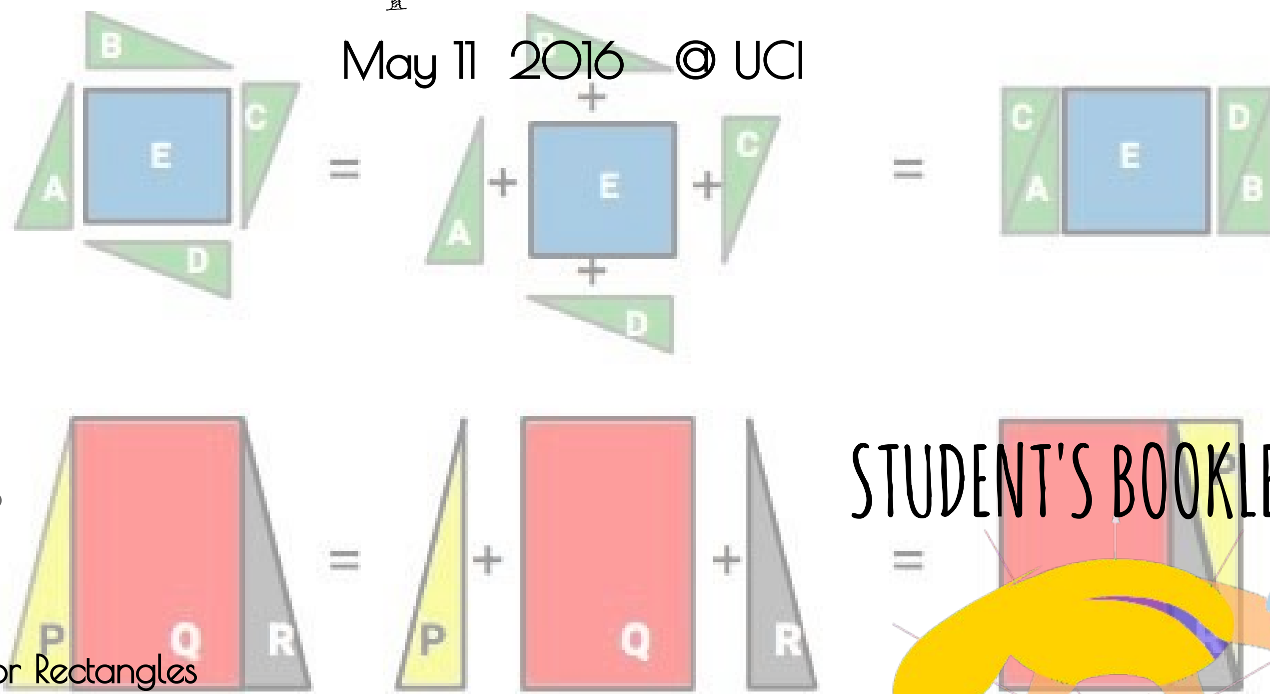
UC IRVINE MATH CEO

Community Educational Outreach

Meeting 22 Student's Booklet

Explorations in 2D

May 11 2016 @ UCI



STUDENT'S BOOKLET

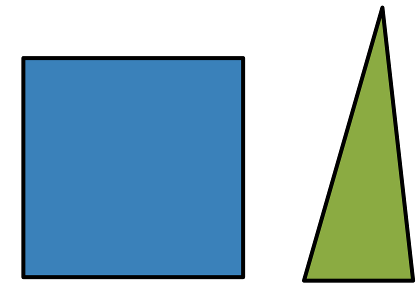
Contents

UC IRVINE MATH CEO
http://www.math.uci.edu/mathceo/

1 Puzzles

Shapes A, B and C are made of the following two elements (shown to the right): a specific square and a specific triangle.

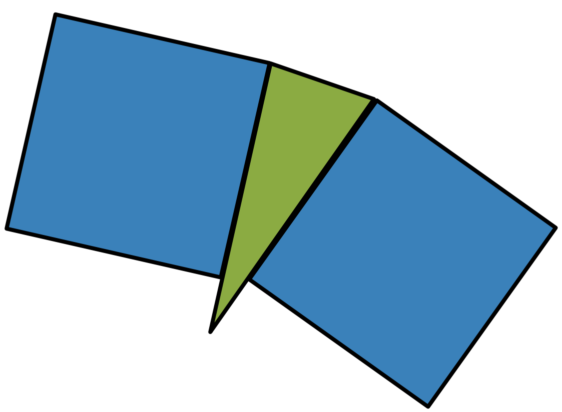
We know that the triangle has an area of 5 square units. We also know that the area of figure A below is equal to 37 square units. We are asked to find the area of figure B. Can you try? See the solution in the next page to check your answer.



Area of square is not given

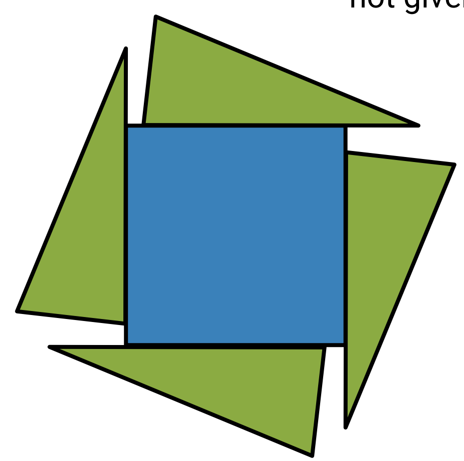
Area of triangle: 5 sq. units

A



Area: 37 sq. units

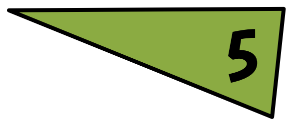
B



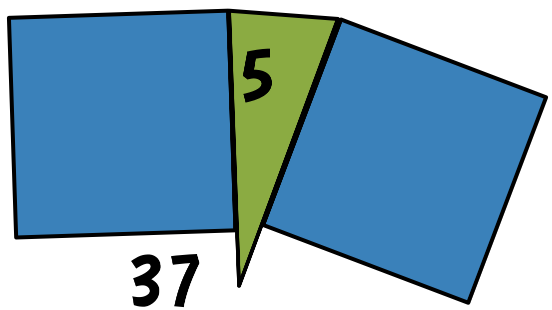
Area: ____ sq. units

Solution

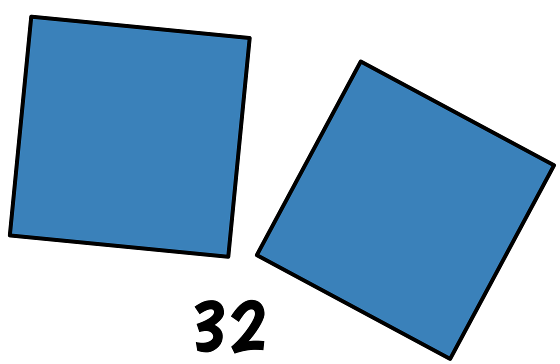
The area of the triangle is 5.



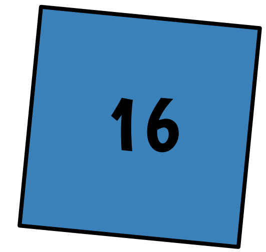
The area of the "bat" is 37:



If we remove the triangle from the bat, we can find the area of the wings: 37 - 5 = 32.

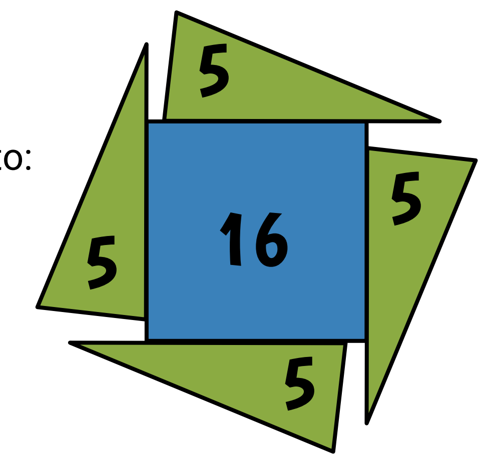


So the area of a single square must be 32/2 = 16.

