COMPLEX ANALYSIS

Qualifying Exam

Thursday, September 19, 2013 — 1:00 pm -3:30 pm, Rowland Hall 114

Problem	1	2	3	4	5	6	7	8	Σ
Points									

Student's name:

Problem 1.

Describe all entire functions $f : \mathbb{C} \to \mathbb{C}$ which satisfy

a) $f\left(\frac{1}{n}\right) = f\left(-\frac{1}{n}\right) = \frac{1}{n^2}$ for all $n \in \mathbb{N}$;

b) $f\left(\frac{1}{n}\right) = f\left(-\frac{1}{n}\right) = \frac{1}{n^3}$ for all $n \in \mathbb{N}$.

Problem 2.

Suppose f is an entire function such that $\lim_{z\to\infty} \frac{|f(z)|}{|z|} = 0$. Prove that f is constant.

Problem 3.

Describe explicitly the automorphism group $Aut(\mathbb{C}\backslash\{0,1\})$.

Problem 4.

Evaluate the integral

$$\int_0^\infty \frac{1}{1+x^5} dx$$

Problem 5.

Prove that the function

$$u(x, y) = y \cos y \sinh x + x \sin y \cosh x$$

is harmonic in $\mathbb{R}^2$ and find its harmonic conjugate.

Problem 6.

How many solutions has the equation

$$z^4 + 3z^2 + z + 1 = 0$$

in the closed upper half unit disc?

Problem 7.

Assume that $z = 0$ is an essential singularity of a holomorphic function f. Show that it is also an essential singularity of f^2, f^3, and, in fact, of f^n for every $n \in \mathbb{N}$.

Problem 8.

Suppose f is holomorphic on $\mathbb{C}\backslash\{0\}$, and suppose that $f\left(\frac{1}{z}\right) = f(z)$ for all $z \in \mathbb{C}\backslash\{0\}$. Suppose further that f is real on the unit circle. Show that f is real for all real $z \neq 0$.