

MATH 2A: SAMPLE MIDTERM #1

- This exam consists of 7 questions and 100 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (10 points) The following limit represents the slope of the tangent line to a curve $y = f(x)$ at the point $(a, f(a))$. Determine the function f and the number a , then calculate the limit.

$$\lim_{x \rightarrow 3} \frac{\frac{1}{x+1} - \frac{1}{4}}{x - 3}$$

2. (7 points each) Evaluate each of the following limits.

a. $\lim_{x \rightarrow 2} \frac{\sqrt{x+7} + 3}{5x + 1}$

b. $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x^2 - 4}$

c. $\lim_{x \rightarrow 3^+} f(x)$ where $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ 1 & \text{if } 0 < x \leq 3 \\ 4x & \text{if } x > 3. \end{cases}$

d. $\lim_{x \rightarrow 4^-} \frac{1 - 2x}{x - 4}$

e. $\lim_{x \rightarrow 0^+} \tan^{-1} \left(\frac{1}{x} \right)$

3. (3 points each) Find all the values at which each function is discontinuous, if any. No work needs to be shown in this problem.

a. $f(x) = \frac{x - 5}{(x - 5)(x - 2)}$

b. $f(x) = \left| \frac{1}{\sqrt{x} - 4} \right|$

c. $f(x) = \frac{1}{\cos(x) - 1}$

d. $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \tan(x) & \text{if } 0 < x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$

e. $f(x) = \begin{cases} x + 4 & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ x^{-2} & \text{if } x > 2 \end{cases}$

4. (10 points) Show that there is a root of the equation $\cos(\pi\sqrt{x}) = e^x - 2$ in the interval $(0, 1)$.

5. (10 points) A movie theater has a differentiable function $t = g(c)$ which models the number of movie tickets sold, t , when the cost of a ticket is c , measured in dollars.

a. What is the meaning of the derivative $g'(c)$? What are its units?

b. Do you expect $g'(10)$ to be positive or negative? Briefly explain.

6. (10 points) Find the inverse of the function $f(x) = \frac{e^x}{3 - 5e^x}$.

7. (10 points) Find all horizontal and vertical asymptotes for the following functions.

a. $f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 3}$

b. $g(x) = \tan(x)$

c. $h(x) = \frac{x}{x^2 - 2x - 3}$

d. $m(x) = \frac{x^2 - x - 6}{x}$

e. $n(x) = \ln x$