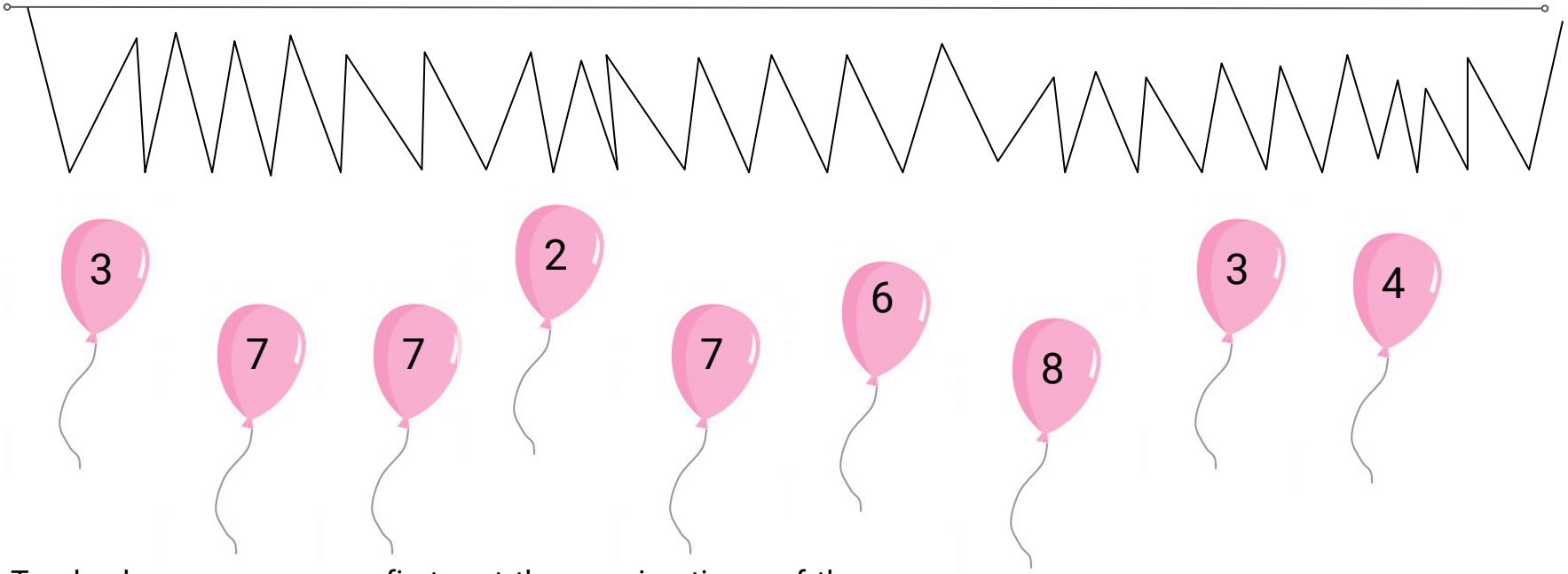
b We define the **mode** to be the time or times where most balloons popped.

What is the mode? ____

c We define the **median** to be the first moment, in seconds, where more than half of the balloons have popped. So the median is that time where we were half-through the process of popping all the balloons.

What is the median? ____

To check your answer or get tips about answering these questions, just move on to the next page.



To check your answer, we first sort the popping times of the nine balloons in increasing order, to make things easier:

2	3	3	4	6	7	7	7	8

We see that the popping time varies between 2 and 8 seconds, so 6 seconds go by between the first and the last pop. Notice that 6 = 8 - 2. We call 6 "the **range".** If we made a movie from the first to the last pop, it would last 6 seconds.

The "**mode"** is the loudest time, that is, the time in which more balloons popped than each other time. It was the "most popular moment".

To find the mode, we go through the list and find the number which repeats the most. In our example, the mode is 7.

The "**median"** is the middle number in our list. Once we list the popping times in increasing order, we see that the middle number is 6. Indeed, 6 is sitting in the middle of our list. Note how you have 4 numbers greater than (or equal to) 6 in the list, and 4 numbers that are smaller than (or equal to) 6.

