Errata for "Almost Free Modules, Revised Edition"

Page 31: in the statement of Proposition 3.4, "subgroup" should be "submodule"

Page 67, line 4: $\sum_{k<n}\varphi(1_k)x(k)$ should be $\sum_{k<n}\varphi(e_k)x(k)$

Page 111, lines 16-17: the domain of φ should be κ; $\mu < \nu$ should be $\mu \leq \nu$

Page 111, line 23: A_ν should be chosen to contain $N_\nu \cup A_{\nu-1}$

Page 113, lines 14-16: the second half of (e) should read:

moreover, given a "basis" X_α of each M_α such that $X_\alpha = X_{\alpha+1}|M_\alpha$ for all $\alpha < \rho$, then $\bigcup_{\alpha<\rho} X_\alpha$ is contained in some "basis" of $\bigcup_{\alpha<\rho} M_\alpha$.

Page 138: a gremlin introduced the wrong exact sequence into the proof of V.2.6. The second displayed sequence should be:

$$\begin{aligned} \mathrm{Hom}(\mathbb{Q}/\mathbb{Z}, D) &\rightarrow \mathrm{Hom}(\mathbb{Q}/\mathbb{Z}, D/A) \rightarrow \\ \mathrm{Ext}(\mathbb{Q}/\mathbb{Z}, A) &\rightarrow \mathrm{Ext}((\mathbb{Q}/\mathbb{Z}, D). \end{aligned}$$

Page 140, Exercise 5: this result is not correct. For a counterexample and more information, see "On a problem of saturation of certain reduced direct products" by A. Laradji and J. Pawlikowski, Colloq. Math. 62 (1991), 189-191

Page 221: in the 5th line from the bottom, $k \leq \ell(\eta)$ should be $k \leq \ell(\nu)$

Page 368: in the first line of the statement of Lemma 1.9, K should be K'

Page 369: in the second line of the statement of Theorem 1.10, $< \kappa$ should be $\leq \kappa$

Page 372, line -2: $\beta + 1 \geq \tau$ should be $\beta + 1 > \tau$

Page 381, statement of Theorem 3.3: κ should be assumed to be uncountable; the hypothesis "$\mathrm{Ext}(M_\alpha, C) = 0$ for all $C \in \mathcal{S}$" in Theorem 3.3 may be omitted.

Page 440, Notes: the reference to 3.12 should be to 3.14

Pages 442–443: there are numerous problems with the statement and proof of Lemma 1.1. (Thanks to Doron Shafrir for pointing them out.) See pages 3 and 4 following for a revised version.

Page 471, line 21: V-algebra should be V-module

Page 491: (✠) is not correct; therefore the proof of Theorem XV.3.1 is defective. By a different method, Göbel and Shelah ("Absolutely Indecomposable Modules" Proc. AMS **135** (2007), 1641-1649) have proved a result that implies Theorem XV.3.1 for $|R|$ and λ less than $\kappa(\omega)$. It remains open whether there are arbitrarily large absolutely indecomposable modules.

Page 499, line 1: ${}^{\perp}\mathcal{F}$ should be $\mathcal{F}^{\perp}$

Page 504, line 16: in the parenthetical remark, add, after "projective dimension ≤ 1", "(respectively, injective dimension ≤ 1)"

Page 525, Notes: add 1.2 is due to Eklof-Trlifaj 2001.

Page 562, Problem F2: "free" should be "projective"

Page 562, Problem F4: the last occurrence of "free" should be replaced by "Whitehead"

Page 575, reference [1993]: the letter á is omitted from Szekszárd and from János

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Corrections are welcome; please email peklof@math.uci.edu

Corrected Lemma XIV.1.1

We use H_* to denote the p-pure closure of H.

