MATH 2A: SAMPLE FINAL #1

- This exam consists of 15 questions and 100 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (5 points) For what value of a is the function

$$f(x) = \begin{cases} x^2 & x < 3 \\ 2ax & x \geq 3 \end{cases}$$

continuous at every x? As always, justify your answer.

2. (5 points) The theory of relativity predicts that an object whose mass is m_0 when it is at rest will appear heavier when it is moving at speeds near the speed of light. When the object is moving at speed v, its mass m is given by

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}$$

where c is the speed of light. Find $\frac{dm}{dv}$ and explain in terms of physics what this quantity tells you.

3. (5 points) Show that the equation $3x + 2\cos(x) + 5 = 0$ has exactly one real root.

4. (5 points) A hyperbola is given by the equation $x^2+2xy-y^2+x = 2$. Use implicit differentiation to find an equation of the tangent line to this curve at the point $(1, 2)$.

5. (5 points) Little Susie is enjoying a nice spherical lollipop. She sucks the lollipop in such a way that the circumference decreases by 1 centimeter per minute. How fast is the volume of her lollipop changing when the lollipop has a radius of 5 centimeters? Remember to include units.

6. (5 points) Find the linear approximation of the function $f(x) = x^{3/4}$ at the point $a = 16$.

7. (5 points) If $f(3) = 4$, $g(3) = 2$, $f'(3) = -5$, $g'(3) = 6$, find the following values:

a. $(f + g)'(3) =$ ____________

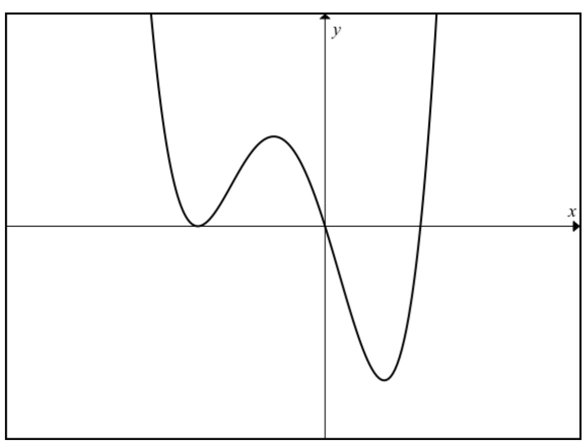
b. $(fg)'(3) =$ ____________

c. $\left(\frac{f}{g}\right)'(3) =$ ____________

8. (5 points) Find the absolute maximum and minimum values of the function $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

9. (5 points) A balloon ascending at a rate of 12 ft/s is at a height of 80 ft above the ground when a package is dropped. How long does it take the package to reach the ground? (Hint: the acceleration due to gravity is 32 ft/s^2 downward. Use antiderivatives.) You may leave your answer in radical form.

10. (5 points) The graph of $f(x)$ is below. Sketch graphs for $f'(x)$ and $f''(x)$.



11. (10 points) Complete each of the following definitions and statements.

a. A function f is continuous at a number a if ________________________.

b. The derivative of a function f at a number a is $f'(a) =$ ________________________, if this limit exists.

c. The Intermediate Value Theorem says ____

____.

d. ________________________ Theorem says "If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

e. A function F is called an antiderivative of f on an interval I if ________________________ for all x in I.

12. (10 points) Find the dimensions of the rectangle with largest area that can be inscribed in a semicircle of radius 2 inches.

13. (10 points) For the following problems, find the limit if it exists or explain why the limit does not exist.

a. $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

b. $\lim_{x \to \infty} \frac{5x+2}{7x^2-4x+8}$

c. $\lim_{x \to 3^-} \frac{x}{x-3}$

d. $\lim_{x \to 1} \frac{x-1}{x^4-1}$

e. $\lim_{x \to 1} \frac{x^2-1}{|x-1|}$

14. (10 points) Compute each of the following:

a. $\dfrac{dy}{dt}$ for $y = \dfrac{1}{\sqrt{t}} + 5t + 3e^t$

b. $f'(4)$ for $f(x) = \sqrt{9 + 4x}$

c. $f'(x)$ for $f(x) = \sin(x \tan^{-1}(x))$

d. $h'(r)$ for $h(r) = r \ln(3r)$

e. $\dfrac{dy}{dx}$ for $y = x^{\tan(x)}$

15. (10 points) Consider the function

$$f(x) = \frac{(x+1)^2}{1+x^2}.$$

a. Find the domain of $f(x)$.

b. Find the x and y intercepts.

c. Determine if $f(x)$ is even, odd, periodic (or none of these).

d. Find any vertical, horizontal, or slant asymptotes of $f(x)$.

e. Find intervals on which $f(x)$ is increasing and on which it is decreasing.

f. Find any local maximum and minimum values.

g. Find intervals on which $f(x)$ is concave up and on which it is concave down.

h. Find any points of inflection.

i. Sketch a graph of $f(x)$.