

Mathematics Colloquium
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Spectral asymptotics for non-local operators

Abstract

In October 1910, the physicist, Hendrik Antoon Lorentz, delivered a series of six lectures (the Paul Wolfskehl lectures) at the University of Göttingen titled “old and new problems in physics.” During the fourth lecture, with David Hilbert and his student Hermann Weyl in the audience, he raised the problem of proving that the number of eigenvalues for the Laplacian for a region D in three space not exceeding the positive number λ grows like the volume of D times $\lambda^{3/2}$, as λ gets large. The problem had been raised a month earlier by Arnold Sommerfeld at a lecture in Königsberg. Hilbert predicted that this would not be proved in his lifetime. He was wrong by many years as the assertion was proved by Weyl in 1912.

Weyl’s celebrated theorem, commonly referred to as *Weyl’s Law*, has been extended and refined in many directions with connections to many areas of mathematics and physics. In this talk we first give an overview of some of the classical results and discuss the elegant connections to Brownian motion first explored by Mark Kac in the 50’s and 60’s. We will then discuss problems that arise when the Brownian motion, which “goes” with the Laplacian, is replaced by other stochastic processes that share important properties with Brownian motion. While the problems here remain quite open, there is progress to report on heat asymptotics.

This talk is designed for a general mathematical audience.