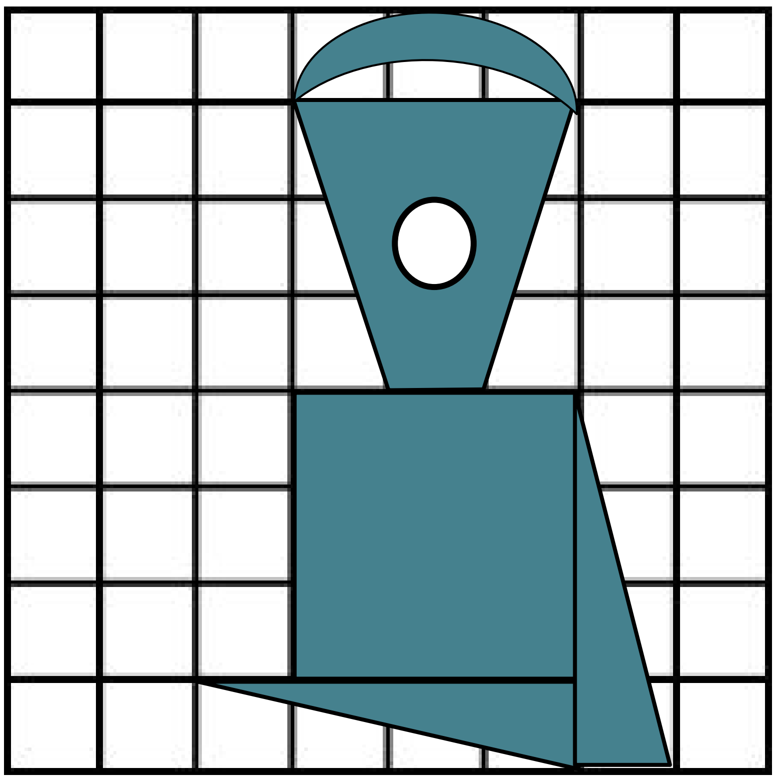


Therefore, the area of figure B must be equal to:

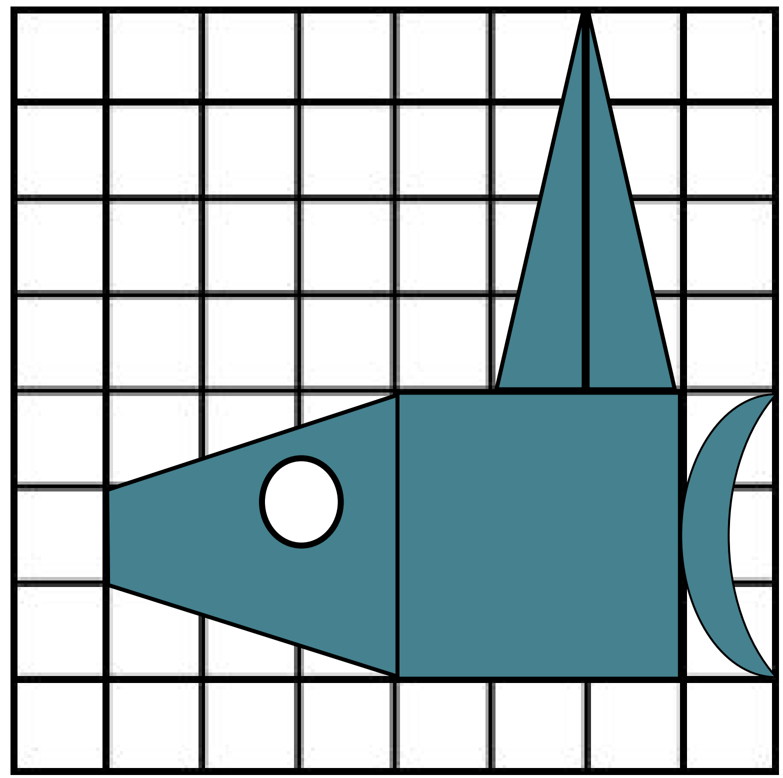
  16 + 4x5
= 16 + 20
= **36**



a We know that figure A has an area of 20 square units. USING THIS INFORMATION, find the area of figure B.

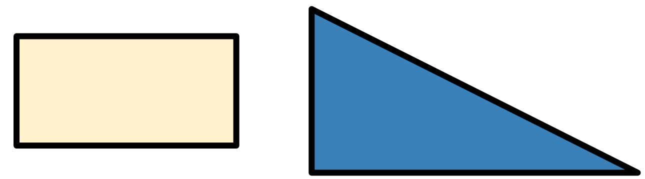


A Area = 20

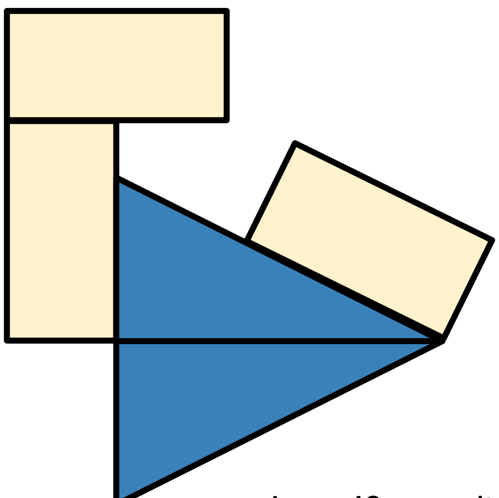


B Area =

b Figures A, B, and C are made using the following two elements (shown to the right) as pieces: a specific rectangle and a specific right triangle. We know the areas of Figures A and B, as indicated below. Based on that, can you find the area of figure C?

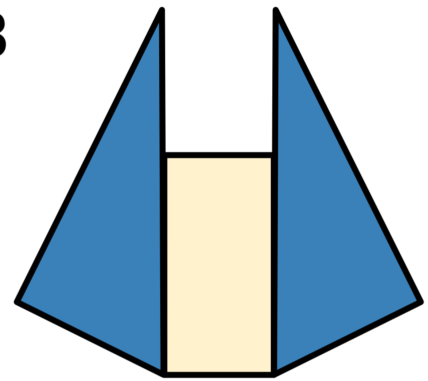


A



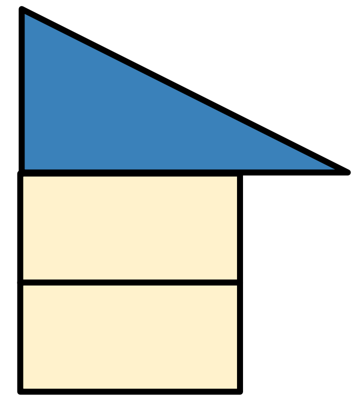
Area: 42 sq. units

B



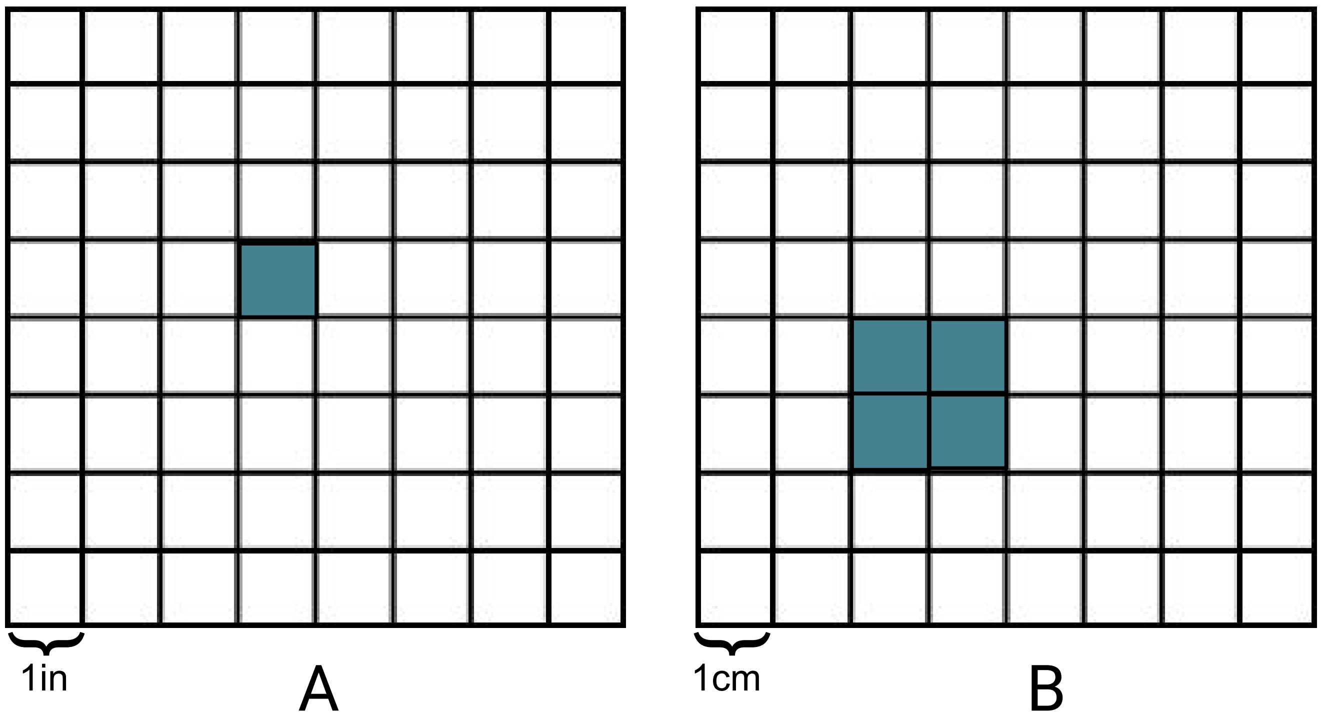
Area: 26 sq. units

C



Area: ______ sq. units

c The two grids use different scales. Which shaded area is bigger?

