MATH 2A: SAMPLE MIDTERM #2

- This exam consists of 7 questions and 100 total points.

- Read the directions for each problem carefully and answer all parts of each problem.

- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.

- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.

- Clearly indicate your final answer to each problem.

1. (20 points) Compute the indicated derivative of each of the following functions.

    a. For $f(x) = x^5 - \sqrt{x} + \pi^6 + \frac{3}{x^2}$, find $f'(x)$.

    b. For $w(t) = \tan^{-1}(t)$, find $w'(3)$.

    c. For $y = \dfrac{e^x}{1 + 3x}$, find $\dfrac{dy}{dx}$.

    d. For $g(x) = \ln(x)$, find $g''(x)$.

    e. For $r(\theta) = \sin(\theta)$, find $r^{(25)}(\theta)$.

2. (15 points) Find the equation of the tangent line to the curve $x^2 + \dfrac{2}{\pi}\cos(\pi y) + 4xy = 3$ at the point $\left(1, \frac{1}{2}\right)$.

3. (15 points) A cylindrical tank with radius $5\,m$ is being filled with water at a rate of $3\,m^3/min$. How fast is the height of the water increasing?

4. (18 points) Find the derivative $f'(x)$ for each of the following functions. (You don't have to simplify your answers.)

a. $f(x) = \ln(1 + 3x^2)$

b. $f(x) = e^x(x^3 - x)^{12}$

c. $f(x) = \sin(\cos(\tan(x)))$

5. (12 points) Find the linearization of the function $f(x) = \sqrt{x + 99}$ at $a = 1$. Use this to approximate $\sqrt{102}$.

6. (10 points) The half-life of anteatertonium is 50 years. Suppose that we have a $1024 \, mg$ sample of anteatertonium.

   a. Find the mass of anteatertonium remaining after 150 years.

   b. How long would it take the original sample to decay to $30 \, mg$? You don't have to simplify your answer.

7. (10 points) Suppose $C(s)$ is the total cost (in dollars) of building a home with s square feet of space.

   a. What are the units of $C'(s)$?

   b. What is the practical meaning of $C'(s)$ for this problem?

   c. Is $C'(s)$ positive or negative? Why?