

Math 2A: Sample Midterm

This exam consists of 9 questions. The point value is listed beside each problem. The exam is out of 100 total points. Read the directions for each problem carefully. Please show all work needed to arrive at your solutions. Label all graphs. Give units when appropriate. Clearly indicate your final answers.

1) (10 points) Find the derivative of $f(x) = \frac{1}{x}$ by using the definition of the derivative.

2) (5 points each) Evaluate each of the following limits.

a) $\lim_{x \rightarrow 2} \frac{x-2}{\sqrt{6-x} - 2}$

b) $\lim_{x \rightarrow 3^+} f(x)$ where $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ 1 & \text{if } 0 < x \leq 3 \\ 4x & \text{if } x > 3 \end{cases}$

c) $\lim_{x \rightarrow 4^-} \frac{1-2x}{x-4}$

d) $\lim_{x \rightarrow 0^+} \tan^{-1}\left(\frac{1}{x}\right)$

3) (5 points each) Find all the values at which each function f is discontinuous, if any. (No work is required to be shown.)

$$a) f(x) = \frac{x-5}{(x-5)(x+2)}$$

$$b) f(x) = \frac{1}{\cos x - 1}$$

$$c) f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \tan x & \text{if } 0 < x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$$

4) (5 points) Show that there is at least one root of the equation $\cos \sqrt{x} = e^x - 2$ in the interval $(0, 1)$.

5) (5 points) The table below gives the number of cars (in millions), $C = f(t)$, in the US in year t .

t (year)	2002	2003	2004	2005	2006
C (cars in millions)	135.9	135.7	136.4	136.6	135.4

a) During the period 2002-2006, when is $f'(t)$ positive? When is it negative?

b) Estimate $f'(2003)$. Using units, interpret your answer in terms of cars.

6) (5 points) Find the equation of the tangent line to the curve $x^2 + \frac{2}{\pi} \cos(\pi y) + 4xy = 3$ at the point $(1, \frac{1}{2})$.

7) (5 points each) State any horizontal and vertical asymptotes for the following functions.

a) $f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 3}$

b) $h(x) = \frac{x}{x^2 - 2x - 3}$

c) $m(x) = \frac{x^2 - x - 6}{x}$

8) (5 points) Suppose $C(s)$ is the total cost (in dollars) of building a home with s square feet of space.

a) What are the units of $C'(s)$?

b) What is the practical meaning of $C'(s)$ for this problem?

c) Is $C'(s)$ positive or negative? Why?

9) (5 points each) Compute the indicated derivative of each of the following functions.

a) If $f(x) = x^5 - \sqrt{x} + \pi^6 + \frac{3}{x^2} + \tan^{-1} x$, find $f'(x)$.

b) If $y = e^x(x^3 - x)^{12}$, find $\frac{dy}{dx}$.

c) If $g(x) = \ln(1 + 3x^2)$, find $g''(x)$.

d) If $f(x) = \sin(\cos(\tan x))$, find $f'(x)$.