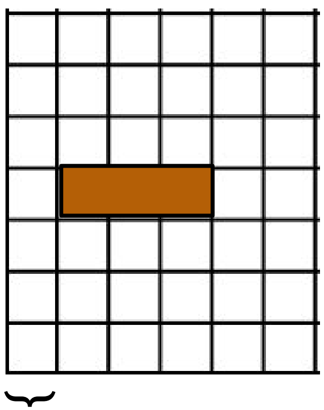


Remember:
1 in = 2.54 cm

Different Scales

We now introduce different scales, and look at how the areas and perimeters of the pictures change.

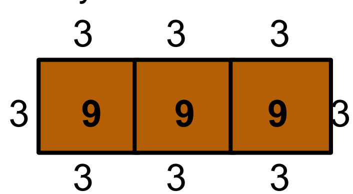
EXAMPLE



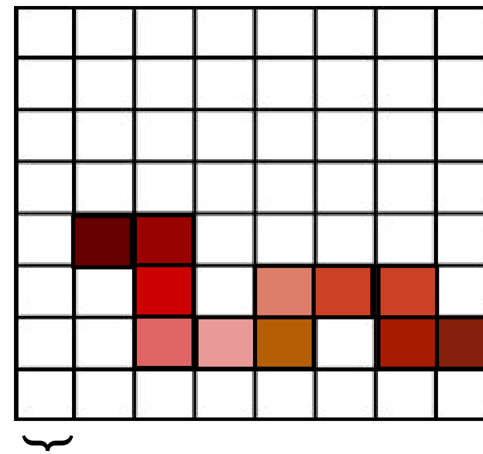
3 cm

In this scale, a unit represents 3 cm. This means that the length of each small square (□) is 3 cm.
The shaded 3 x 1 rectangle has sides of length 9 cm and 3 cm, so it has an area of 9 x 3 = 27 cm squared.
The perimeter of the rectangle is (9+3+9+3) = 24 cm.

This becomes very clear when we label the picture:



d The puzzle below shows a *MATH BUG*. The puzzle box says that every puzzle piece is 5 cm long.



5 cm

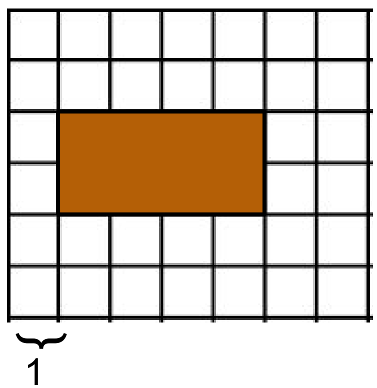
Find the area and perimeter of the actual *MATH BUG*.

Area of MATH BUG

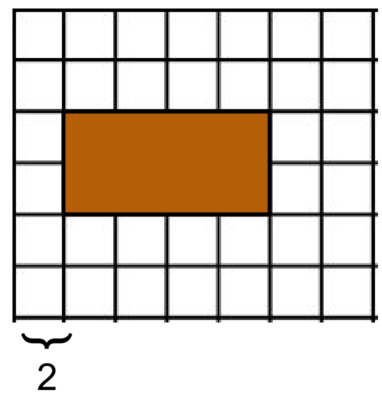
Perimeter of MATH BUG

e The three pictures below represent a 2 x 4 rectangle in three different scales.

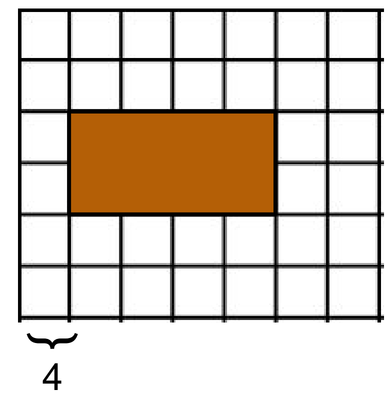
UNIT = 1 in



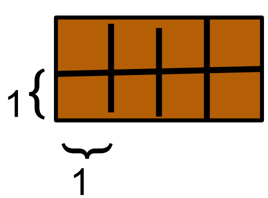
UNIT = 2 in



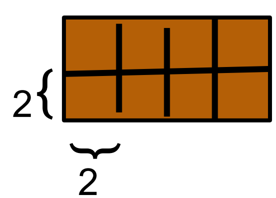
UNIT = 4 in



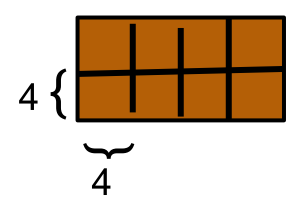
When we change scale, do the **area** and the **perimeter** of the rectangle change in the same way?
Label the boxes, and find out.



Area:
Perimeter:



Area:
Perimeter:



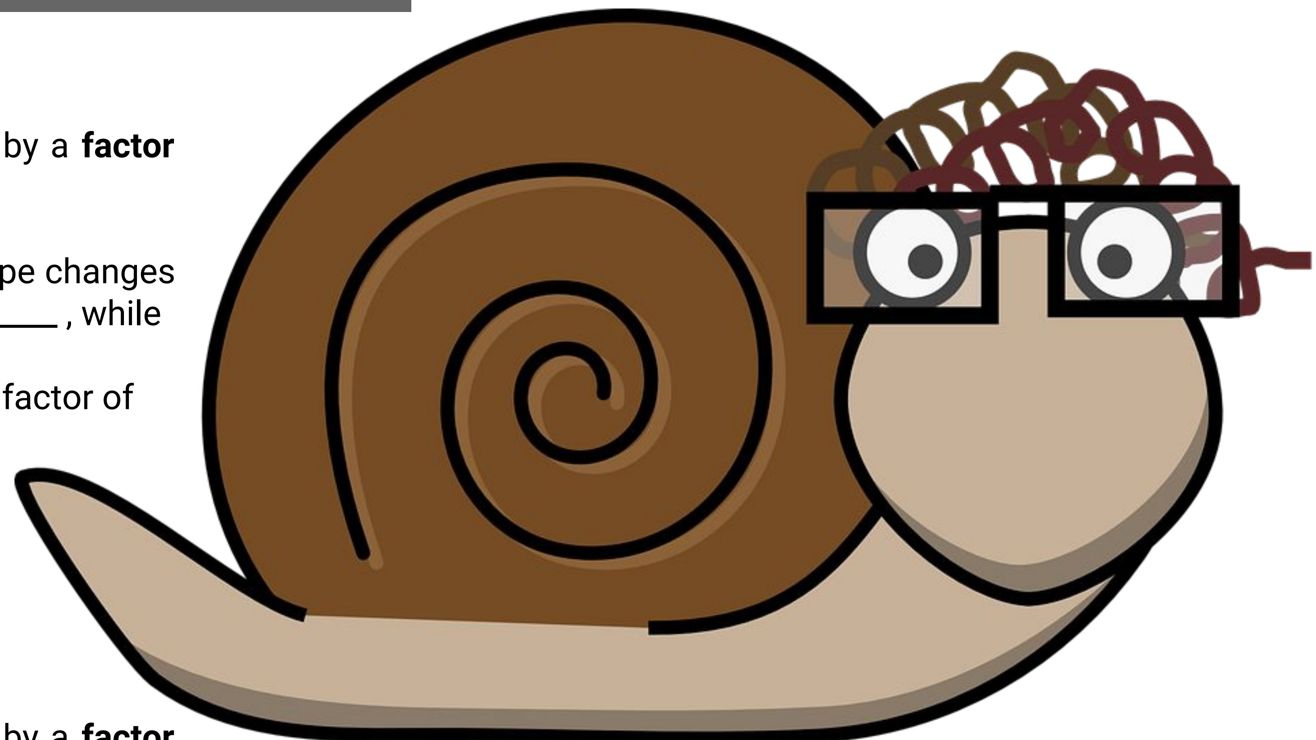
Area:
Perimeter:

MATH FACTS

The factor 2 theorem

When we multiply the scale by a **factor of 2,**

- the perimeter of a shape changes by a factor of __________ , while
- the area changes by a factor of __________ .



