

## MATH 2B: SAMPLE MIDTERM #2

- This exam consists of 5 questions and 90 total points.
- Read the directions for each problem carefully and answer all parts of each problem.
- Please show all work needed to arrive at your solutions (unless instructed otherwise). Label graphs and define any notation used. Cross out incorrect scratch-work.
- No calculators or other forms of assistance are allowed. Do not check your cell phones during the exam.
- Clearly indicate your final answer to each problem.

1. (10 points each) Evaluate each of the following integrals.

a.  $\int \frac{\ln t}{t^5} dt$

b.  $\int e^x \sin(3x) dx$

c.  $\int \sin^5 \theta \, d\theta$

d.  $\int_{2\sqrt{2}}^4 \frac{1}{x\sqrt{x^2-4}} \, dx$

2. (15 points) Determine whether the following improper integrals are convergent or divergent. Evaluate those that are convergent.

a.  $\int_2^{\infty} \frac{dx}{\sqrt{x}}$

b.  $\int_0^3 \frac{1}{\sqrt{9-x^2}} dx$

c.  $\int_0^{\infty} \frac{dz}{z^2 + 3z + 2}$

3. (10 points) Find the length of the curve  $f(x) = x^3 + \frac{1}{12x}$  on the interval  $[\frac{1}{2}, 2]$

4. (10 points) Determine whether each of the following statements is true or false. Briefly justify your answers.

a. True/False: If  $\{a_n\}$  is decreasing and  $a_n > 0$  for all  $n$ , then  $a_n$  is convergent.

b. True/False: If  $f(x) \leq g(x)$  and  $\int_0^\infty g(x) dx$  diverges, then  $\int_0^\infty f(x) dx$  also diverges.

c. True/False: The integral  $\int_1^\infty \frac{1}{x^\pi} dx$  converges.

d. True/False:  $\int_0^3 e^{x^2} dx = \int_0^5 e^{x^2} dx + \int_5^3 e^{x^2} dx$ .

5. (15 points) Determine whether each of the following sequences is convergent or divergent. If a sequence is convergent, find its limit.

a.  $a_n = n \sin\left(\frac{1}{n}\right)$

b.  $a_n = \sin^{-1}\left(\frac{3n}{3n+8}\right)$

c.  $a_n = -5 + (0.9)^n$

d.  $a_n = 4 + (-1)^n$

e.  $a_n = \frac{n^2+2n-12}{n+2}$