Algebra Qualifying Exam Syllabus

I.   Groups

Groups and group homomorphisms and isomorphisms; subgroups; cyclic groups; centralizers and normalizers; cosets; Lagrange's Theorem; normal subgroups; quotient groups; the isomorphism theorems; groups acting on sets; Sylow theory; semi-direct products; fundamental theorem of finitely generated abelian groups; free groups; permutation groups; solvable groups.

II. Rings

Rings, ideals and homomorphisms; quotient rings; isomorphism theorems for rings; rings of fractions; polynomial rings; group rings; principal ideal domains; Euclidean domains; unique factorization; Gauss's Lemma; rings of fractions.

III. Modules

Modules and module homomorphisms; quotient modules; modules over PIDs; free modules and direct sums of modules; characteristic and minimal polynomials; rational and Jordan canonical forms; exact sequences; tensor products of modules.

IV. Fields

Field extensions; finite fields; algebraic and transcendental extensions; splitting fields; normal extensions; separable and inseparable extensions; Galois theory; algebraic closure; transcendence bases.

V. Representation Theory

Representation theory and character theory of finite groups.

References:

Algebra by S. Lang
Abstract Algebra, Third Edition, by David Dummit and Richard Foote