Buying balloons

Suppose that we bought nine balloons at the following prices (some were really expensive):

$10	$2	$11.5	$5	$9	$2	$1	$12.5	$10

The first step that we will take in order to analyze the data, is to sort it in increasing order:

$1	$2	$2	$5	$9	$10	$10	$11.5	$12.5

d We define the **range** to be the price difference between the most expensive balloon and the cheapest balloon.

What is the range?

e We define the **mode** to be the most common cost (or costs) of the balloons.

What is the mode?

f If we imagine that all nine balloons had the same cost and that we bought them all by paying the same total that we actually paid, what would be that imaginary price?

We define this imaginary price to be the **mean** (or **average)**.

Collecting numbers

Ask one friend at your table for numbers between 1 and 6, to fill up the blank spaces of the grid (in **increasing order,** repeating values is OK):

	2			4			5	

These **nine** numbers may represent many things in the real world, for example ages, prices or times.

g We define the **range** of our data to be the difference between the largest number and the smallest number.

What is the range? ☐

h We define the **mode** of our data to be the number (or numbers) that repeat the most. What is the mode? ☐

i We define the **median** of our data to be the number that appears in the middle of the increasing list. What is the median? ☐

j Create a list of nine numbers *all equal to each other*, such that the sum of these seven numbers is equal to the sum of the numbers in our original list above .

☐☐☐☐☐☐☐☐☐

The repeated number that appears in this list is called the **mean** or **average** of our original data.

k If we have the three numbers 1, 2 and 9, the mean is 4, because 1 + 2 + 9 = 4 + 4 + 4.

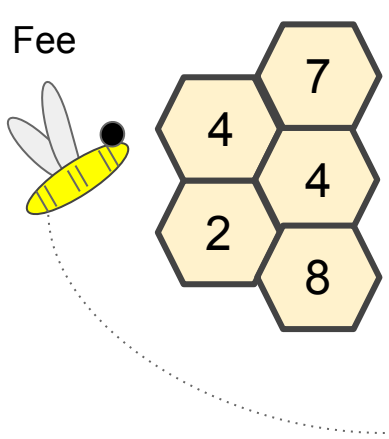
For the data consisting on the numbers 4, 7, 10, what is the mean? ☐

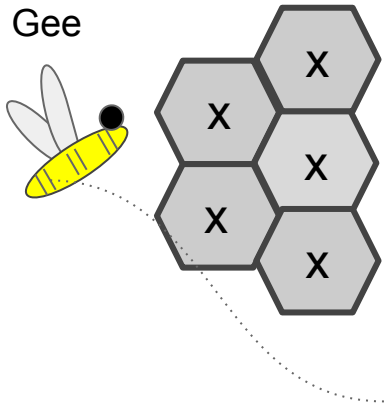
5 TWO BEES AND A HUNGRY CATERPILLAR

Two Bees...

a Gee tells Fee: "All five numbers in my beehive are equal. The sum of my five numbers is equal to the sum of your five numbers". Find the numbers in Gee's beehive.

What is the **mean** of Fee's beehive?

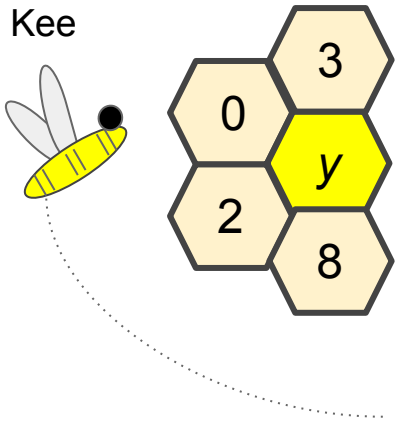


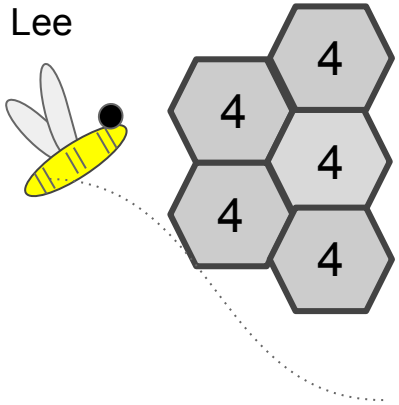


b Kee tells Lee: "The sum of the five numbers of my beehive is equal to the sum of the five numbers in yours".

