MATH 2B: SAMPLE FINAL #1

- This exam consists of 14 questions and 100 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (6 points) Suppose that $\int_{-1}^{1} f(x)\,dx = 6$, $\int_{1}^{4} f(x)\,dx = -2$ and $\int_{-1}^{1} h(x)\,dx = 9$. Use this information to compute the following.
   a. $\int_{4}^{1} 6f(x)\,dx$

   b. $\int_{-1}^{1} \left[2f(x) + 3h(x)\right]\,dx$

   c. $\int_{-1}^{4} f(x)\,dx$

2. (6 points)
   a. Evaluate the following derivative

$$\frac{d}{dx}\int_{\sin(x)}^{x^2} t^3\tan(t)\,dt.$$

   b. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ representing January 1, 2000, and $r(t)$ is measured in barrels per year. What does $\int_{0}^{13} r(t)\,dt$ represent and what are its units?

3. (6 points) Evaluate $\int x^2 \tan^{-1} x \, dx$

4. (6 points) Evaluate $\displaystyle\int \frac{1}{x \ln(3x)} \, dx$

5. (6 points) Evaluate $\int \sin^5(x)\cos^2(x)\,dx$

6. (6 points) Evaluate $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,dx$, where $x > 5$

7. (6 points) Determine whether each of the following improper integrals are convergent or divergent. Evaluate the integral if it is convergent.

   a. $\int_0^2 \frac{1}{(x-2)^2}\, dx$

   b. $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$

8. (6 points)

   a. Find the average value of the function $f(x) = \sec^2(x)$ on the interval $[0, \frac{\pi}{4}]$.

   b. Find the arc length of the curve given by $y = 2x^{3/2}$ from $x = 0$ to $x = 1$.

9. (6 points) Find the first 5 non-zero terms in the Maclaurin series for $f(x) = (1-x)^{-2}$. Find the associated radius of convergence of this power series.

10. (6 points) Determine whether each of the following sequences converges or diverges. If it converges, find the limit.

    a. $a_n = \left(\frac{2}{3}\right)^n + 3$

    b. $b_n = n^3 e^{-n}$

    c. $c_n = \tan^{-1}(\ln(n))$

11. (10 points) Find the area of the region(s) bounded by the curves $y = x^3$ and $y = 4x$.

12. (10 points)

a. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the line $y = 5$ to generate a solid. Find the volume of that solid.

b. Let R be the region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$. Find the volume of the solid with base R and cross-sections perpendicular to the x-axis are squares.

13. (10 points) Answer True or False to each of the following and briefly explain your answers.

a. True/False: We have $\int_0^5 |x^2 - 3x - 4|\, dx \geq 0$.

b. True/False: We have $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\, \pi^{2k} = -1$.

c. True/False: We have

$$\frac{d}{dx}\left(\int_0^{\pi/4} \cos(x)\, dx\right) = \frac{\sqrt{2}-2}{2}.$$

d. True/False: There is a positive integer m such that $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{m-1} + \frac{1}{m} > 20$.

14. (10 points) Determine whether each of the following series is convergent or divergent. Indicate test used.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

c. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

d. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$

e. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n-1}$