Last Name:_____

Math 2A Final Exam

Sample # 2

First Name:_____

Last Name:_____

Student ID #:_____

Section:_____

I certify that this exam was taken by the person named and done without any form of assistance including books, notes, calculators and other people.

_____

Your signature

(For instructor use only!)

Problem	**Score**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

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- *This exam consists of 10 questions worth 10 points each.*
- ***Read** the directions for each problem carefully and answer all parts of each problem.*
- *Please **show all work** needed to arrive at your solutions. **Label** graphs and define any notation used.*
- *Clearly **indicate your final** answer to each problem.*

1.) Determine the value of each of the following limits.

$a)\ \lim_{x\to 1} \dfrac{2-x}{(x-1)^2}$

$b)\ \lim_{x\to 3} f(x) \quad where \quad f(x) = \begin{cases} x^2+5 & if\ x \neq 3 \\ 7 & if\ x = 3 \end{cases}$

$c)\ \lim_{x\to 0} \dfrac{\sqrt{x+1}\ -\ 1}{x}$

$d)\ \lim_{r\to \infty} \dfrac{\ln\sqrt{r}}{r^2}$

$e)\ \lim_{x\to -\infty} \dfrac{\sqrt{7x^2+3x}}{3x-5}$

2.) Compute the indicated derivative of each of the following functions. (You do **not** need to simplify the result algebraically.)

a.) $y = 2x + 6 - 4x^2 + \frac{5}{x^2} + \ln x$, Find $\frac{dy}{dx}$

b.) $f(\theta) = 2\sqrt{\theta} + \frac{2}{\sqrt{\theta}}$, Find $f'(4)$

c.) $y = \frac{3t-1}{t^2+t-2}$, Find y'

d.) $r(t) = 5^t \sin t$, Find $r'(t)$

e.) $y = \sec x + 3\cos x$, Find y''

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3.) Compute the indicated derivative of each of the following functions.

a.) $y = (5x^2 - 2x)^{\frac{3}{4}}$, Find $\left.\frac{dy}{dx}\right|_{x=2}$

b.) $f(x) = (3x)^{\tan^{-1}x}$, Find $f'(x)$

4.) a.) Given that the tangent line to $y = f(x)$ at $(4,3)$ passes through the point $(0,2)$, find $f(4)$ and $f'(4)$.

b.) Sketch a graph of a continuous, differentiable function $f(x)$ which satisfies:

$$f'(x) > 0 \quad for\ x < -2\ and\ x > 3$$
$$f'(x) < 0 \quad for -2 < x < 3$$
$$f'(x) = 0 \quad for\ x = -2\ and\ x = 3$$

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5.) Use implicit differentiation to find the equation of the tangent line to the curve $e^y \sin x + x - xy = \pi$ at the point $(\pi, 0)$.

6.) For the function $f(x) = \frac{2x^2-1}{x^2-4x-21}$, answer each of the following:

a.) Is the function $f(x)$ continuous at $x = 2$? Justify your answer using the definition of continuity.

b.) Find all of the points of discontinuity of $f(x)$.

c.) Use the Intermediate Value Theorem to verify that $f(x)$ has a zero on the interval $[0, 1]$.

d.) Find the equations for any horizontal and vertical asymptotes of this function.

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7.) Suppose that $f(0) = 4$ and $f'(x) \leq 2$ for $x > 0$.
Apply the Mean Value Theorem on the interval [0,3] to prove that $f(3) \leq 10$.
Also, prove more generally that $f(x) \leq 4 + 2x$ for all $x > 0$.

8.) A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 2 ft/sec. When the total area of the disturbed water is 16π square feet, at what rate is the total area of the disturbed water changing?

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9.) According to postal regulations, a carton is classified as “oversized” if the sum of its height and girth (the perimeter of its base) exceeds 108 inches. Find the dimensions of a carton with a square base that is not oversized and has maximum volume.

10.) For the function $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$, answer each of the following:

a.) Find all intervals on which f is increasing and intervals on which f is decreasing.

b.) Find any local maximum and minimum values.

c.) Find all intervals on which f is concave up and intervals on which f is concave down.

d.) Find any points of inflection.

e.) Graph the function $f(x)$.