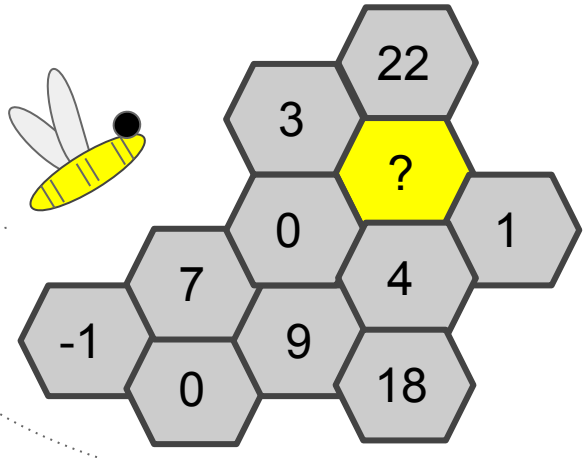
What is the **mean** of Kee's beehive?

Find the mysterious number in Kee's beehive (y).

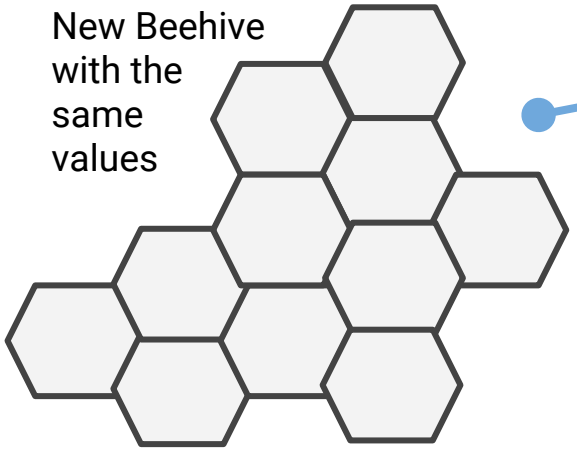




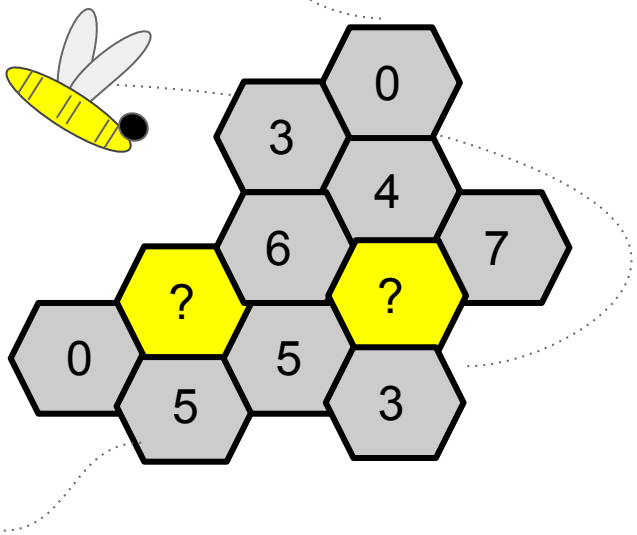
c A beehive of eleven values is shown. Find the value of the missing hexagon so that the **mean** of the beehive is equal to 7. Verify that you got the right answer by building a new beehive of 11 equal values (all equal to the number you got) so that the sum of the values in the first beehive equals to the sum of the values on the second beehive.



New Beehive with the same values



d Find the value of the missing hexagons so that the **mode** of the beehive shown is equal to 4.

