Math 2B Final Exam
Sample # 2

First Name: _________________________________

Last Name: _________________________________

Student ID #: _______________________________

Section: _________________________________

I certify that this exam was taken by the person named and done without any form of assistance including books, notes, calculators and other people.

________________________________________
Your signature

(For instructor use only!)

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<th>Problem</th>
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1.) Consider continuous functions $f$ and $f'$ (where $f'$ denotes the derivative of function $f$) with values given by the following table: [8 pts]

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
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<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>$f(x)$</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

Use the information in the table above to find the following:

a.) Find $\int_{0}^{4} f'(x)dx$

b.) Estimate $\int_{1}^{4} f(x)dx$ using a left-hand Reimann sum with 3 equal subintervals.

c.) Evaluate the following derivative at the point $x = 3$.

$$\frac{d}{dx} \left( \int_{2}^{x} f(t)dt \right)$$

d.) Supposed $f(x)$ is function which gives the height of a rocket, measured in yards, $x$ minutes after its launch. What are the units of $\int_{0}^{4} f'(x)dx$ and what does this quantity represent?

Evaluate each of the following indefinite integrals: [7 pts each]

2.) $\int \frac{x}{1 + x^4} \, dx$
3.) \( \int \frac{x^2}{e^{2x}} \, dx \)

4.) \( \int \sin^3 4t \, dt \)
5.) Evaluate the following integral by making an appropriate trigonometric substitution [7 pts]

\[ \int \frac{dx}{x^2 \sqrt{x^2 - 9}} \]

6.) Determine whether the following integral is convergent or divergent. Evaluate the integral if it is convergent. [8 pts]

\[ \int_{0}^{\infty} \frac{dz}{z^2 + 3z + 2} \]
7.) Find the area of the region bounded by the curves $y = \frac{3}{2} - \frac{x^2}{2}$ and $y = |x|$.

8.) Find the volume of the solid obtained by rotating about the $x$-axis the region bounded by the curves $y = \sqrt{4 - x^2}$ and $y = 2 - x$. 


9.) Determine whether each of the following sequences is convergent or divergent. Find the limit of the convergent sequences. [6 pts]

a) \( a_n = \frac{e^{2n}}{\sqrt{n}} \)

b) \( a_n = \frac{(-1)^n}{n!} \)

c) \( a_n = \tan^{-1} n \)

10.) Compute the arc length of the curve \( y = \ln(\cos x) \) over the interval \( [0, \frac{\pi}{4}] \). [6 pts]

\[ \text{Hint: } \int \sec(x) \, dx = \ln|\sec x + \tan x| + C. \]
11.) Use the indicated test to determine whether each of the following series is convergent or divergent. [12 pts]

a.) \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}} \] (integral test)

b.) \[ \sum_{n=1}^{\infty} \frac{100^n}{n!} \] (ratio test)

c.) \[ \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n + 5} \] (alternating series test)

d.) \[ \sum_{n=2}^{\infty} \frac{n^2}{n^3 - 1} \] (comparison test)
12.) Find the sum of each of the following convergent series. [6 pts]

a.) \[ \sum_{n=1}^{\infty} \frac{3^n}{5^{n+1}} \]

b.) \[ \sum_{n=1}^{\infty} \frac{1}{n(n+3)} \]

13.) Find a power series representation for the function \( f(x) = \frac{2}{3-x} \) and determine the interval of convergence. [6 pts]