Print Your Name: ______________ ______________
last first

Print Your I.D. Number: __________ ______________

Real Analysis Qualifying Examination, Tuesday, September 19, 2023

1:00 PM – 3:30 PM, Room RH 306

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1. Let f be a positive continuous function on $[0, 1]$. Prove that

$$\lim_{p\to\infty} \|f\|_{L^p([0,1])} = \|f\|_{L^\infty([0,1])}.$$

2. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is absolutely continuous and $g : [a, b] \to \mathbb{R}$ is absolutely continuous and strictly monotonic. Prove that $f \circ g : [a, b] \to \mathbb{R}$ is absolutely continuous.

3. Suppose that $f \in L^\infty(0,1)$ is such that

$$\int_{(0,1)} g'(x) f(x)\, dx = 0$$

for every absolutely continuous function g on $[0,1]$ for which $g(0) = g(1) = 0$. Prove that f equals to a constant almost everywhere.

4. Fix $f \in L^{6/5}(\mathbb{R})$ and $g \in L^6(\mathbb{R})$. Let h be the function on $\mathbb{R}$ defined by

$$h(x) = \int_{-\infty}^{\infty} f(x-y)g(y)\,dy.$$

Prove that h is continuous.

5. Fix $1 \leq p \leq \infty$ and $f \in L^p(\mathbb{R})$. Show that

$$\int_{-\infty}^{\infty} \frac{|f(t)|}{1+t^2}\, dt < \infty.$$

6. Fix $f \in L^1(\mathbb{R})$. Show that the series

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} f(x + \sqrt{k})$$

converges absolutely for almost every $x \in \mathbb{R}$.