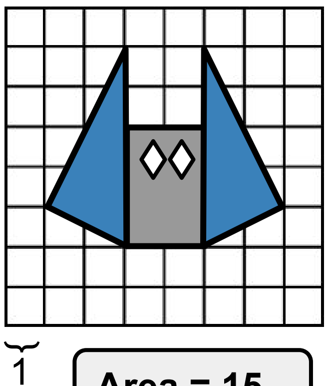
The factor 5 theorem

When we multiply the scale by a **factor of 5,**

- the perimeter of a shape changes by a factor of __________ , while
- the area changes by a factor of __________ .

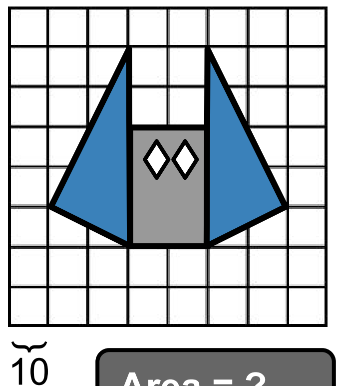
f Compare the animals in Figures 1 and 2 and find the missing information. **Try to use ONLY the math facts you learned in the previous page.** *All lengths are in inches.*

FIGURE 1



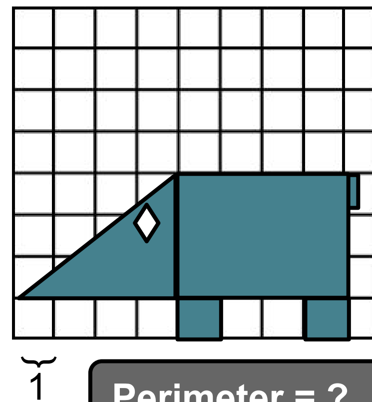
Area = 15

FIGURE 2



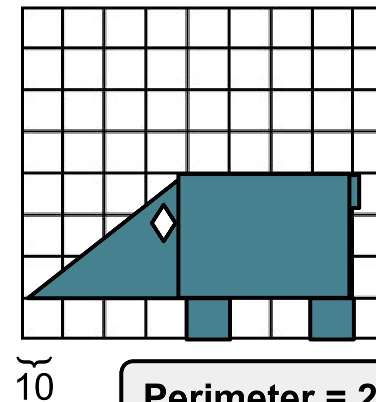
Area = ?

FIGURE 1



Perimeter = ?

FIGURE 2



Perimeter = 29

Scale = 1

Area = 25

Perimeter = ?

FIGURE 1

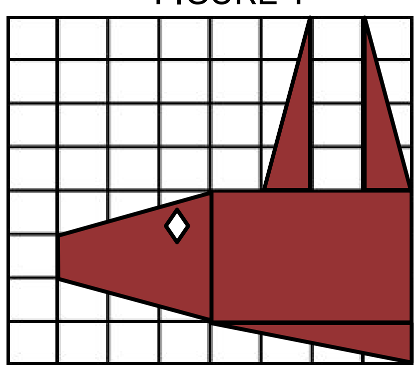
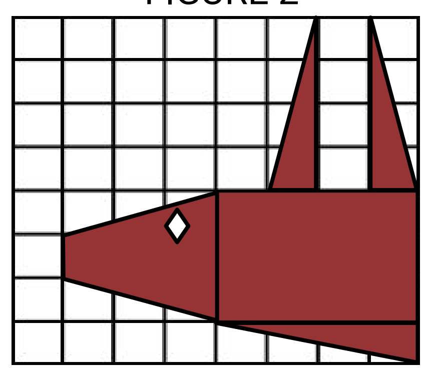


FIGURE 2



Scale = ?

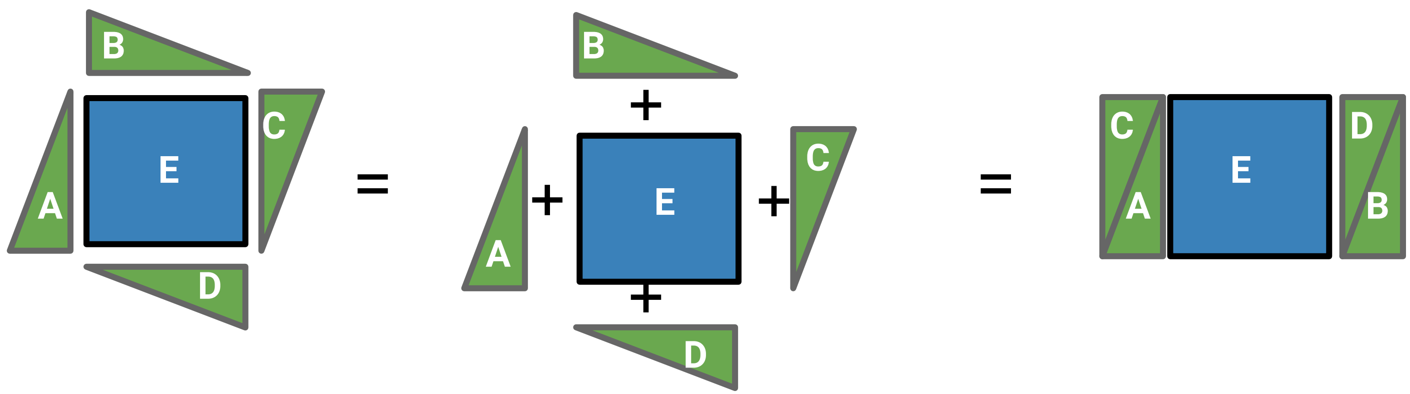
Area = 100

Perimeter = 30

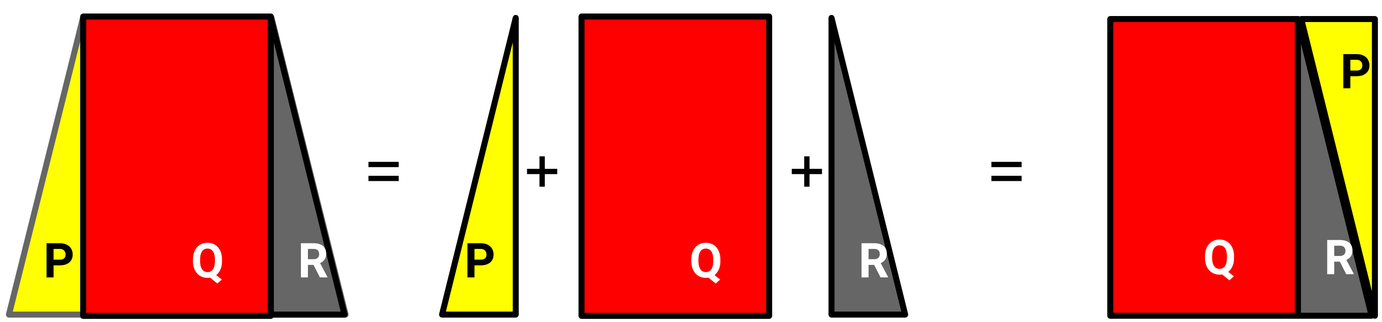
2 Searching for rectangles

We want to find the areas of complicated shapes. A very effective strategy is to "search for rectangles", that is, try to cut a shape in pieces and re-glue them back together to form a rectangle. The new shape will have the same area as the original one, but - since finding the area of a rectangle is so easy - the new area computation will be very simple.

