

ALGEBRA JUMPSTART 2012: SYLLABUS

- Group homomorphisms, subgroups, normal subgroups
- Isomorphism theorems for group homomorphisms
- Lagrange's Theorem
- Euclidean Division Algorithm, Cyclic groups, their subgroups and quotients
- Chinese Remainder Theorem
- Examples of groups: D_n , S_n , A_n
- Cauchy's Theorem and Sylow's Theorems
- Simplicity of A_n
- Classification of groups of small order
- Rings, homomorphisms, ideals, quotient rings
- Factorization in commutative rings, primes and irreducibles
- Euclidean domains, PID and UFD
- Rings of polynomials, irreducibility criteria
- Modules, module homomorphisms, quotient modules
- Modules over PID: classification theorem.
- Elementary divisors and invariant factors. Application to f.g. abelian groups.
- Vector spaces, linearly independent systems of vectors, bases.
- Rank-nullity theorem.
- Characteristic and minimal polynomials, determinant and trace.
- Cayley-Hamilton Theorem
- Eigenvalues and eigenvectors.
- Rational canonical form and Jordan normal form.
- Vector spaces with a scalar product (Euclidean and Hermitian case)
- Orthogonality, Gram-Schmidt orthogonalization
- Riesz Representation Theorem
- Adjoint operators and their properties
- Spectral theorem for normal operators (finite dimension)
- Special cases: self-adjoint, skew-adjoint and unitary/orthogonal operators
- Field extensions, degree of an extension, multiplicative property of degrees.
- Finite fields, existence and uniqueness.
- The multiplicative group of a finite field is cyclic.