Math 2A: Sample Final 3

- Turn off your cell phone and do not check it during the exam.
- No calculators or other forms of assistance allowed.
- This exam consists of 14 questions for 100 total points. Points per question are in brackets.
- Read the directions for each problem carefully and answer all parts of each problem.
- Unless instructed otherwise, show all work for full credit.
- Define any notation used and label any sketches/graphs.

1. Compute the following derivatives:

   (a) $f'(x)$ where $f(x) = \sin(x^2 + 2)$ (2)

   (b) $\dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ where $s = 25t + t^{3/2}$ (3)

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}z} \tan^{-1}(\sqrt{z})$ (4)

   (d) $f'(x)$ where $f(x) = x^{\sin x}$ (5)

2. Find the equation of the tangent line to the curve $x^3 + xy + y^2 = 1$ at the point $(-1, 2)$. (6)

3. Show that the equation $2^{-x} = \frac{x-1}{x+1}$ has at least one solution in the interval $[1, 3]$. Explain your answer and state what theorem you are using. (6)

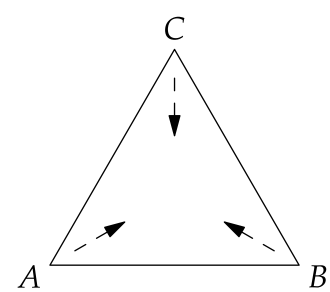
4. Compute the limits.

(a) $\lim_{x \to 3^+} \dfrac{2-x}{\sqrt{x-3}}$ (3)

(b) $\lim_{x \to \infty} \dfrac{x^{1/3}}{\ln x}$ (4)

5. An equilateral triangle $\triangle ABC$ (see picture) initially has sides of length 10 in. Suppose that the side lengths shrink at a constant rate of 2 in/sec.

(*You may use the fact that the area of an equilateral triangle with side length ℓ is $\frac{\sqrt{3}}{4}\ell^2$*)

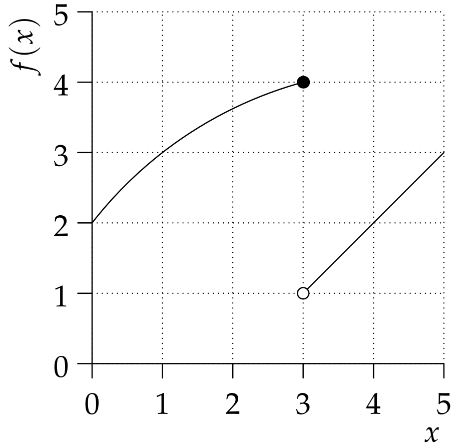


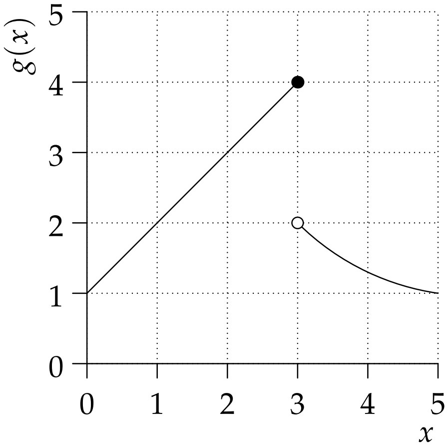
How rapidly is the area of the triangle changing when its area is $4\sqrt{3}$ in^2? (5)

6. Use a linear approximation (or differentials) to estimate $\sqrt[3]{9}$. (6)
(*Hint: Let* $f(x) = \sqrt[3]{x}$ *and recall that* $\sqrt[3]{8} = 2$)

7. Find the anti-derivative $F(x)$ of the function $f(x) = 3x^2 + e^x$ which satisfies $F(1) = e$. (5)

8. You are given the following graphs for functions f and g.





(a) State the following limits. Write DNE if the limit does not exist. (*No working required*)

i. $\lim_{x \to 3} f(x)$ (1)

ii. $\lim_{x \to 3^+} \dfrac{f(x)}{g(x)}$ (2)

