$\boxed{\text{solutions}}$

• Keep eyes on your own paper...

Math 2D Quiz 4 Morning, October 20th

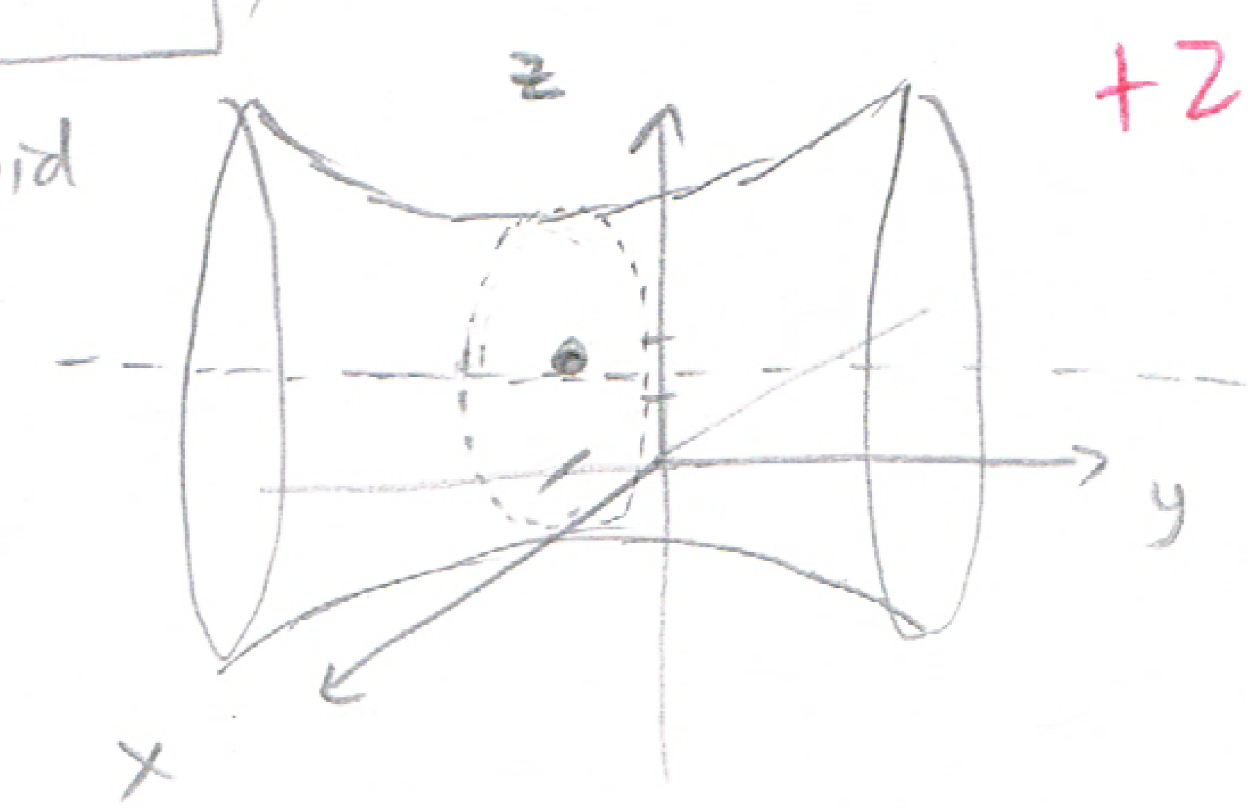
Please put name on front & ID on back for redistribution!

Show all of your work. *There are questions on the back side.*

1. (a) [4pts] Identify and sketch the graph of $4x^2 - 4y^2 - 8y + z^2 - 4z = 12$.