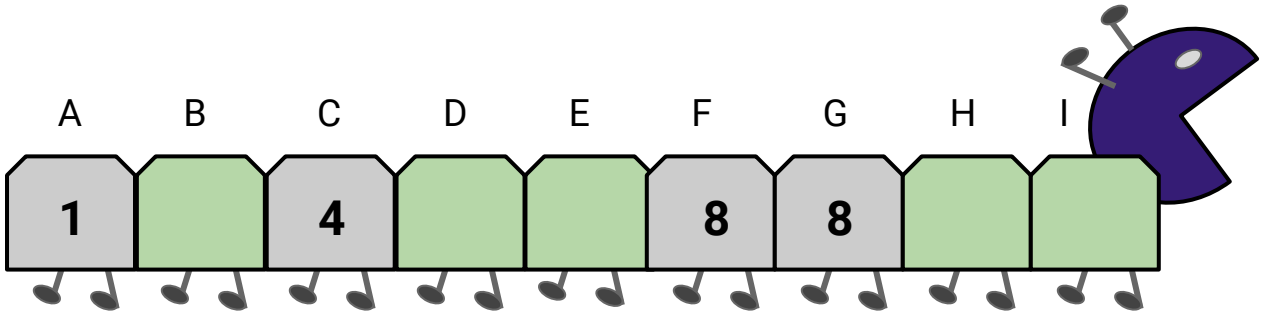


A very hungry caterpillar

A caterpillar has nine numbers listed in **increasing order** from left to right. We can only see four of them (the ones in positions A, C, F and G), the other five are hidden.

Find the missing five numbers so that:

- The range is equal to 19.
- The median is equal to 5.
- The mode is equal to 4.
- The mean is equal to 7.



RANGE:
The value that you need to add to the smallest number to obtain the largest number.
In other words, if you subtract the smallest number from the largest number, you obtain the range.

MEDIAN:
The number that appears **in the middle** of the caterpillar

MODE:
The number or numbers that **most often** appear in the caterpillar

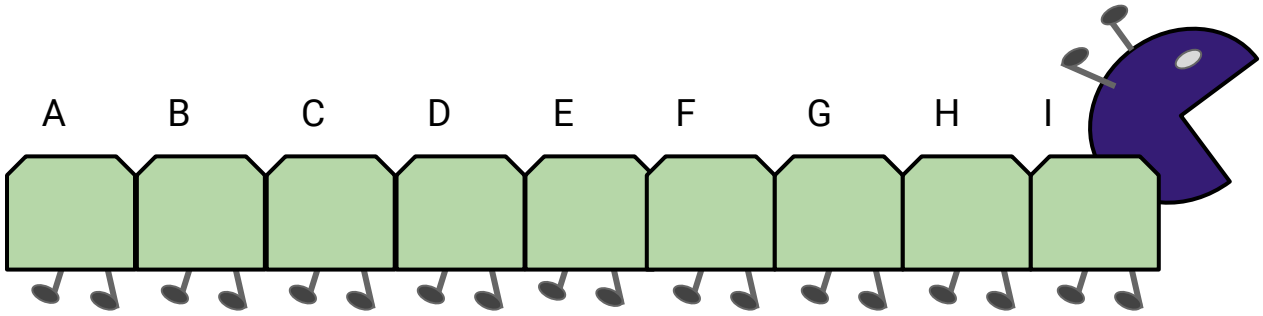
MEAN:
If you add all nine numbers and divide the result by nine, you obtain the mean

... and one more very hungry caterpillar

Fill out the caterpillar's body using numbers 1, 5, 8 (and possibly other numbers of your choice), so that

- The mode is equal to 7.
- The median is equal to 5.
- The mean is equal to 5.
- The range is equal to 7

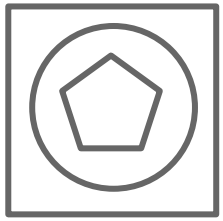
Make sure to list your numbers in increasing order.



CHALLENGE(Trinities)

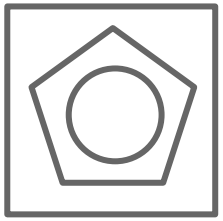
Can you transform trinity 1 into trinity 2 by a sequence of transformations of types A and B? (Use as many transformations as you want and in any order. You may repeat transformations if needed). **Indicate the transformations and the order in which you apply them**.

1



2

Similarly as above, can you transform trinity 3 into trinity 4?



3

4