COMPLEX ANALYSIS

Qualifying Exam

Wednesday, June 20, 2015 — 1:00 pm -3:30 pm, Rowland Hall 114

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Math Exam ID #:

Problem 1.

Determine the number of roots, counted with multiplicity, of the equation

$$2z^5 - 15z^2 + z + 2$$

inside the annulus $1 \leq |z| \leq 2$.

Problem 2.

Let f and g be analytic functions on the open set $U = D(1, 15) \setminus \{i\}$, i.e. the open disc centered at 1 with radius 15 with the point i removed. Suppose $f'(z) = g'(z)$ for all $z \in U$. Prove that f and g differ by a constant, that is, there exists $a \in \mathbb{C}$ such that $f(z) - g(z) = a$ for all $z \in U$.

Problem 3.

Let $L \subset \mathbb{C}$ be the ray $\{t + it \mid t \geq 1\}$, and $U = \{\mathrm{Re}\, z > 0,\ \mathrm{Im}\, z > 0\}$. Find an explicit conformal mapping of $U \backslash L$ to the unit disc.

Problem 4.

Suppose $f(z) = u(x, y) + iv(y)$ is a holomorphic function. Show that there exists $a \in \mathbb{R}$ and $\lambda \in \mathbb{C}$ such that $f(z) = az + \lambda$.

Problem 5.

Suppose $p(z)$ is a polynomial of degree $d \geq 2$ that has only simple zeros $r_1, r_2, \ldots, r_d$. Prove that $\frac{1}{p'(r_1)} + \frac{1}{p'(r_2)} + \ldots + \frac{1}{p'(r_d)} = 0$.

Problem 6.

Evaluate $\int_{-\infty}^{\infty} \frac{\sin x}{x+i} dx$

Problem 7.

Prove that the range of the function $f(z) = \sum_{n=1}^{2018} \cos^n(z)$ is the whole complex plane $\mathbb{C}$.

Problem 8.

Let us say that a holomorphic function $f : \mathbb{D} \to \mathbb{C}$ on the unit disc $\mathbb{D}$ is "good" if for some $n \in \{1, 2, \ldots, 2018\}$ the function f does not take values on the ray $\{te^{\frac{2\pi}{n}i} \mid t \geq 0\}$. Prove that collection of all "good" functions is normal.