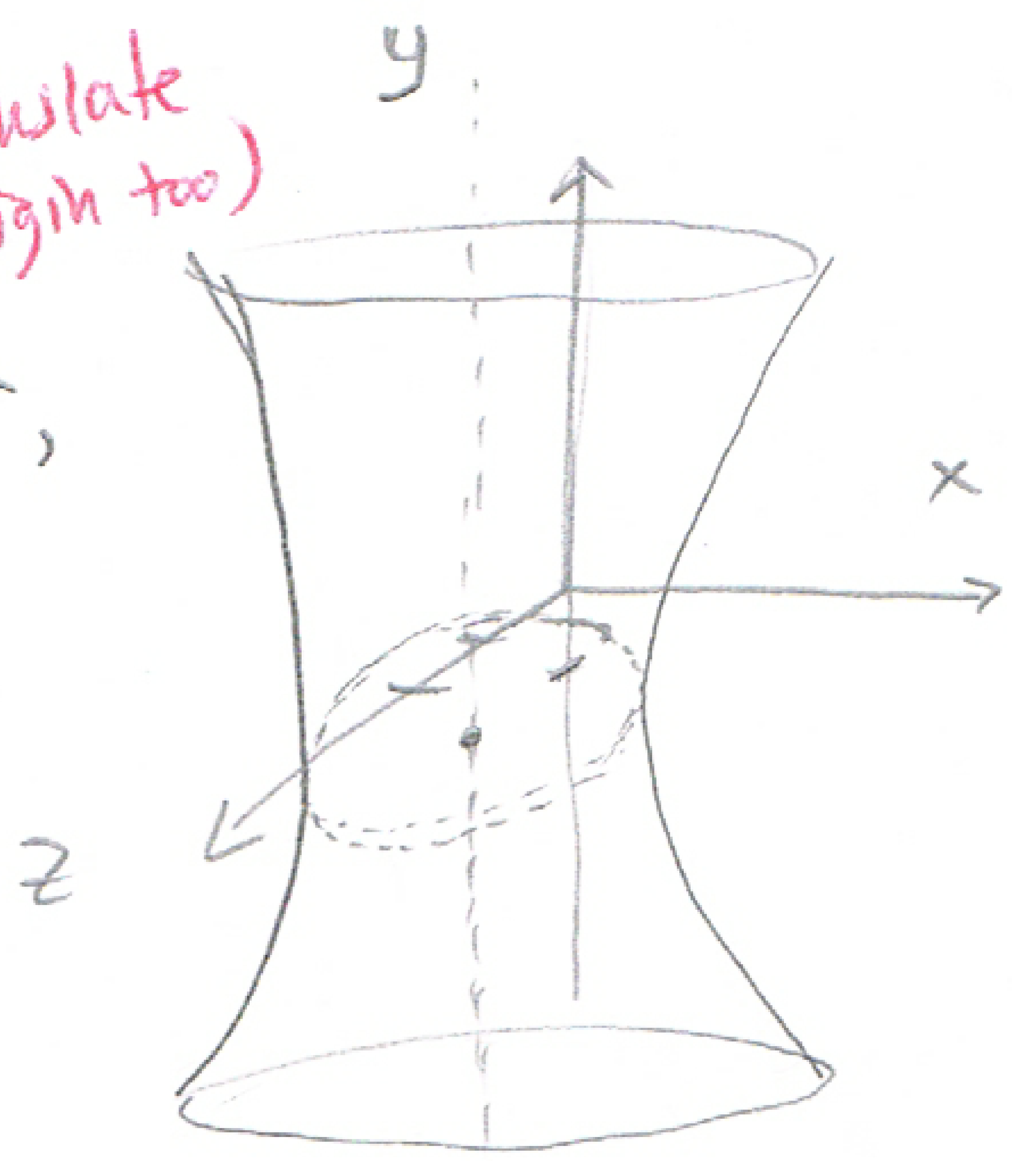
$\boxed{4x^2 - 4(y+1)^2 + (z-2)^2 = 12} - \cancel{4} + \cancel{4}$ +2

1-sheet Hyperboloid (along y-axis)