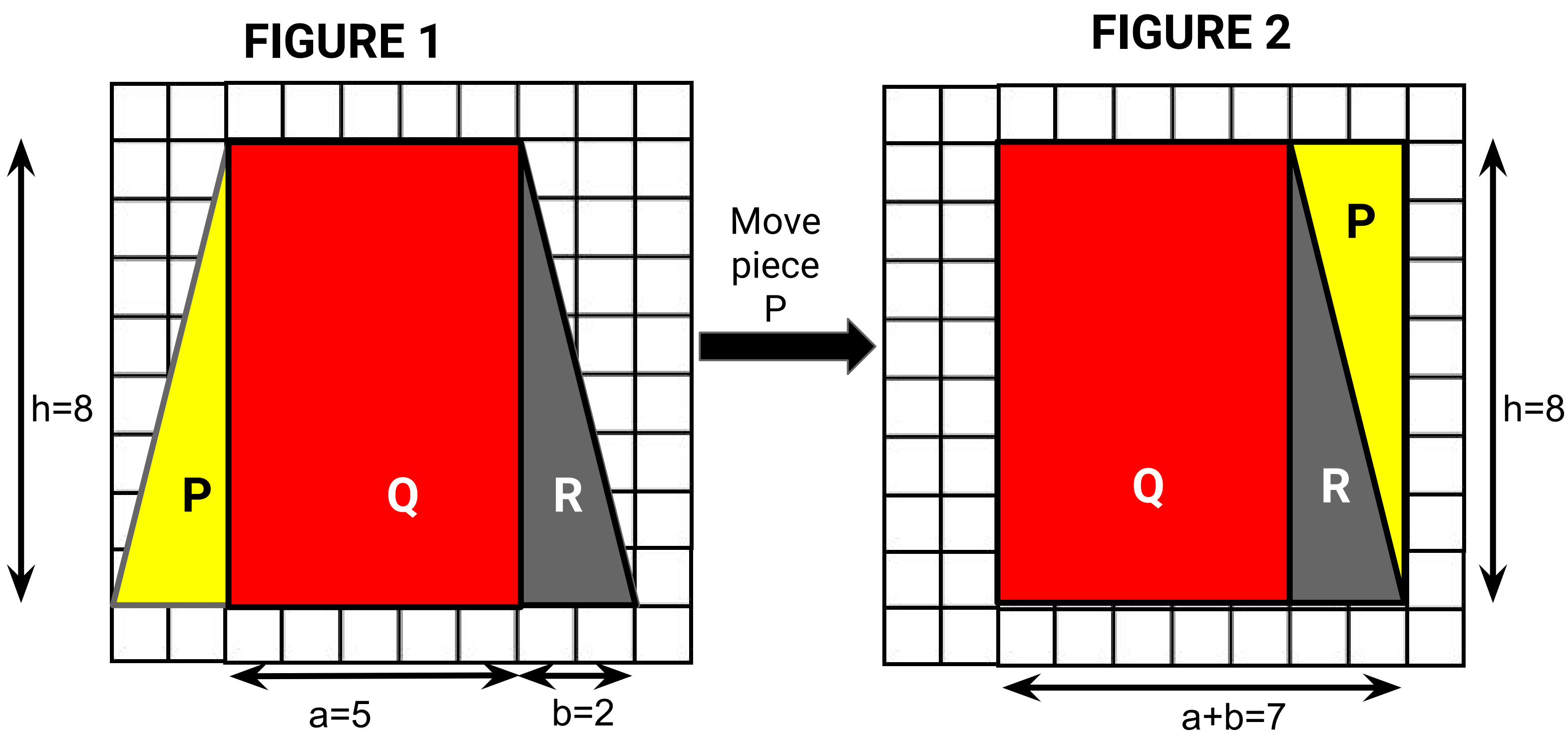
Example 1: turning a wind-wheel into a rectangle



Example 2: turning a trapezoid into a rectangle



a We want to find the area of the trapezoid in Figure 1. In order to do this, we can **rearrange its pieces to obtain a rectangle** in Figure 2, which has the same area as the trapezoid. Moreover, the area of the rectangle can be easily computed. Based on this, what is the area of the trapezoid in Figure 1?



In Figure 2, the area is (a + b) x h = 7 x 8.

b Similarly, we want to find the area of the kite in Figure 1. In order to do this, we can **rearrange its pieces to obtain a rectangle** in Figure 2, which has the same area as the kite. Moreover, the area of the rectangle can be easily computed. Based on this, what is the area of the kite?

FIGURE 1

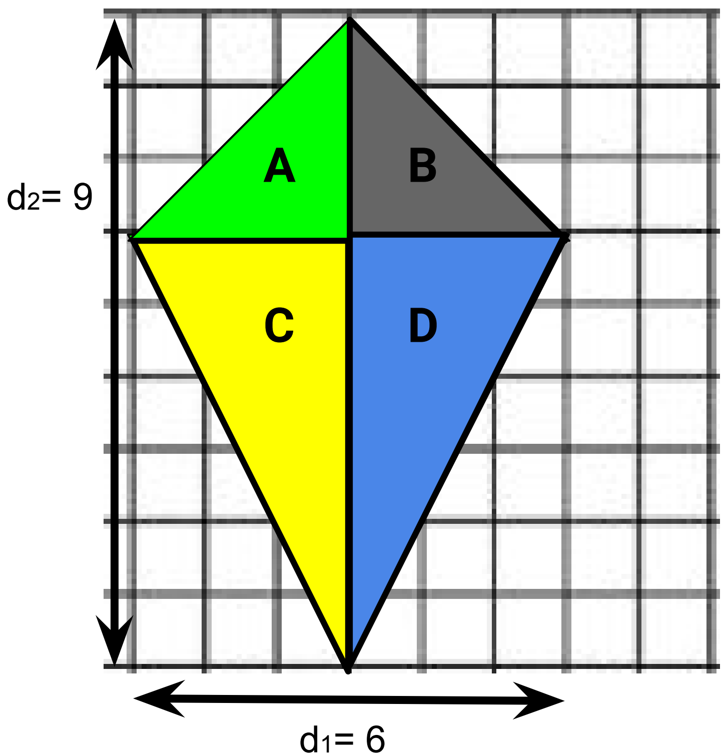
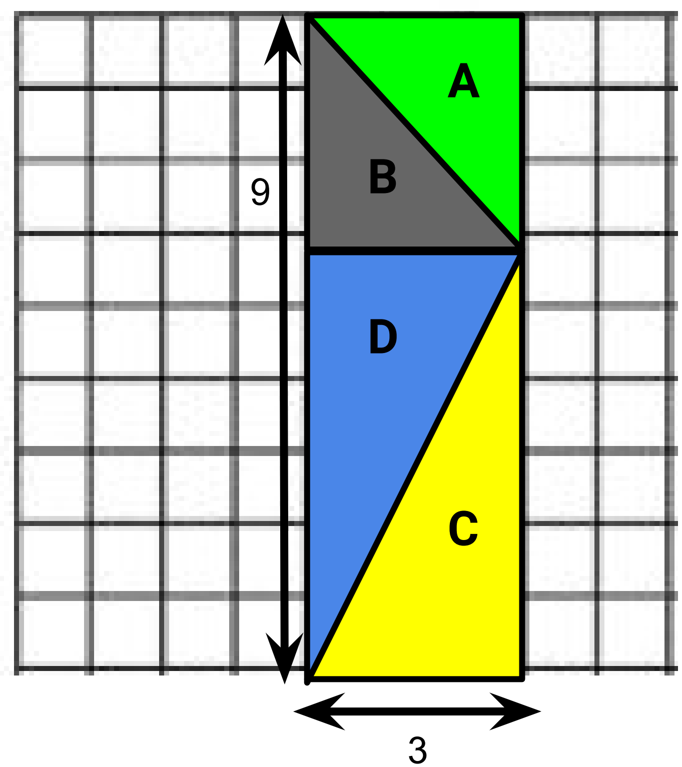
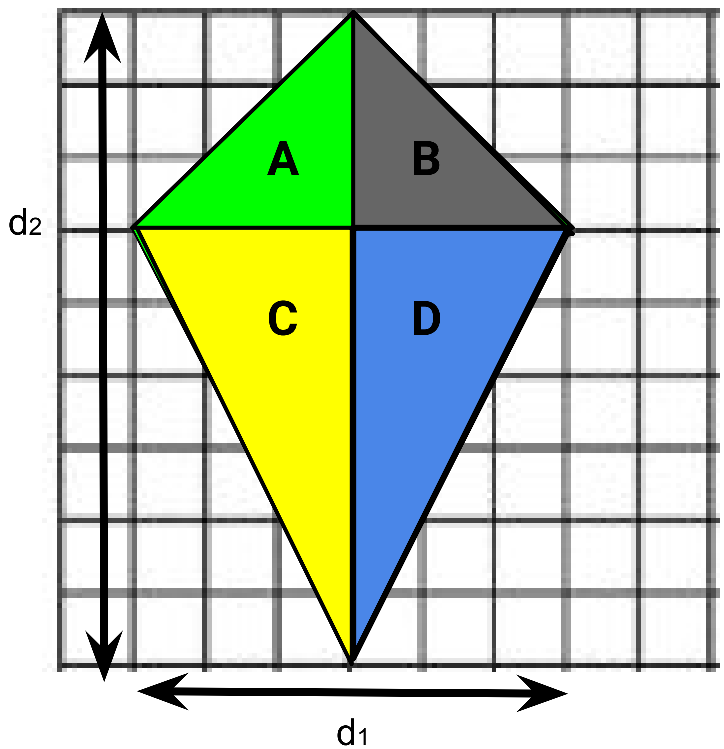


FIGURE 2



c Based on the previous activity, what is the area of a kite with diagonals of lengths d_1 and d_2? (Your answer should be a formula in terms of d_1 and d_2).



Area of a kite:

d Similarly, we want to find the area of the fish in figure 1. For this, **rearrange its pieces to obtain rectangles** and put them in the space for Figure 2. Based on this, what is the area of the fish?

