MATH 2A: SAMPLE FINAL #2

- This exam consists of 10 questions and 100 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (10 points) Determine the value of each of the following limits.

   a. $\lim_{x \to 1} \dfrac{2 - x}{(x-1)^2}$

   b. $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} x^2 + 5 & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}$

   c. $\lim_{x \to 0} \dfrac{\sqrt{x+1} - 1}{x}$

   d. $\lim_{r \to \infty} \dfrac{\ln \sqrt{r}}{r^2}$

   e. $\lim_{x \to -\infty} \dfrac{\sqrt{7x^2 + 3x}}{3x - 5}$

2. (10 points) Compute the indicated derivative of each of the following functions. (You do not need to simplify the result algebraically.)

a. Find $\dfrac{dy}{dx}$, for $y = 2x + 6 - 4x^2 + \dfrac{5}{x^2} + \ln x$

b. Find $f'(4)$, for $f(\theta) = 2\sqrt{\theta} + \dfrac{2}{\sqrt{\theta}}$

c. Find y', for $y = \dfrac{3t - 1}{t^2 + t - 2}$

d. Find $r'(t)$, for $r(t) = 5^t \sin t$

e. Find y'', for $y = \sec(x) + 3\cos(x)$

3. (10 points) Compute the indicated derivative of each of the following functions.

   a. For $y = (5x^2 - 2x)^{\frac{3}{4}}$, find $\frac{dy}{dx}\big|_{x=2}$

   b. For $f(x) = (3x)^{\tan^{-1}(x)}$, find $f'(x)$

4. (10 points)
   a. Given that the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$, find $f(4)$ and $f'(4)$.

   b. Sketch a graph of a continuous, differentiable function $g(x)$ which satisfies
      - $g'(x) > 0$ for $x < -2$ and $x > 3$
      - $g'(x) < 0$ for $-2 < x < 3$
      - $g'(x) = 0$ for $x = -2$ and $x = 3$

5. (10 points) Use implicit differentiation to find the equation of the tangent line to the curve $e^y \sin(x) + x - xy = \pi$ at the point $(\pi, 0)$.

6. (10 points) For the function $f(x) = \dfrac{2x^2 - 1}{x^2 - 4x - 21}$, answer each of the following.

   a. Find all of the points of discontinuity of $f(x)$.

   b. Use the Intermediate Value Theorem to verify that $f(x)$ has a zero on the interval $(0, 1)$.

   c. Find the equations for any horizontal and vertical asymptotes of this function.

7. (10 points) Answer True or False to each of the following and explain your answer. (Unjustified answers will not receive credit.)

   a. True/False: Let $f(x) = \tan(x)$. By the Mean Value Theorem, there exists some c in the interval $(0, \pi)$ such that $f'(c) = 0$.

   b. True/False: Assume the half-life of a certain radioactive substance is 200 years. If we begin with n grams of the substance, then after 600 years we will have $\frac{1}{8}n$ grams.

   c. True/False: $F(x) = \dfrac{\sin^3(x)}{3} + 10$ is an antiderivative of $\sin^2(x)$.

8. (10 points) A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of $2\ ft/sec.$ When the total area of the disturbed water is 16π square feet, at what rate is the total area of the disturbed water changing? Remember to include units.

9. (10 points) According to postal regulations, a carton is classified as "oversized" if the sum of its height and girth (the perimeter of its base) exceeds 108 inches. Find the dimensions of a carton with a square base that is not oversized and has maximum volume.

10. (10 points) For the function $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$, answer each of the following.

a. Find all intervals on which f is increasing and all intervals on which f is decreasing.

b. Find any local maximum and minimum values.

c. Find all intervals on which f is concave up and all intervals on which f is concave down.

d. Find any points of inflection.

e. Graph the function $f(x)$.