COMPLEX ANALYSIS

Qualifying Exam

Tuesday, June 18, 2013 — 1:00pm - 3:30pm, Rowland Hall 114

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Student's name:

Problem 1.

Find the largest set D where the power series

$$\sum_{n=1}^{\infty} \frac{n}{2^n} z^{n^2}$$

converges.

Problem 2.

Let $\{f_j\}$ be a sequence of holomorphic functions from $D(0,1)$ to $D(0,1) \setminus \{0\}$. Prove that if $\sum_{j=1}^{\infty} |f_j(0)|$ converges, then $\sum_{j=1}^{\infty} f_j(z)^2$ converges absolutely and uniformly on compact sets in $D(0,1/3)$.

Problem 3.

Suppose f is holomorphic on the upper half plane $\mathbb{H} = \{\operatorname{Im} z > 0\}$, $f(i) = 0$, and $|f(z)| \leq 1$ for all $z \in \mathbb{H}$. Prove that $|f(2i)| \leq \frac{1}{3}$.

Problem 4.

Suppose $f(z) = u(x, y) + iv(y)$ is a holomorphic function. Show that there exists $a \in \mathbb{R}$ and $\lambda \in \mathbb{C}$ such that $f(z) = az + \lambda$.

Problem 5.

Determine the number of roots, counted with multiplicity, of the equation

$$2z^5 - 6z^2 + z + 1 = 0$$

inside the annulus $1 \leq |z| \leq 2$.

Problem 6.

Suppose f is analytic in an annulus $r < |z| < R$, and there exists a sequence of polynomials p_n converging to f uniformly on any compact subset of the annulus. Show that f is an analytic function on the disc $\{ |z| < R\}$.

Problem 7.

Evaluate the integral for $a > 0$

$$\int_{-\infty}^{+\infty} \frac{\cos^3 x}{a^2 + x^2} dx$$

Problem 8.

Find explicitly a conformal mapping of the domain

$$U = \{|z| < 1, z \notin [1/2, 1)\} = \mathbb{D} \backslash [1/2, 1)$$

to the unit disc $\mathbb{D} = \{|z| < 1\}$.