FIGURE 1

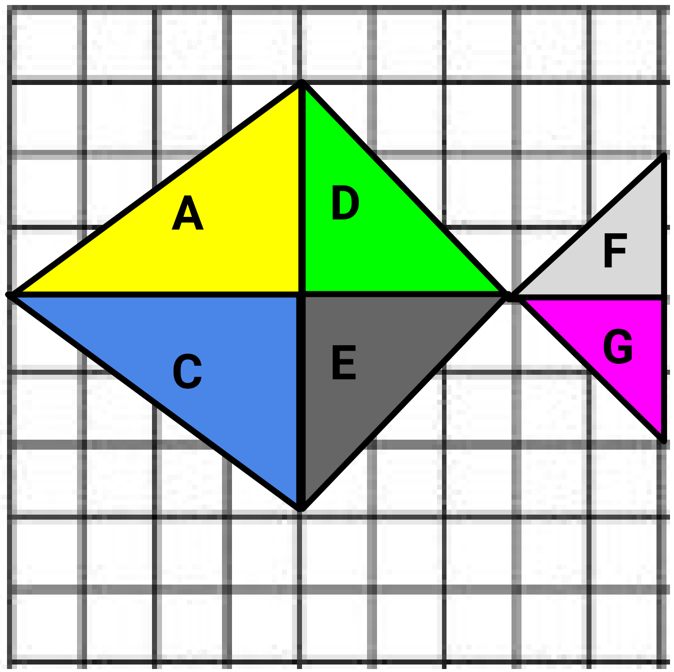
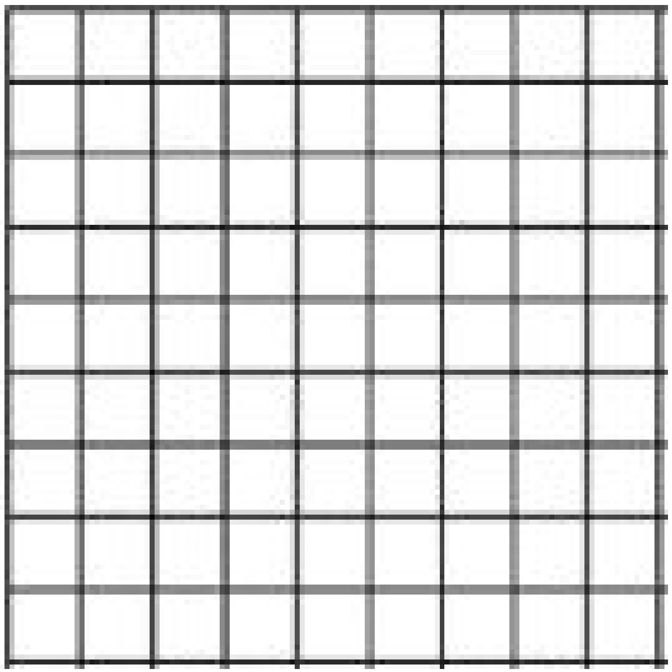


FIGURE 2



CHALLENGE

Figure 1 shows two identical trapezoids, one next to the other.

Figure 2 shows a rectangle formed by a rearrangement of the pieces in Figure 1.

Use the two pictures to find a general formula for the area of a (single) trapezoid.

FIGURE 1

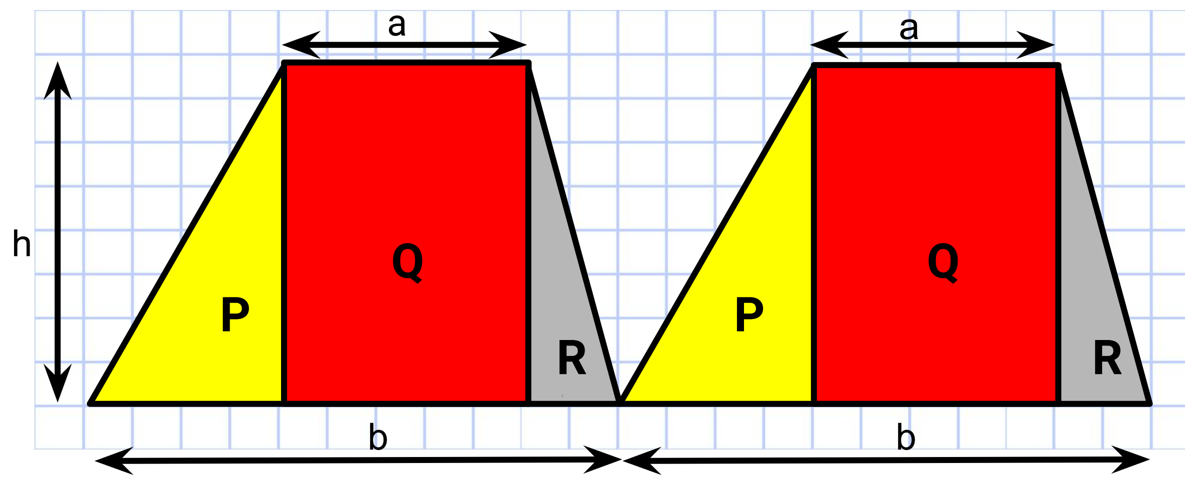
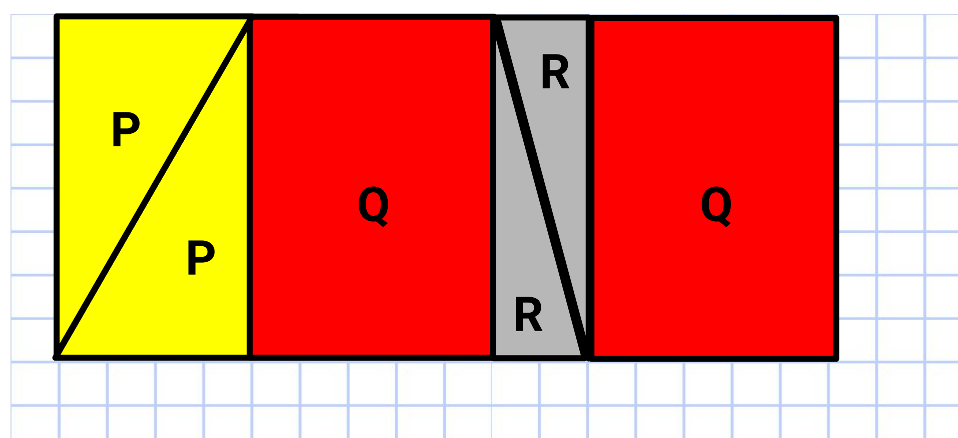
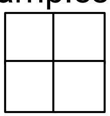


FIGURE 2

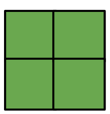


3 Quarnies

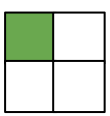
A quarny is a square divided into four smaller squares (top left, top right, bottom left and bottom right), with some of them shaded (sometimes none of them, sometimes all of them). Here are some examples:



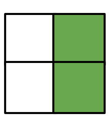
The empty quarny



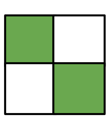
The full quarny



A one-fourth quarny

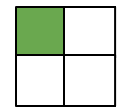


A one-half quarny

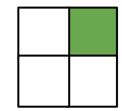


Another one-half quarny

Watch out: **orientation matters** when considering quarnies. The following quarnies A and B are not equal:



A



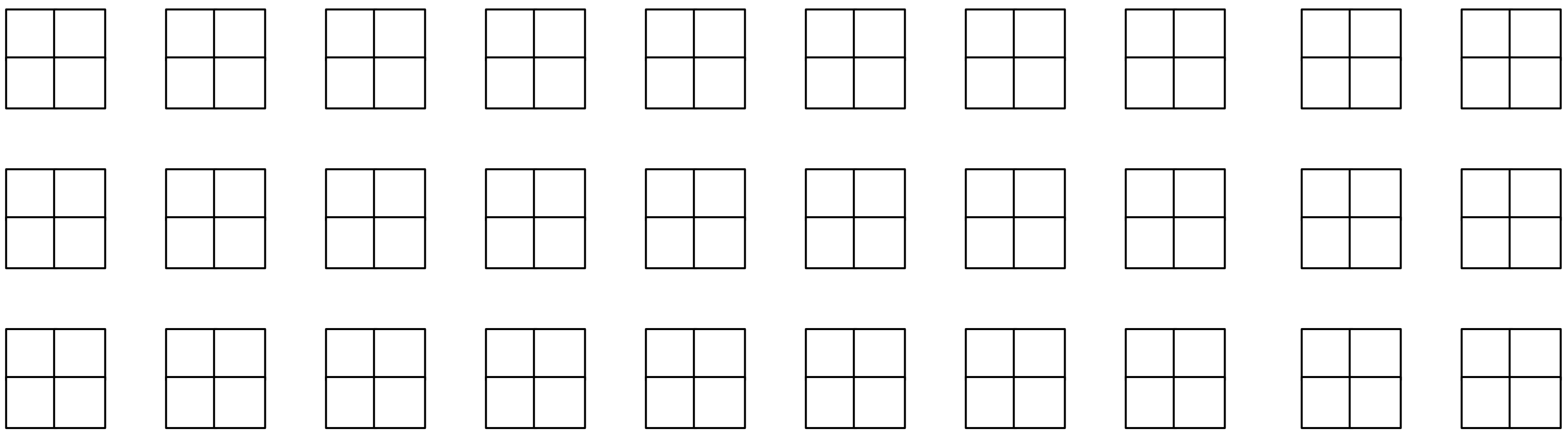
B

a How many quarnies are one-fourth full? Draw all of them.

b How many quarnies are one-half full? Draw all of them.

