

Suggested Syllabus for Math 205A-B-C

Revised: September 7, 2004

Mathematics 205A-B-C is a year-long course called ‘Introduction to Graduate Analysis’. It is intended for two primary audiences:

(a) New graduate students whose scores on the Department’s *Advisory Examination in Analysis*, taken upon entry into the graduate program, indicate a need for solidifying their analysis background before moving on to more advanced courses such as Math 210 and Math 218. (The corresponding algebra course arising from the Advisory Examination in Algebra is Math 206A-B-C, *Introduction to Graduate Algebra*.)

(b) Undergraduate students in the Department’s Honors Program, who are required to take either Math 205 or Math 206 as part of that program.

The purpose of Math 205 is ensure that the student become thoroughly familiar with the concepts, facts, and methods found in Math 140A-B-C-D (‘Undergraduate Analysis’), but at a much higher level of understanding and abstraction than is expected from the undergraduate courses. Thus, the topics in the course, as listed in the catalog description of Math 205, may sound familiar: continuity, differential and integral calculus in \mathbf{R} and \mathbf{R}^n , sequences and series of functions, implicit functions, and so on; but these topics will be covered more deeply, and from a more abstract viewpoint, than is done in undergraduate courses.

The expectation is that the student who succeeds in Math 205 not only does well on the homeworks and exams, but actually has the ideas and results of the course permanently engraved in the quick-access portions of their brains, ready to be invoked instantaneously, as needed, for the rest of their mathematical careers. In particular, even though much of the material might be considered ‘undergraduate’, the course will be taught at an intense graduate level.

The following ‘Suggested Syllabus for Math 205’ is based on the structure of the course in the 2004-2005 academic year; the syllabus in other years may differ in the details, but probably not in the overall spirit.

Required Text: ‘Mathematical Analysis (Second Edition)’ by T. Apostol (Addison-Wesley Publisher)

In addition, at various times there will be supplementary material presented in the lectures that is not found in the text is or presented differently from the approach taken by the text.

Material for Math 205A: The fall quarter covers essentially everything in Chapters 1 through 6. This includes some basic set theory and a review of the structure of the real numbers; the point-set topology of the Euclidean spaces \mathbf{R} and \mathbf{R}^n , and the topology of metric spaces; limits and continuity, including uniform continuity; the differential calculus of functions of a single real variable; the theory of functions of bounded variation, and rectifiable curves.

Material for Math 205B: In the winter quarter Chapters 7, 8, 9 and 12 fill most of the schedule. Thus, the main topics are the Riemann-Stieltjes integral; infinite series and infinite products of numbers; the convergence properties, including uniform convergence, of infinite sequences and series of functions (including, in supplemental lecture material, the concept of ‘equicontinuity’ and the Arzela-Ascoli Theorem); the basics of multivariable differential calculus.

Material for Math 205C: By this point in the course, the students should have developed their mathematical muscles well enough to handle the difficult material of Chapter 13 (‘Implicit Functions and Extremum Problems’) and Chapter 14 (‘Multiple Riemann Integrals’) in a reasonable number of weeks, leaving time for other topics to be chosen by the instructor. These topics will probably include Vector Calculus (including surface integrals and the theorems of Green, Gauss, and Stokes); if so, there will be Supplementary Notes made available. Other possible topics, depending on the available time, might include an introduction to submanifolds of \mathbf{R}^n , or the theory of tensors in \mathbf{R}^n , or whatever.

NOTE: Among the topics that traditionally are not covered in Math 205 are those considered in Chapters 10 and 11 of the text, ‘The Lebesgue Integral’ and (its Applications to) ‘Fourier Series and Fourier Integrals’; and in Chapter 16, ‘Cauchy’s Theorem and the Residue Calculus’. The reason for omitting from Math 205 these topics of admittedly great importance is simple: They are covered thoroughly in other basic graduate courses (Math 210 and Math 220, respectively) that most graduate students will go on to take. (A basic principle for choosing ‘optional’ material for Math 205 is that it should be analysis that is important and which later basic courses might assume the students should know, but which is not normally taught in those later courses.)