(b) Define the function $h(x) = 2f(x) - 3g(x)$. Is h continuous or discontinuous at $x = 3$? Explain. (5)

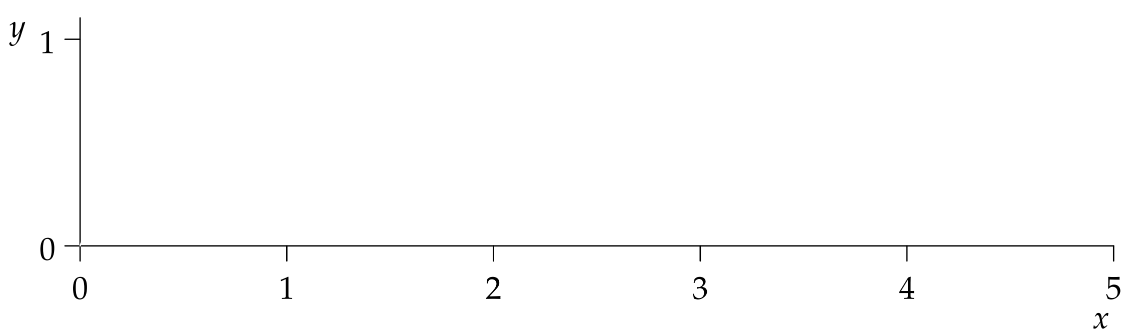
9. Consider the function $f(x) = 2xe^{-x}$ with domain $[0, \infty)$.

(a) Compute the first two derivatives $f'(x)$ and $f''(x)$. (4)

(b) Complete the following table. State 'none' if applicable. No working necessary. (9)

Absolute/local minima	
Absolute/local maxima	
Interval(s) of increase	
Interval(s) of decrease	
Inflection point(s)	
Interval(s) of upwards concavity	
Interval(s) of downwards concavity	
Horizontal Asymptote(s)	
Vertical Asymptote(s)	

(c) Sketch the curve $y = f(x)$ on the axes below. (2)



10. Use the limit definition to compute the derivative of $f(x) = \dfrac{1}{x}$ (6)

11. The profit made by EZ-motors when it sells n cars is given by a function $P(n)$. Suppose, when $n = 100$, that $\mathrm{d}P = \$1,000$. Which of the following sentences must be true? Circle all that apply. (4)

(a) If EZ-motors sells 102 cars, then it will make exactly $2,000 profit.

(b) If EZ-motors sells 97 cars, then it will make approximately $3,000 less profit than if it sold 100 cars.

(c) If $P(100) = \$50,000$, then $P(104) \approx \$54,000$.

(d) The rate of change of $P(n)$ is $\frac{1000}{100} = 10\ \$/\text{car}$.

12. Answer *true* or *false* to each of the following and give a short explanation. Unjustified answers will receive no credit.

(a) $\lim_{x \to -\infty} \frac{\sqrt{x^6 + 5x + 9}}{3x^3 + 2} = \frac{1}{3}$ (3)

(b) Let $f(x) = x^2$. Then there exists some $c \in (2, 4)$ for which $f'(c) = 6$. (4)

13. The rate of change of the temperature (T °F) of a cup of coffee is proportional to the temperature difference between the coffee and the surrounding air (70°F). Suppose, at time $t = 0$ minutes, that the coffee has temperature 180°F, and is decreasing at a rate of 5°F/min. Which of the following is a correct model for the temperature of the coffee? (3)
(*No working necessary*)

(a) $\frac{\mathrm{d}T}{\mathrm{d}t} = -22(T - 70)$

(b) $\frac{\mathrm{d}T}{\mathrm{d}t} = 22(T - 70)$

(c) $\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{1}{22}(T - 70)$

(d) $\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{22}(T - 70)$

14. A cylinder has base radius r and height h. Its volume and surface area are given by the formulas

$$V = \pi r^2 h \quad \text{and} \quad A = 2\pi r(r + h)$$

Find the dimensions of the cyclinder with maximum volume if its surface area is $A = 6\pi$. (8)