CHALLENGE (Booklet)

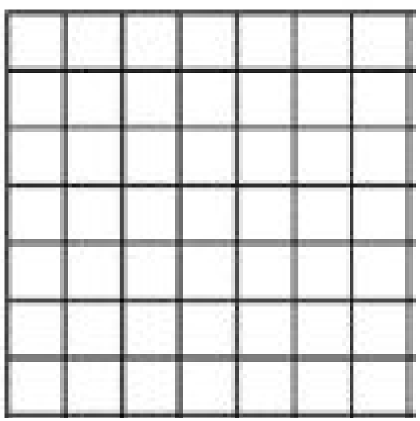
How many quarnies are there? Count and draw them all.



Note: To trick you, we may have drawn more templates than you will need! You can cross out the ones that you did not use.

4 Game of Squares

Consider the following large 7 x 7 Square:



a We choose two positive integers p and q whose sum is equal to 7. We choose p = 2 and q = 5 to illustrate.

Answer the following two questions, first without doing any drawing:

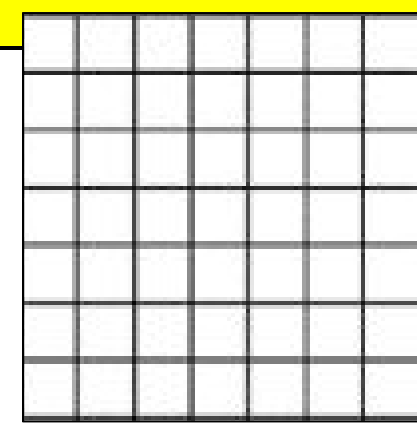
Question 1 Do you think that we can cover the *full* 7 x 7 square by using one 2 x 2 square and one 5 x 5 square?

◯ Yes ◯ No

Question 2 Is it true that if we add the length of a diagonal of the 2 x 2 and the length of a diagonal of the 5 x 5 square, we obtain the length of a diagonal of the 7 x 7 square?

◯ Yes ◯ No

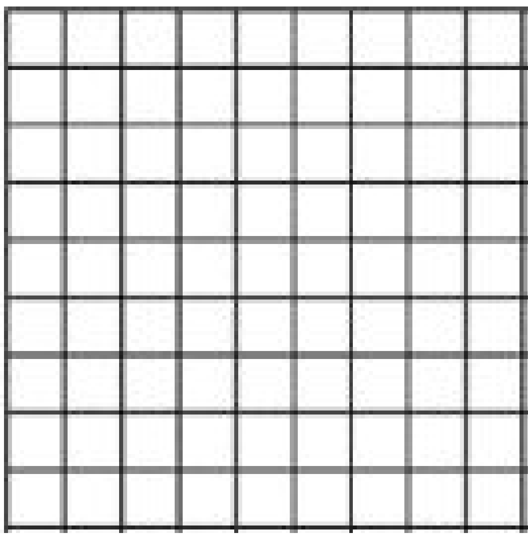
Now draw the 2 x 2 and 5 x 5 squares (inside the 7 x 7 grid) to verify your answers. Modify your answers if you need to.



If it was not possible to cover the *full* 7 x 7 square using your 2 smaller squares, how many square units did you fail to cover?

Game 1: Cover THE MOST

Consider the following large 9 x 9 square:



9 x 9

These games can be played in teams of two players!

b Read all instructions before you play.

1. Team-up with a friend at your table. Each team chooses one positive integer p and one positive integer q, whose sum is equal to 9. All teams have 1 minute to do so....

2. The players in each team draw a p x p square and a q x q square inside the 9 x 9 square, trying to cover as much area as possible. The total area covered will be the score of the team.

3. The team at the table that gets the highest score wins the game. If there is a tie, the teams share the victory.

Once everyone understands the rules, play begins.

Scoring sheet

p = ____ **q** = ____ Remember that p plus q should equal 9

Draw your squares in the grid to the left

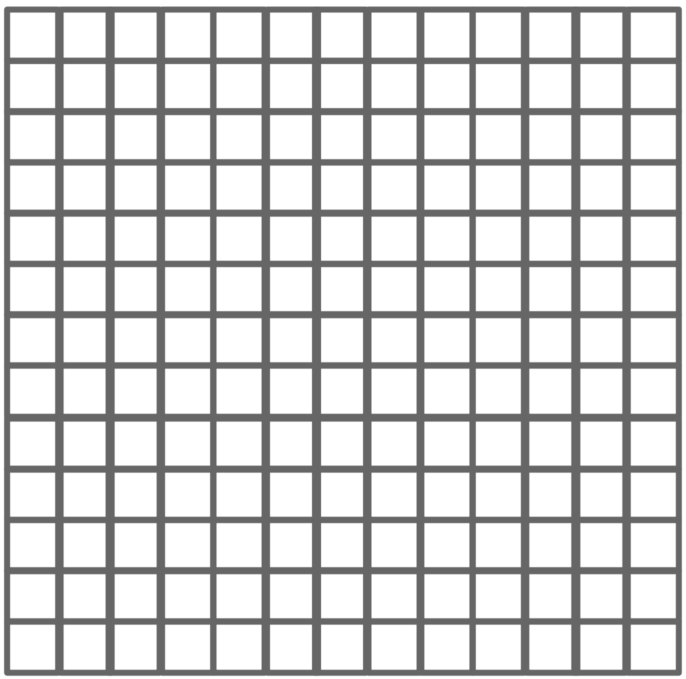
Team score = ____ **Winning score** = ____

Discuss

What would be a **perfect score**? Can it be achieved? What is the highest score possible?

Game 1: Cover THE MOST

Consider the following large 13 x 13 square:



13 x 13

c **Play the game again, but use a 13 x 13 grid this time.**

Remember, the team that covers the **most area** wins the game.

Scoring sheet

p = ____ **q** = ____

Remember that p plus q should equal 13

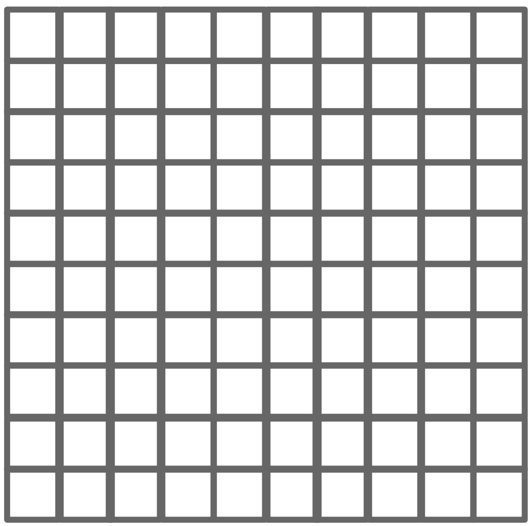
Draw your squares in the grid to the left

Team score = ____

Winning score = ____

Game 2: Cover THE LEAST

Consider the following large 10 x 10 square:



10 x 10

d

This game is similar to the previous one, but now **you want to cover the least amount of area.**

The two squares must be drawn inside the grid, and cannot overlap.

Your score is equal to the area inside the grid that was *not covered*. Highest score wins.

Scoring sheet

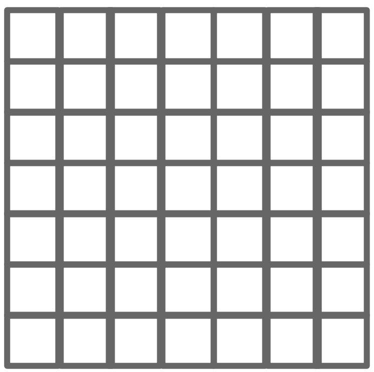
p = ☐ **q** = ☐ Remember that p plus q should equal 10

Draw your squares in the grid to the left

Team score = ☐ **Winning score** = ☐

Game 2: Cover THE LEAST

Consider the following large 7 x 7 square:



7 x 7

e Play the game again, this time with an 7 x 7 grid. **You want to cover the least amount of area.**

The two squares must be drawn inside the grid, and cannot overlap.

