1. Suppose $G$ is a group of order 80. Prove that $G$ is not simple.

2. Prove that the additive group $\mathbb{R}/\mathbb{Z}$ is isomorphic to the multiplicative group \{\(z^2\in\mathbb{C}\) : \(|z|=1\)}.

3. Let $R$ be an integral domain. A nonzero nonunit element $p \in R$ is prime if $p | ab$ implies that $p | a$ or $p | b$. A nonzero nonunit element $p \in R$ is irreducible if $p = ab$ implies that $a$ or $b$ is a unit.
   (a) Show that every prime element is irreducible.
   (b) Show that if $R$ is a Unique Factorization Domain (UFD), then every irreducible element is prime.

4. Let $G$ and $H$ be finite abelian groups, and suppose that the order of $G$ is relatively prime to the order of $H$. Show that $G \otimes_{\mathbb{Z}} H = 0$.

5. Let $D_8$ be the dihedral group of order 8.
   (a) Compute the center of $D_8$.
   (b) Compute the commutator subgroup of $D_8$.
   (c) Compute the conjugacy classes of $D_8$.

6. Let $R$ be a commutative ring with 1, and let $M$ be an $R$-module. Show that if $M \oplus M$ is a finitely generated $R$-module, then $M$ is a finitely generated $R$-module.

7. (a) Find a polynomial $f(x) \in \mathbb{Q}[x]$ whose splitting field $L_f$ has Galois group $\text{Gal}(L_f/\mathbb{Q})$ isomorphic to $\mathbb{Z}/2\mathbb{Z}$.
   (b) Find a polynomial $g(x) \in \mathbb{Q}[x]$ whose splitting field $L_g$ has Galois group $\text{Gal}(L_g/\mathbb{Q})$ isomorphic to $S_3$.
   (c) Find a polynomial $h(x) \in \mathbb{Q}[x]$ whose splitting field $L_h$ has Galois group $\text{Gal}(L_h/\mathbb{Q})$ isomorphic to $\mathbb{Z}/2\mathbb{Z} \times S_3$.
   Justify your answers.

8. Suppose $F$ is a field and $f(x) \in F[x]$ is a nonconstant polynomial. Show that $F[x]/(f(x))$ is a direct product of fields if and only if $f(x)$ is a separable polynomial.

9. Suppose $p$ is a prime.
   (a) Show that all matrices $A \in \text{GL}_2(\mathbb{F}_p)$ of order exactly $p$ have the same characteristic polynomial, and find that polynomial.
   (b) Show that all matrices $A \in \text{GL}_2(\mathbb{F}_p)$ of order exactly $p$ have the same minimal polynomial, and find that polynomial.

10. Let $K = \mathbb{F}_3(\sqrt{2})$ and let $f(x) = x^4 + 1 \in \mathbb{F}_3[x]$.
    (a) Show that $K$ is the splitting field of $f$.
    (b) Find a generator $\alpha$ of the multiplicative group $K^\times$.
    (c) Express the roots of $f$ in terms of $\alpha$. 