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Comprehensive Examination of Analysis

9:00AM-11:30AM, June 16, 2015; Rowland Hall 114

Choose 8 from the 9 problems

You need to cross out the problem you don't want to be graded

 Problem 1 — / 10

 Problem 2 — / 10

 Problem 3 — / 10

 Problem 4 — / 10

 Problem 5 — / 10

 Problem 6 — / 10

 Problem 7 — / 10

 Problem 8 — / 10

 Problem 9 — / 10

Total ———/ 80

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1. Prove that

$$\sum_{n\geq 2} \frac{1}{n(\log n)^2} < +\infty.$$

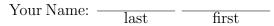
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2. Compute

 $\lim_{n \to +\infty} \int_0^1 \sin(nx) e^{-x^2} \, dx.$

Justify your answer.





3. Assume that $f \in C^1(\mathbb{R})$ and $\lim_{|x|\to+\infty} \frac{f(x)}{|x|} = +\infty$. Show that for any $p \in \mathbb{R}$, there exists $y \in \mathbb{R}$ such that f'(y) = p (i.e., $f' : \mathbb{R} \to \mathbb{R}$ is onto). Hint: Consider g(x) = f(x) - px and $\lim_{x\to\pm\infty} g(x)$.

4. (a) State the Stokes' Theorem(b) Evaluate the following integral:

$$\int_{\partial D} \frac{x^3}{3} dy \wedge dz + \sin(yz) dy \wedge dz + x^{10} dx \wedge dz$$

where

$$D = \left\{ (x, y, z) : \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} < 1 \right\}$$

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5. Prove that $\sin(\sqrt{x})$ is uniformly continuous on $[0, \infty)$.

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6. Let

$$f_n(x) =: \frac{nx}{1 + n^2 x^3}.$$

- (a) Prove $f_n(x) \to 0$ as $n \to \infty$ pointwisely on $[0, \infty)$; (b) Prove or disprove $f_n(x) \to 0$ as $n \to \infty$ uniformly on $[0, \infty)$.

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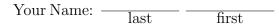
7. Let (X, d) be a metric space and $E \subset X$ is a compact set. Prove that E is closed.





8. Let f(x, y) be a function on the unit disc $D = \{(x, y) : x^2 + y^2 < 1\}$ with $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist for all $(x, y) \in D$. Prove or disprove the following each statement.

(a) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are bounded on D, then f is continuous on D; (b) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are continuous on D, then f is differentiable on D.



9. Let $1 < p, q < \infty$ satisfy $\frac{1}{p} + \frac{1}{q} = 1$. Prove (a) For any $x, y \in (0, \infty)$

$$xy \le \frac{x^p}{p} + \frac{y^q}{q}$$

(b) If f and g are in $L^p[a, b]$ and $L^q[a, b]$, respectively, then f(x)g(x) is Lebesgue integrable and we have

$$\int_{a}^{b} f(x)g(x)dx \le \Big(\int_{a}^{b} |f(x)|^{p} dx\Big)^{1/p} \Big(\int_{a}^{b} |g(x)|^{q} dx\Big)^{1/q}.$$

Problem 9 (continued)