

Last Name: _____

Math 2B Final Exam

Sample # 2

First Name: _____

Last Name: _____

Student ID #: _____

Section: _____

I certify that this exam was taken by the person named and done without any form of assistance including books, notes, calculators and other people.

Your signature

(For instructor use only!)

Problem	Score	Problem	Score
1		8	
2		9	
3		10	
4		11	
5		12	
6		13	
7		TOTAL	

ID #: _____

- This exam consists of 13 questions. The point value is indicated in brackets beside each problem.
- Read the directions for each problem carefully and answer all parts.
- Please **show all work** needed to arrive at your solutions.
- Clearly **indicate your final answer** to each problem.

1.) Consider continuous functions f and f' (where f' denotes the derivative of function f) with values given by the following table: [8 pts]

x	0	1	2	3	4	5
$f(x)$	3	4	6	9	13	18
$f'(x)$	1	2	4	6	7	5

Use the information in the table above to find the following:

- a.) Find $\int_0^4 f'(x) dx$
- b.) Estimate $\int_1^4 f(x) dx$ using a left-hand Reimann sum with 3 equal subintervals.
- c.) Evaluate the following derivative at the point $x = 3$.
- $$\frac{d}{dx} \left(\int_2^x f(t) dt \right)$$
- d.) Supposed $f(x)$ is function which gives the height of a rocket, measured in yards, x minutes after its launch. What are the units of $\int_0^4 f'(x) dx$ and what does this quantity represent?

Evaluate each of the following indefinite integrals:

[7 pts each]

2.) $\int \frac{x}{1+x^4} dx$

3.) $\int \frac{x^2}{e^{2x}} dx$

4.) $\int \sin^3 4t \, dt$

- 5.) Evaluate the following integral by making an appropriate trigonometric substitution [7 pts]

$$\int \frac{dx}{x^2 \sqrt{x^2 - 9}}$$

- 6.) Determine whether the following integral is convergent or divergent. Evaluate the integral if it is convergent. [8 pts]

$$\int_0^{\infty} \frac{dz}{z^2 + 3z + 2}$$

7.) Find the area of the region bounded by the curves $y = \frac{3}{2} - \frac{x^2}{2}$ and $y = |x|$. [10 pts]

8.) Find the volume of the solid obtained by rotating about the x -axis the region bounded by the curves $y = \sqrt{4 - x^2}$ and $y = 2 - x$. [10 pts]

9.) Determine whether each of the following sequences is convergent or divergent. Find the limit of the convergent sequences. [6 pts]

a) $a_n = \frac{e^{2n}}{\sqrt{n}}$

b) $a_n = \frac{(-1)^n}{n!}$

c) $a_n = \tan^{-1} n$

10.) Compute the arc length of the curve $y = \ln(\cos x)$ over the interval $\left[0, \frac{\pi}{4}\right]$. [6 pts]

Hint: $\int \sec(x) dx = \ln|\sec x + \tan x| + C$.

11.) Use the indicated test to determine whether each of the following series is convergent or divergent. [12 pts]

a.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$ (*integral test*)

b.) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ (*ratio test*)

c.) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+5}$ (*alternating series test*)

d.) $\sum_{n=2}^{\infty} \frac{n^2}{n^3-1}$ (*comparison test*)

12.) Find the sum of the each of the following convergent series.

[6 pts]

a.)
$$\sum_{n=1}^{\infty} \frac{3^n}{5^{n+1}}$$

b.)
$$\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$$

13.) Find a power series representation for the function $f(x) = \frac{2}{3-x}$ and determine the interval of convergence.

[6 pts]