Math 2A : Sample Final Exam #1

This exam consists of 15 questions. # 1-10 are worth 5 points each and # 11-15 are worth10 points each. Read directions for each problem carefully. Please show all work needed to arrive at your solutions. Label all graphs. Clearly indicate your final answers.

1.) For what value of a is the function $f(x) = \begin{cases} x^2 & x < 3 \\ 2ax & x \geq 3 \end{cases}$ continuous at every x?

2.) The theory of relativity predicts that an object whose mass is m_0 when it is at rest will appear heavier when it is moving at speeds near the speed of light. When the object is moving at speed v, its mass m is given by

$$m = \frac{m_0}{\sqrt{1 - \left(v^2/c^2\right)}}$$

where c is the speed of light. Find $\frac{dm}{dv}$ and explain in terms of physics what this quantity tells you.

3.) Show that the equation $3x + 2\cos x + 5 = 0$ has exactly one real root.

4.) A hyperbola is given by the equation $x^2 + 2xy - y^2 + x = 2$. Use implicit differentiation to find an equation of the tangent line to this curve at the point $(1,2)$.

5.) Little Susie is enjoying a nice spherical lollipop. She sucks the lollipop in such a way that the circumference decreases by 1 centimeter per minute. How fast is the volume of her lollipop changing when the lollipop has a radius of 5 centimeters?

6.) Find the linear approximation of the function $f(x) = x^{3/4}$ at the point $a = 16$.

7.) If $(3) = 4$, $g(3) = 2$, $f'(3) = -5$, $g'(3) = 6$, find the following values:

a.) $(f + g)'(3) =$ _____

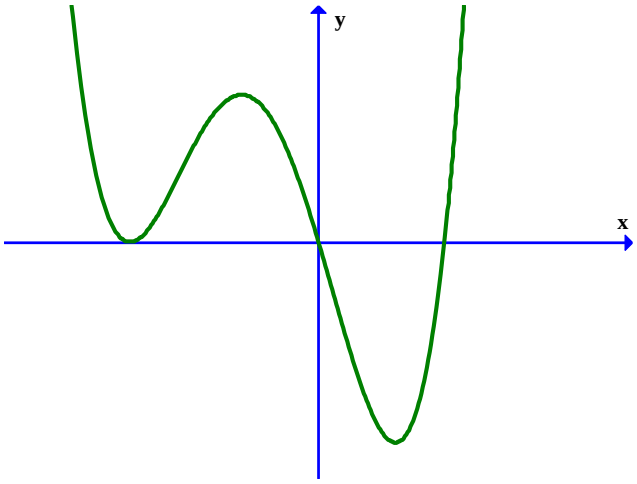
b.) $(fg)'(3) =$ _____

c.) $\left(\frac{f}{g}\right)'(3) =$ _____

8.) Find the absolute maximum and minimum values of the function $f(x) = 3x^4 - 4x^3$ on the interval $[-1,2]$.

9.) A balloon ascending at a rate of $12\ ft/s$ is at a height of $80\ ft$ above the ground when a package is dropped. How long does it take the package to reach the ground? (Hint: the acceleration due to gravity is $32\ ft/s^2$ downward. Use antiderivatives.) You may leave your answer in radical form.

10.) The graph of $f(x)$ is below. Sketch graphs for $f'(x)$ and $f''(x)$.



11.) Complete each of the following definitions and statements.

a.) A function f is continuous at a number a if _____.

b.) The derivative of a function f at a number a is $f'(a) =$_____ if this limit exists.

c.) The Intermediate Value Theorem says _____ _____.

d.) _____ Theorem says "If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$."

e.) A function F is called an antiderivative of f on an interval I if _____ for all x in I.

12.) Find the dimensions of the rectangle with largest area that can be inscribed in a semicircle of radius 2 inches.

13.) For the following problems, find the limit if it exists or explain why the limit does not exist.

a.) $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

b.) $\lim_{x\to \infty} \frac{5x+2}{7x^2-4x+8}$

c.) $\lim_{x\to 3^-} \frac{x}{x-3}$

d.) $\lim_{x\to 1} \frac{x-1}{x^4-1}$

e.) $\lim_{x\to 1} \frac{x^2-1}{|x-1|}$

14.) Compute each of the following:

a.) $\frac{dy}{dt}$ for $y = \frac{1}{\sqrt{t}} + 5t + 3e^t$

b.) $f'(4)$ for $f(x) = \sqrt{9 + 4x}$

c.) $f'(x)$ for $f(x) = \sin(x \tan^{-1} x)$

d.) $h'(r)$ for $h(r) = r \ln 3r$

e.) $\frac{dy}{dx}$ for $y = x^{\tan x}$

15.) Consider the function

$$f(x) = \frac{(x+1)^2}{1+x^2}$$

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a.) Find the domain of $f(x)$.

b.) Find the x and y intercepts.

c.) Determine if $f(x)$ is even, odd, or periodic.

d.) Find any vertical, horizontal or slant asymptotes of $f(x)$.

e.) Find intervals on which $f(x)$ is increasing and on which it is decreasing.

f.) Find any local maximum and minimum values.

g.) Find intervals on which $f(x)$ is concave up and on which it is concave down.

h.) Find any points of inflection.

i.) Sketch a graph of $f(x)$.