### Algebra Cheat Sheet

#### Arithmetic Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ab + ac = a(b + c) )</td>
<td>Add the coefficients of ( a )</td>
</tr>
<tr>
<td>( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} )</td>
<td>Multiply the numerators and denominators</td>
</tr>
<tr>
<td>( a + c = \frac{ad + bc}{bd} )</td>
<td>Add fractions with different denominators</td>
</tr>
</tbody>
</table>

#### Exponent Properties

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>( a^0 = 1 )</td>
<td>Any number raised to the power of 0 is 1</td>
</tr>
<tr>
<td>( a^m \cdot a^n = a^{m+n} )</td>
<td>Multiply powers with the same base</td>
</tr>
<tr>
<td>( (a^m)^n = a^{mn} )</td>
<td>Power of a power</td>
</tr>
</tbody>
</table>

#### Distance Formula

If \( P_1(x_1, y_1) \) and \( P_2(x_2, y_2) \) are two points, the distance between them is

\[
d(P_1, P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}
\]

#### Completing the Square

Solve \( 2x^2 - 6x - 10 = 0 \)

1. Divide by the coefficient of the \( x^2 \)
2. Move the constant term to the other side.
3. Take half the coefficient of \( x \), square it and add it to both sides.
4. Factor the left side
5. Use Square Root Property
6. Solve for \( x \)

#### Logarithms and Log Properties

**Definition**

\[ y = \log_a x \] is equivalent to \( x = a^y \)

**Example**

\( \log_{125} 3 \) because \( 5^3 = 125 \)

<table>
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<tbody>
<tr>
<td>( \log_b b^x = x )</td>
<td>Logarithm of a base raised to a power</td>
</tr>
<tr>
<td>( \log_a \left( x \right) = \log_a x - \log_a y )</td>
<td>Difference of logarithms</td>
</tr>
<tr>
<td>( \log_a x )</td>
<td>Natural log</td>
</tr>
</tbody>
</table>

### Factoring and Solving

#### Quadratic Formula

Solve \( ax^2 + bx + c = 0 \), \( a \neq 0 \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Example**

If \( b^2 - 4ac > 0 \) - Two real unequal solutions.
If \( b^2 - 4ac = 0 \) - Repeated real solution.
If \( b^2 - 4ac < 0 \) - Two complex solutions.
### Functions and Graphs

**Constant Function**

\[ y = a \text{ or } f(x) = a \]

Graph is a horizontal line passing through the point \((0, a)\).

**Line/Linear Function**

\[ y = mx + b \text{ or } f(x) = mx + b \]

Graph is a line with point \((0, b)\) and slope \(m\).

- **Slope**
  - Slope of the line containing the two points \( (x_1, y_1) \) and \( (x_2, y_2) \) is
  \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
  - **Slope – intercept form**
    - The equation of the line with slope \( m \) and \( y \)-intercept \((0, b)\) is
    \[ y = mx + b \]

- **Parabola/Quadratic Function**
  - \( y = ax^2 + bx + c \)
  - \( f(x) = ax^2 + bx + c \)

  \[ x = ay^2 + by + c \]

  The graph is a parabola that opens right if \( a > 0 \) or left if \( a < 0 \) and has a vertex at \( \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \).

- **Circle**
  \[ (x-h)^2 + (y-k)^2 = r^2 \]

  Graph is a circle with radius \( r \) and center \((h, k)\).

- **Ellipse**
  \[ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \]

  Graph is an ellipse with center \((h, k)\) with vertices \( a \) units right/left from the center and vertices \( b \) units up/down from the center.

- **Hyperbola**
  \[ \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \]

  Graph is a hyperbola that opens left and right, has a center at \((h, k)\), vertices \( a \) units left/right of center and asymptotes that pass through center with slope \( \pm \frac{b}{a} \).

- **Hyperbola**
  \[ \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1 \]

  Graph is a hyperbola that opens up and down, has a center at \((h, k)\), vertices \( b \) units up/down from the center and asymptotes that pass through center with slope \( \pm \frac{a}{b} \).

### Common Algebraic Errors

<table>
<thead>
<tr>
<th>Error</th>
<th>Reason/Correct/Justification/Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{x} = 0 ) and ( \frac{2}{x} = 2 )</td>
<td>Division by zero is undefined!</td>
</tr>
<tr>
<td>( 3^2 = -9 ) and ( (-3)^2 = 9 )</td>
<td>Watch parenthesis!</td>
</tr>
<tr>
<td>( (xy)^2 \times x )</td>
<td>A more complex version of the previous error.</td>
</tr>
<tr>
<td>( \frac{a}{x} + \frac{a}{y} = \frac{a}{x+y} )</td>
<td>Beware of incorrect canceling!</td>
</tr>
</tbody>
</table>
| \( a(x-1) = -ax + a \) | Make sure you distribute the “-”!
| \( (x+a)^2 = x^2 + a^2 \) | See previous error. |
| \( \sqrt{x^2 + a^2} = x + a \) | More general versions of previous three errors. |
| \( 2(x^2 + x) = 2x^2 + 4x \) | See the previous example. You can not factor out a constant if there is a power on the parethesis! |
| \( \sqrt{x^2 + a^2} = -\sqrt{x^2 + a^2} \) | \( \sqrt{x^2 + a^2} = \sqrt{-x^2 + a^2} \) |

For a complete set of online Algebra notes visit [http://tutorial.math.lamar.edu](http://tutorial.math.lamar.edu) © 2005 Paul Dawkins