1.1 Lemma. *Suppose G is a p-pure subgroup of the p-adic completion of $\bigoplus_{\nu\in N}\mathbb{Z}a_\nu$ containing $\bigoplus_{\nu\in N}\mathbb{Z}a_\nu$ such that for all $\xi\in\widehat{\mathbb{Z}}\setminus\mathbb{Z}_{(p)}$, $\xi G\cap G=\{0\}$. Further suppose that $\{\nu_n\colon n\in\omega\}$ is such that for all $g\in G$, $\{n\colon \nu_n\in\operatorname{supp}(g)\}$ is finite. Let $\varphi\in\operatorname{End}(G)$ and $b\in G$ be such that for all $k\in\mathbb{Z}$, $\varphi(b)\neq kb$. Then there is $z_0\in\widehat{G}$ so that φ does not extend to an endomorphism of $G_1\overset{\text{def}}{=}\langle G,\, z_0\rangle_*$. In fact, there is an element $y\in\widehat{G}\setminus G_1$ so that if $H\supseteq G_1$ and $\psi\in\operatorname{End}(H)$ extends φ, then $\psi(z_0)=y$.*

Furthermore, z_0 can be chosen such that for all $\xi\in\widehat{\mathbb{Z}}\setminus\mathbb{Z}_{(p)}$, $\xi G_1\cap G_1=\{0\}$.

Proof. We can assume b is not divisible by p. (Otherwise replace b by c such that $p^nc=b$ and p does not divide c.)

Choose any $\zeta_0=\sum_{n\in\omega}u_np^n$ in $\widehat{\mathbb{Z}}\setminus\mathbb{Z}_{(p)}$. Let

$$\zeta_m=\sum_{n\geq m}u_np^{n-m}$$

For $m\in\omega$, let $x_m=\sum_{n\geq m}p^{n-m}a_{\nu_n}$ and

$$y_m=x_m+\zeta_mb. \tag{1.1.1}$$

So

$$p^my_m=x_0-g_m+p^m\zeta_mb \tag{1.1.2}$$

for some $g_m\in G$. Then $\langle G,\,x_0\rangle_*=\langle G\cup\{x_m\colon m\in\omega\}\rangle$ and $\langle G,\,y_0\rangle_*=\langle G\cup\{y_m\colon m\in\omega\}\rangle$. If z_0 is either x_0 or y_0 and $G_1=\langle G,\,z_0\rangle_*$, then $\xi G_1\cap G_1=\{0\}$ for all $\xi\in\widehat{\mathbb{Z}}\setminus\mathbb{Z}_{(p)}$ (by an argument on supports, using the fact that for all $g\in G$, $\{n\colon\nu_n\in\operatorname{supp}(g)\}$ is finite).

Let φ denote also the unique extension of φ to a homomorphism from $\widehat{G}$ to $\widehat{G}$. It suffices to prove that either $\varphi(x_0)\notin\langle G,\,x_0\rangle_*$ or $\varphi(y_0)\notin\langle G,\,y_0\rangle_*$. Suppose that $\varphi(x_0)\in\langle G,\,x_0\rangle_*$. Then $\varphi(x_0)=kx_m+g$ for some $m\in\omega$, $k\in\mathbb{Z}$ and $g\in G$, such that k is not divisible by p. So by (1.1.2)

$$\varphi(p^my_m)=kx_m+g-\varphi(g_m)+p^m\zeta_m\varphi(b). \tag{1.1.3}$$

Subtracting k times (1.1.1) from (1.1.3) we get that

$$\varphi(p^my_m)-ky_m-g+\varphi(g_m)=\zeta_m(p^m\varphi(b)-kb).$$

If $\varphi(y_m)$ were in $\langle G, y_0\rangle_*$, then since $\zeta_m\langle G, y_0\rangle_* \cap \langle G, y_0\rangle_* = \{0\}$, we would have $p^m\varphi(b) = kb$. But this is a contradiction since p does not divide k or b. Therefore φ does not extend to an endomorphism of $\langle G, y_0\rangle_*$ and we are done. $\square$

Lemma 1.5 should be modified accordingly.