Summer Program in Analysis/2012

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- This is a four-weeks (32 hours) course based on the Rudin's book:

Principles of Mathematical analysis

- The materials will be covered as follows:

1. Set topology in $\mathbb{R}^n$ or Metric spaces (Chapters 1–3)
2. Continuity of functions in $\mathbb{R}^n$ (Chapters 4 and 7)
3. Differentiability (Chapters 5 and 9)
4. Integrability and special function (Chapters 6, 8 and 10)

- Note: Homework will be assigned at the end of each lecture.

1 Set topology in $\mathbb{R}^n$ and Metric spaces

- Lecture 1: Examination (cover basic materials in analysis)
- Lecture 2: Real number field and the limits of sequences
  - a) Rational number field
  - b) Order number fields
  - c) Supremum Existing Axiom/ Real number field
  - d) Limit of sequences
  - e) Completion of real number field/Cauchy sequence
  - f) Properties and examples for the limits of sequences
- Lecture 3: Metric spaces
  - a) Definitions and examples for metric spaces
  - b) Contractive sequence in metric space
  - c) Set topology
- Lecture 4:
  - a) Convex sets
  - b) Series of numbers

2 Continuity of functions in $\mathbb{R}^n$

- Lecture 1:
  - a) Limit and continuity of functions in $\mathbb{R}^n$
  - b) Examples, including special functions
- Lecture 2: Properties of continuous functions
  - a) Existence of Maxima and Minima
  - b) Intermediate Value Theorem
  - c) Examples
- Lecture 3: Uniformly continuity of functions
  - a) Test for uniformly continuity
  - b) Properties for uniform continuous functions
  - c) Examples
- Lecture 4: Sequences, series of functions in $\mathbb{R}^n$ (Chapter 7)
  - a) Sequences of continuous functions in $\mathbb{R}^n$
  - b) Series of continuous functions in $\mathbb{R}^n$
  - c) Equi-continuity and Arzela-Ascoli theorem
  - d) Stone-Weierstrass theorem (approximation)

3 Differentiability (Chapters 5 and 9)

- Lecture 1: Differentiability of functions in $\mathbb{R}$
  - a) Differentiable functions in $\mathbb{R}$ and Mean Value Theorem
  - b) l'Hospital rule
  - c) Taylor theorem
  - d) Convex function and extremal problems
  - e) Examples
- Lecture 2: Differentiability of functions in $\mathbb{R}^n$
  - a) Partial derivatives
  - b) Definition and condition for differentiability
  - c) Condition for order exchange of partial derivatives
  - d) Examples

- Lecture 3: Inverse Function Theorem
  a) Jacobian
  b) Inverse Function Theorem
  c) Open mapping theorem
  d) Examples
- Lecture 4: Implicit Functions and application
  a) Implicit function theorem
  b) Lagrange multiplier
  c) Examples

4 Integrability (Chapters 6 and 10)

- Lecture 1: Riemann integrals of one variable
  a) Test for integrability
  b) Integral mean value theorem and the integration by parts
  c) Examples
- Lecture 2: Riemann integrals in $\mathbb{R}^n$
  a) Reimann integrals in rectangular coordinates
  b) Reimann integrals in spherical coordinates
  c) Formula for change of variables
- Lecture 3: Stokes and Divergence Theorems
  a) Stokes and Divergence Theorem
  b) Applications
- Lecture 4: Final Examination (cover all materials)