MATH 2B: SAMPLE MIDTERM #2

- This exam consists of 5 questions and 90 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (10 points each) Evaluate each of the following integrals.
   a. \( \int \frac{\ln t}{t^5} \, dt \)
   b. \( \int e^x \sin(3x) \, dx \)
c. \[\int \sin^5 \theta \, d\theta\]

d. \[\int_{2\sqrt{2}}^{4} \frac{1}{x\sqrt{x^2-4}} \, dx\]
2. (15 points) Determine whether the following improper integrals are convergent or divergent. Evaluate those that are convergent.

a. \[ \int_{2}^{\infty} \frac{dx}{\sqrt{x}} \]

b. \[ \int_{0}^{3} \frac{1}{\sqrt{9-x^2}}\,dx \]

c. \[ \int_{0}^{\infty} \frac{dz}{z^2 + 3z + 2} \]
3. (10 points) Find the length of the curve \( f(x) = x^3 + \frac{1}{12x} \) on the interval \([\frac{1}{2}, 2]\).

4. (10 points) Determine whether each of the following statements is true or false. Briefly justify your answers.
   a. True/False: If \( \{a_n\} \) is decreasing and \( a_n > 0 \) for all \( n \), then \( a_n \) is convergent.
   b. True/False: If \( f(x) \leq g(x) \) and \( \int_0^\infty g(x) \, dx \) diverges, then \( \int_0^\infty f(x) \, dx \) also diverges.
   c. True/False: The integral \( \int_1^\infty \frac{1}{x^8} \, dx \) converges.
   d. True/False: \( \int_0^3 e^{x^2} \, dx = \int_0^5 e^{x^2} \, dx + \int_5^3 e^{x^2} \, dx \).
5. (15 points) Determine whether each of the following sequences is convergent or divergent. If a sequence is convergent, find its limit.

a. \( a_n = n \sin \left( \frac{1}{n} \right) \)

b. \( a_n = \sin^{-1} \left( \frac{3n}{3n+8} \right) \)

c. \( a_n = -5 + (0.9)^n \)

d. \( a_n = 4 + (-1)^n \)

e. \( a_n = \frac{n^2 + 2n - 12}{n^2 + 2} \)