Complex Analysis

Suggested Syllabus for Complex Analysis Qualifying Examination

I. Complex Numbers and Functions
The field of complex numbers, geometry of the complex plane, polar representation, the extended plane and spherical representation, analytic functions, power series, rational functions, elementary functions (exponential, trigonometric and logarithmic), Cauchy-Riemann equations, M`` obius transformations, cross ratio.

II. Complex Integration and Cauchy's Theorem
Line integrals, power series representation of analytic functions, Cauchy's estimate, Cauchy's theorem.

III. Applications of Cauchy's Theorem
Liouville's theorem, Fundamental theorem of Algebra, identity (=uniqueness) theorem, maximum modulus theorem, Schwarz's lemma, Morera's theorem, index (=winding number) of a closed curve, Cauchy's integral formula, argument principle, open mapping theorem.

IV. Singularities
Removable singularities, poles, order and singular part of a pole, Laurent expansions, essential singularities, Casorati-Weierstrass theorem, residues, residue theorem, evaluation of real integrals, Rouche's theorem and applications.

V. Normal families, Montel theorem, the Riemann mapping theorem, Automorphism groups of the unit disc, punch disk, etc. Conformal mappings (or angle preserving maps) between two given regions.

VI. Harmonic functions
Mean value property, Maximum principles, Jensen's formula,
Poisson's formula, Dirichlet problem for disk, and Harnack's theorem.

References--Complex Analysis

Functions of One Complex Variable, by J. B. Conway 2nd edition, 1978
Chapter 1 pp. 1-10; Chapters 3, 4, 5 pp. 30-127; Chapter 6, sections 1, 2 pp. 128-133; Chapter 7, sections 1, 2, 4 pp. 142-154, 160-163; Chapter 10, sections 1, 2 pp. 252-263.

Complex Analysis, by J. Bak and D.J. Newman 1982
Chapters 1, 2, 3, 4, 5, 6, 7 pp. 1-85; Chapters 9, 10 pp. 96-118;
Chapter 11 section 1 pp. 119-127; Chapter 14, pp. 169-174; Chapter 16 pp. 184-190

Function Theory of One Complex Variable by R. E. Greene and S. G. Krantz

Complex Analysis, by Lars V. Ahlfors