ON THE NORMALIZED P-LAPLACIAN EVOLUTION

Abstract

I will talk on my results surrounding the normalized $p$-Laplacian evolution

$$|\nabla u|^{2-p} \text{div}(|\nabla u|^{p-2}\nabla u) = u_t$$

(0.1)

obtained in a series of joint work with Prof. Nicola Garofalo. This equation generalizes motion by mean curvature which corresponds to the case $p = 1$ and the heat equation when $p = 2$. Equation (0.1) also arises in image processing. It has gained a lot of attention recently due to its connection to tug of war games with noise where the number of rounds is bounded. The value functions for certain discrete step games approximate a solution to the PDE (0.1) when the parameter that controls the size of the possible steps goes to zero. Compared to the familiar parabolic $p$-Laplacian

$$\text{div}(|\nabla u|^{p-2}\nabla u) = u_t$$

(0.2)

(0.1) has the advantage that it is $1$-homogeneous but also the severe disadvantage that it is not of divergence form. I also plan to indicate some interesting directions of future study for this equation and its variants.