Your score is equal to the area inside the grid that was *not covered*. Highest score wins.

Scoring sheet

p = ☐ **q** = ☐

Remember that p plus q should equal 7

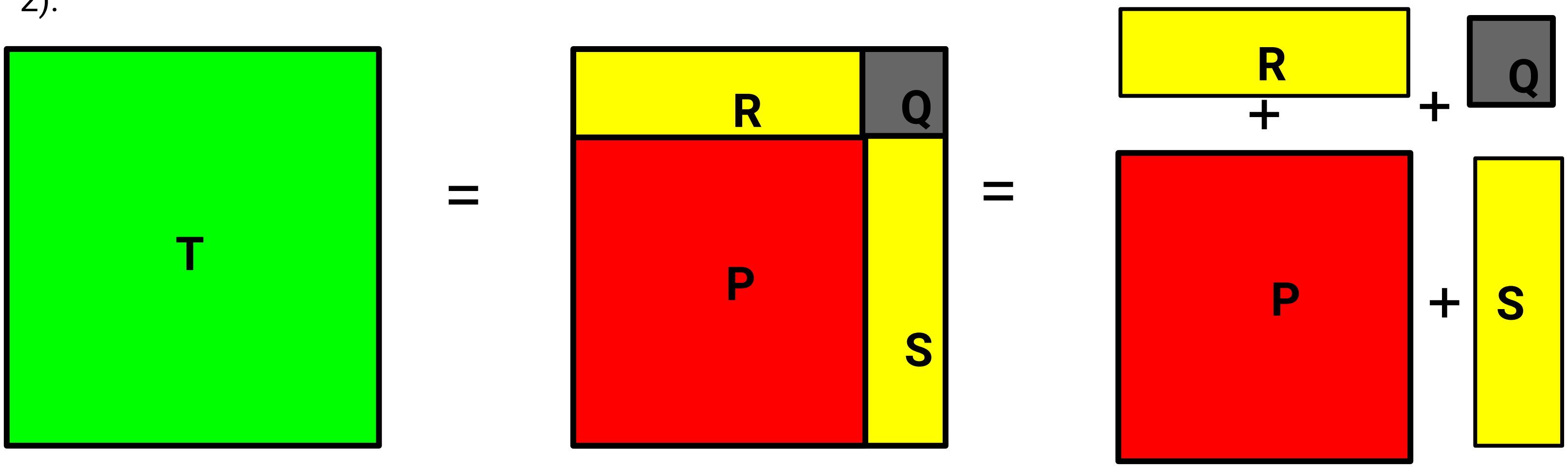
Draw your squares in the grid to the left

Team score = ☐

A method for squaring a number

In the previous games we were trying to cover a big square using two non-overlapping smaller squares, in a way that the leftover area was as big (small) as possible.... Let's now take a different approach.

We think of our big square as the union of 4 pieces, cut and glued back together (like we did in problem 2).



Since the process of cutting and gluing does not change the area:

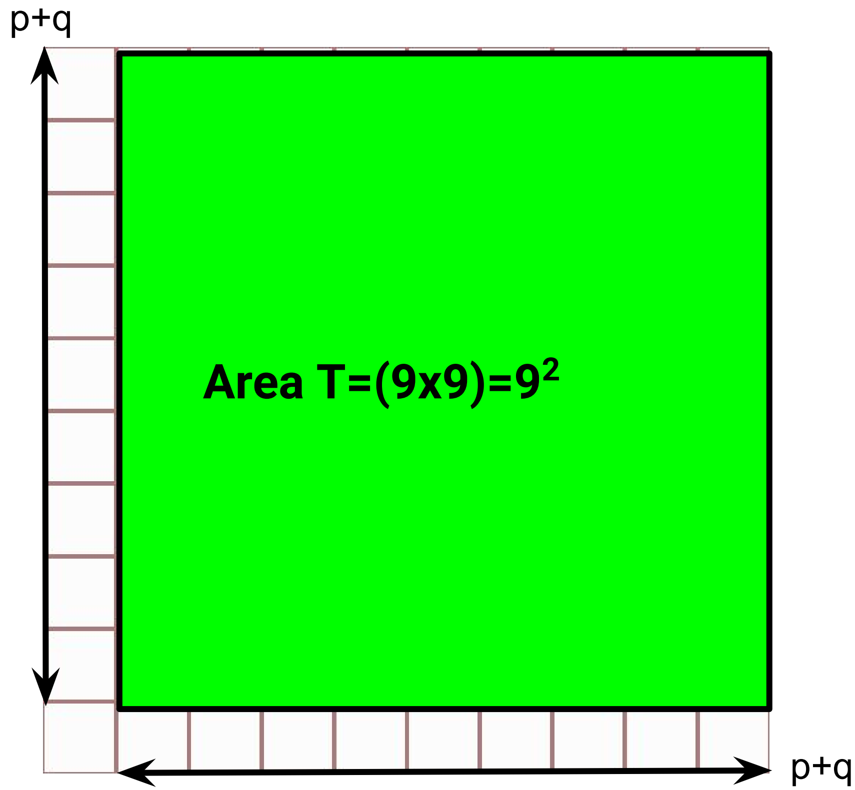
(AREA of T) = (AREA of P) + (AREA of Q) + (AREA of R) + (AREA of S) =

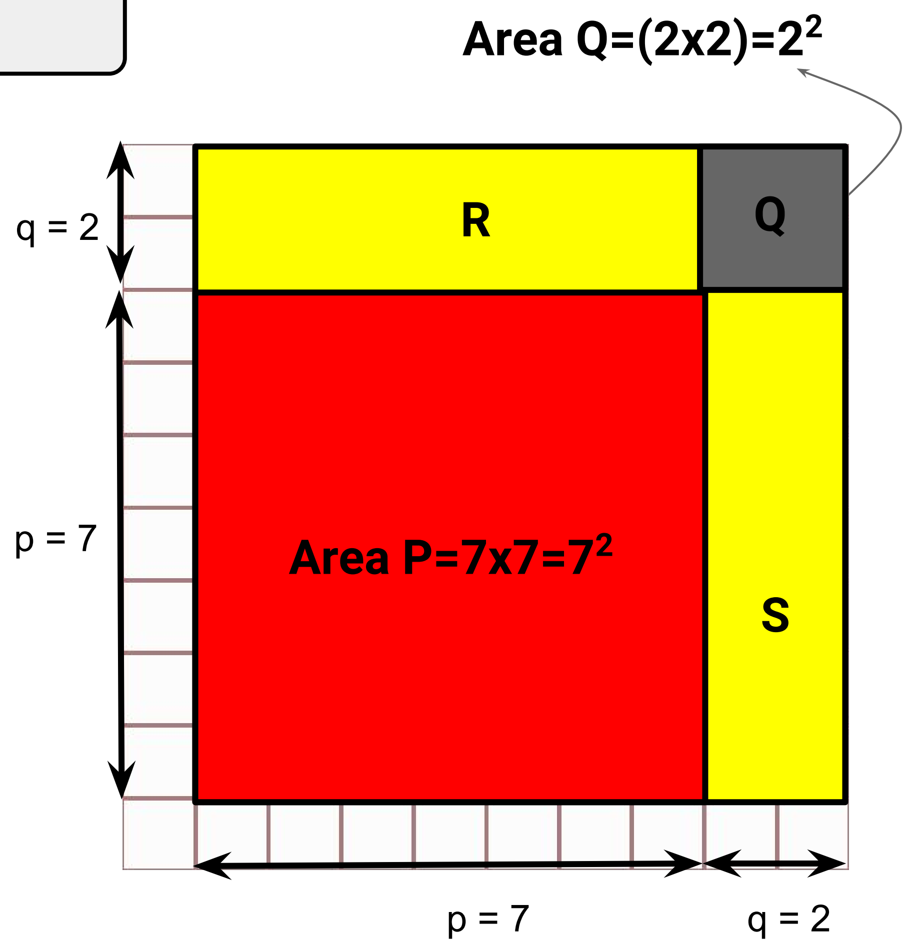
= (AREA of P) + (AREA of Q) + 2 (AREA of R).

We are interested in the case where P is a square of side p, and Q is a square of side q. (The original square then has side length p+q).

f If p=7 and q=2 (so that p+q=9), express the area of T in terms of the areas of P, Q, R and S. That is, write 9^2 as a sum of 7^2, 2^2 and... the left-overs!

$$9^2 = 7^2 + 2^2 + \ldots.$$





A formula for squaring a number

g Use the following pictures to find a formula to express $(a + b)^2$ (the area of T) as a sum of (area of P), (area of Q) and … the left-overs! (areas of R and S).

$$(p + q)^2 = p^2 + q^2 + \ldots.$$