MATH 206 Syllabus

* Groups, homomorphisms, subgroups, normal subgroups
* Isomorphism theorems for group homomorphisms
* Lagrange's Theorem
* Cosets and Quotient groups
* Actions and orbits, orbit-stabilizer formula.
* Euclidean Division Algorithm, Cyclic groups, their subgroups and quotient groups
* Examples of groups: D\_n, S\_n, A\_n, GL\_n(F), SL\_n(F) and Q\_8
* Direct sums, free abelian groups
* Finitely generated abelian groups. Classification theorem without proof.
* Cauchy's Theorem and Sylow's Theorems
* Simplicity of A\_n
* Classification of groups of small order
* Rings, homomorphisms, ideals, quotient rings
* Factorization in commutative rings, primes and irreducibles
* Euclidean domains, PID and UFD
* Fields of quotients
* Rings of polynomials, irreducibility criteria
* Gauss Lemma
* If R is a UFD then R[x] is a UFD
* Chinese Remainder Theorem
* Modules, module homomorphisms, quotient modules
* Free modules, rank
* Modules over Euclidean Domains: classification theorem. Elementary divisors and invariant factors. Application to f.g. abelian groups.
* Vector spaces, linearly independent systems of vectors, bases.
* Matrix of a linear transformation.
* Rank-nullity theorem.
* Characteristic and minimal polynomials, determinant and trace.
* Cayley-Hamilton Theorem
* Eigenvalues and eigenvectors.
* Rational canonical form and Jordan normal form.
* Vector spaces with a scalar product (Euclidean and Hermitian case)
* Orthogonality, Gram-Schmidt orthogonalization
* Riesz Representation Theorem
* Adjoint operators and their properties
* Spectral theorem for normal operators (finite dimension, complex and real cases)
* Special cases: self-adjoint, skew-adjoint and unitary/orthogonal operators
* Field extensions, degree of an extension, multiplicative property of degrees.
* Ruler and compass constructions.
* Separable polynomials and splitting fields. Algebraic closure.
* Finite fields, existence and uniqueness.
* The multiplicative group of a finite field is cyclic.