Meeting 6 Student's Booklet

Alchemy

November 4 2015 @ UCI

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1 The Magic Flute

We start building the magic flute at noon and we add a new piece every hour. Can you find out what piece was added at 7 pm? 8 pm? 9 pm?

Continue the pattern. Draw the magic flute at each missing hour.
Complete the following table:

<table>
<thead>
<tr>
<th>Time</th>
<th>Number of dots</th>
<th>Number of blank squares</th>
<th>Ratio DOTS : BLANK SQUARES</th>
<th>Ask your own question::</th>
<th>Length of flute</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 pm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 pm</td>
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<tr>
<td>6 pm</td>
<td>12</td>
<td>6</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N pm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2 Transmutation

The Magic Flute can change different objects. But you have to combine the right melodies...

Alchemists were interested in transforming materials. Suppose that we start with a perfect balance of materials: 10 units of copper, 10 units of silver and 10 units of gold (these units may be grams, pounds or any other type). We are doing math, so we can and will allow negative units of material, which is kind of funny!

The letters g, c, s will denote the following basic transmutation events, produced by playing notes in the magic flute:

- g : 1 unit of silver turns into gold.
- s : 1 unit of copper turns into silver.
- c : 1 unit of gold turns into copper.

New events

We may create new events by using combinations. Here are some examples:

- 3g : 3 units of silver become gold.
- g + 2c : 1 unit of silver becomes gold, and 2 units of gold become copper.
- – g : 1 unit of gold becomes silver.

Example:

The event 3g
**Event 1**

Let \( y \) be the following event:
\[
y = 3c + 4s + 5g.
\]
Complete the picture for the event \( y \):

From copper to gold

Consider the following event \( z \): “5 units of copper were turned into gold”.
How can you write this event formally (using some of the letters \( c, s, g \))? Can you find more than one answer?

Answer: \( z = \)

**Event 2**

Let \( z \) be the following event:
\[
z = 3(g + c + s + 4c + 4g).
\]
Complete the picture for the event \( z \):

Eliminating \( g \)

Express the following events using only the letters \( c \) and \( s \):

- \( g + c \)  
  Answer:

- \( -3g + 2c - s \)  
  Answer:

- \( -900g \)  
  Answer:
This is a scale in perfect balance. Some objects have their weights labeled. Each cube of silver weighs 10. All copper moons weigh the same. All gold coins weigh the same.

Can you find the weight of each gold coin?
The left and right plates share the following objects: 2 cooper moons, one silver cube and one gold coin. Thus we can remove them and keep the balance. So we have the following equation (where \( x \) represents the weight of a gold coin):

\[ 2x + 4 = 10 + 8 \]

so \( x = 7 \).
Equality and Comic Solve

Think of equality as a scale: each number represents the weight of a coin. We place coins on the two plates of the scale, so that we have a perfect balance.

Ex 1:

\[
\begin{align*}
1 & \quad 7 \\
\hline
5 & \quad 3
\end{align*}
\]

Equation (quick way to express the balance): \(1 + 7 = 5 + 3\). Or: \(1 + 7 = 3 + 5\).

We can also use objects of mysterious weight, and place of the scale in a way that leads to a perfect balance.

Ex 2:

\[
\begin{align*}
1 & \quad 7 \\
\hline
\text{green object} & \quad 6 & \quad 1
\end{align*}
\]

Equation: \(1 + \text{green object} + 7 = \text{green object} + 6 + 1 + 1\).

Or, using the letter N for the weight of \(1 + N + 7 = N + 6 + 1 + 1\).

Another example with a mystery object

Ex 3:

\[
\begin{align*}
\text{5} & \quad \text{3} \\
\hline
\text{8}
\end{align*}
\]

Equation: \(\text{5} + \text{3} + 5 + 3 = \text{8} + \text{8}\). Or, using a letter: \(2K + 5 + 3 = 2K + 8\), where \(2K\) means \(K + K\).

Working Backwards

Ex 4:

Suppose that María knows that each weighs 9, but she does not tell Edwin. Instead, she shows him this scale:

\[
\begin{align*}
\text{4} & \quad \text{4} \\
\hline
27
\end{align*}
\]

Equation: \(N + N + N + 4 = 27 + 4\)
To solve this problem, Edwin will use the method of Comic Solve. His goal is to solve the mystery of finding the weight of each cube. He will write a comic in which he tells a story, and in the end he must discover the weight of each cube.

Here is how Edwin does it:

Three cubes plus 4 weighs the same as 27 plus 4

Break the 27 into three equal pieces (there are three cubes)

Remove the “4” from both plates keeps balance

Verification step: Replace each cube with 9 in the first scale to see if it is balanced

So each cube must weight 9
Suppose that all the boxes have the same number of (gold) coins. Maria bought one box and already had 16 coins at home. Edwin bought three boxes, and already had 2 coins at home. Later they discovered that they had the same total number of coins. How many coins were in each box?

Maria has 1 box and 16 coins. Edin has 3 boxes and 2 coins. Both have the same number of coins.

Each person spends 2 coins and we keep balance

Thus, each box must have 7 coins.

Verification step: Replace each box with 7 in the first scale to see if it is balanced
Now create your own Comic Solve!

Ex 6:
All the boxes have the same number of tennis balls. Each can has 3 balls. Suppose that Alan buys 2 boxes and 6 cans, and Betty buys 4 boxes and 1 cans (none had tennis balls before). After this, Alan has 1 ball less than Betty. How many balls are in each box?

Answer:

Verification step
Among other properties, The Philosopher’s Stone allowed its owner to live forever. Meet the great (and very, very old) alchemists Agathodaemon, Trevisan and Fulcanelli.

These three alchemists claim that they found the Philosopher’s Stone. They seem old enough to support this assertion. Agathodaemon has lived 3 times as much as Trevisan, while Trevisan is 200 years younger than Fulcanelli. However, Fulcanelli points out that he is 50 years older than Agathodaemon.

Can you find the age of each alchemist? To do this, follow the steps in the next pages...
Step 1: The youngest of the pack

Although all of them being pretty old, is it possible to identify with the given information who is the youngest?

Step 2: Pairing

Use the stripes below to assign to each alchemist a symbolic length of their age. In other words:, which alchemist corresponds to each stripe? Put their initials on the blanks.

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Step 3: A Link to the Past

Find the length of the each part with a question mark.

Step 4: Solving the problem

Finally, use the representations to find the age of each alchemist.

Funny bonus: Find which of the alchemists is not old enough to drive a horse carriage...

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Funny bonus: Find which of the alchemists is not old enough to drive a horse carriage.
Mariano poured the three potions into a small 100 oz. cauldron to and filled one quarter of it.

How many more potions does he need to buy in order to fill in the cauldron?

Wizard Mariano bought the following three growth potions at the “Night Cat” store, each having a certain unknown amount of ounces of a precious liquid known as Bigade. He discovered that one potion had one ounce less of Bigade than usual, while other potion had 2 ounces more than usual. The other two were fine. Next time Mariano will shop at the “Happy Owl”, where all growth potions are guaranteed to have the same amount.