Print Your Name: ______________ ________________
last first

Print Your I.D. Number: __________ ______________

Qualifying Examination, September 23, 2014
1:00 pm–3:30pm, Room RH 114

Table of your scores

Problem 1 ________/ 10

Problem 2 ________/ 10

Problem 3 ________/ 10

Problem 4 ________/ 10

Problem 5 ________/ 10

Problem 6 ________/ 10

Problem 7 ________/ 10

Problem 8 ________/ 10

Total ________/ 80

Notation: $D(z_0, r)$ denotes the open disc in $\mathbf{C}$ centered at z_0 and radius r.

1. Let f be an entire holomorphic function such that $f(z) \notin \mathbf{R}$ for all $z \in \mathbf{C}$, where $\mathbf{R}$ is the real line in the complex plane $\mathbf{C}$. Prove or disprove f is a constant.

2. Evaluate the real integral

$$\int_0^\infty \frac{\ln x}{1+x^4}dx$$

3. Let f be entire holomorphic such that $f(x+ix) \in \mathbf{R}$ for all $x \in \mathbf{R}$. If $f(2) = 1 - i$ then find $f(2i)$, where $i^2 = -1$.

4. Let $h(x)$ be a twice differentiable function on $[-1, 1]$ such that $h(0) = h'(0) = 0$ and $h''(0) \neq 0$. Prove

$$\sum_{n=1}^{\infty} h(\frac{1}{n}) z^n$$

defines a holomorphic function on $D(0, 1)$ which is continuous on $\overline{D(0, 1)}$.

5. Let D be a bounded domain in $\mathbf{C}$ with piecewise C^1 boundary. Let $f(z)$ be holomorphic in a bounded domain D and $f \in C(\overline{D})$ with all zeros $\{z_1, \cdots, z_n\} \subset D$ counting multiplicity. Let g be holomorphic in D and continuous on $\overline{D}$. Evaluate

$$\int_{\partial D} \frac{f'(z)}{f(z)} g(z) dz$$

6. Let $D = \{z \in \mathbf{C} : |z| < 1, \Re z > 0, \Im z > 0\}$. Construct a conformal holomorphic map which maps D onto the unit disc $D(0,1)$

7. Let D be a simply connected domain in $\mathbf{C}$ and $z_0 \in D$. Let $\mathcal{F}$ be the set of all $f : D \to D(0,1)$ such that (i) $f(z_0) = 0$; (ii) $f'(z_0) > 0$ and (iii) f is one to one. Then prove $\mathcal{F}$ is not empty set.

8. Let $u(z)$ be harmonic in $D =: D(0,1) \setminus \{0\}$ such that

$$\lim_{z \to 0} \frac{u(z)}{\log |z|} = 0$$

Prove that u can be extended to be harmonic in $D(0,1)$.