COMPLEX ANALYSIS

Qualifying Exam

Wednesday, June 17, 2015 — 1:00 pm -3:30 pm, Rowland Hall 114

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Student's name:

Problem 1.

Prove that for each $n \in \mathbb{N}$ every solution of the equation $(1 - iz)^n + z^n = 0$ must satisfy $\mathrm{Im}(z) = -\frac{1}{2}$.

Problem 2.

Classify all the singularities and find the associated residues for

$$f(z) = \frac{e^{-\frac{1}{z}}}{(z-1)(z+1)^2}$$

Problem 3.

Expand in a series of powers of w each of the branches of $z(w)$ defined by the equation $w = 2z + z^2$ (for one branch $z(0) = 0$, for the other $z(0) = -2$).

Problem 4.

Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx$$

Problem 5.

Suppose $f : D(0,1) \to D(0,1)$ is a holomorphic mapping and $f(0) = \frac{1}{5}$. Give an upper bound for $|f'(0)|$, and characterize the functions for which the upper bound is an equality.

Problem 6.

Let the function $f(z)$ be meromorphic in a neigbourhood of the unit disk $\{|z| \leq 1\}$ and suppose it has only one singular point z_0 on the circle $|z| = 1$ which is a simple pole. Show that $\frac{f^{(n)}(0)}{n!} = \frac{A}{z_0^n}(1 + \phi_n)$ where $\lim_{n\to\infty} \phi_n = 0$.

Problem 7.

TRUE or FALSE: There exists a bounded harmonic function on the upper half plane $\mathbb{H}$ that cannot be extended to any larger domain. Explain your answer.

Problem 8.

Suppose f is analytic in an annulus $r < |z| < R$, and there exists a sequence of polynomials p_n converging to f uniformly on compact subsets of the annulus. Show that f is an analytic function on the disc $|z| < R$.