+2 (translate origin too)

or,



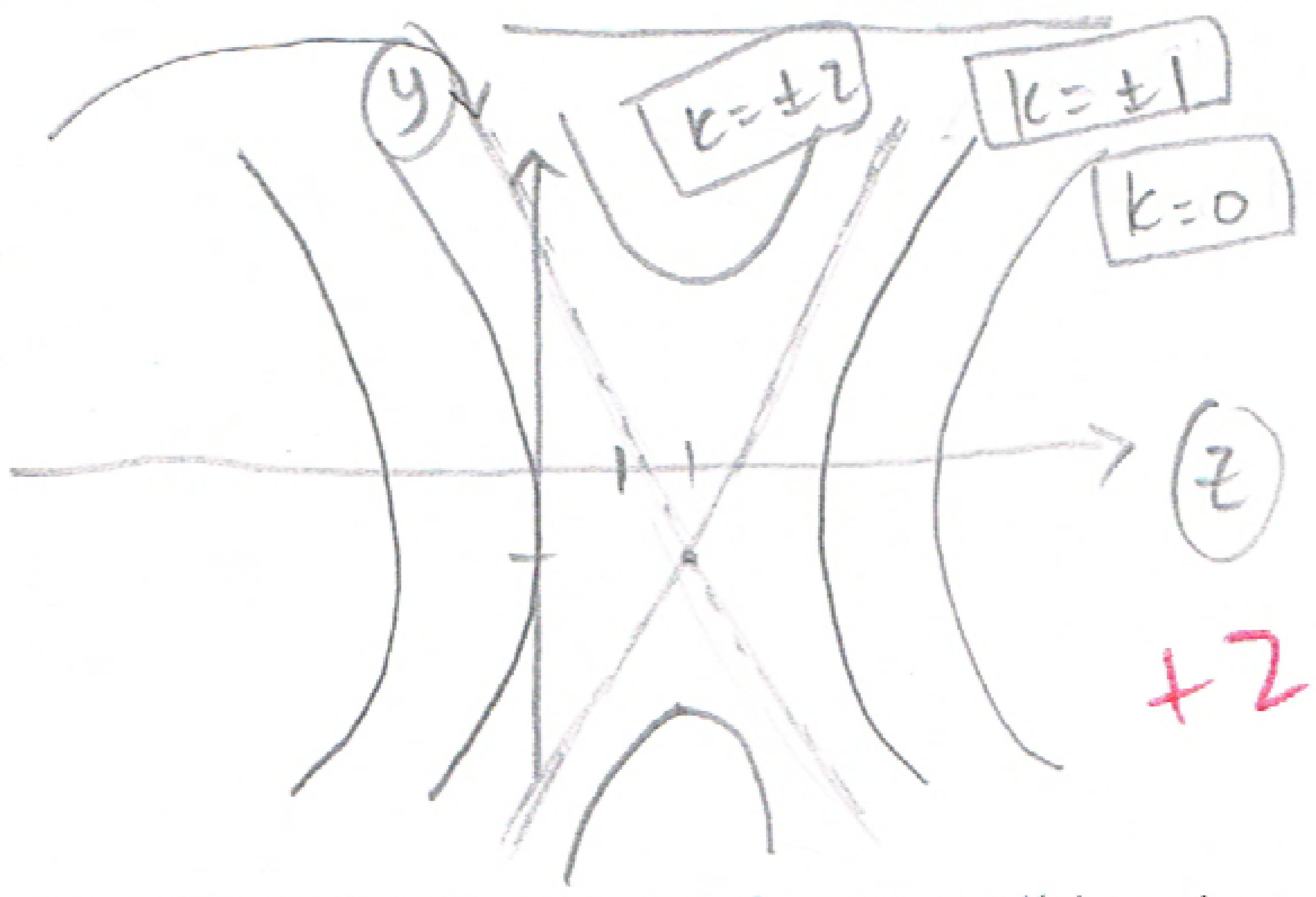
(b) [4pts] Sketch the following traces for the above surface:

(i) yz traces when $x \equiv k$ for $k = 0, \pm 1, \pm 2$. (ii) xz traces when $y \equiv k$ for $k = -4, -3, -2, -1, 0$.

(i) $\to$ $(z-2)^2 - 4(y+1)^2 = 12 - 4k^2$

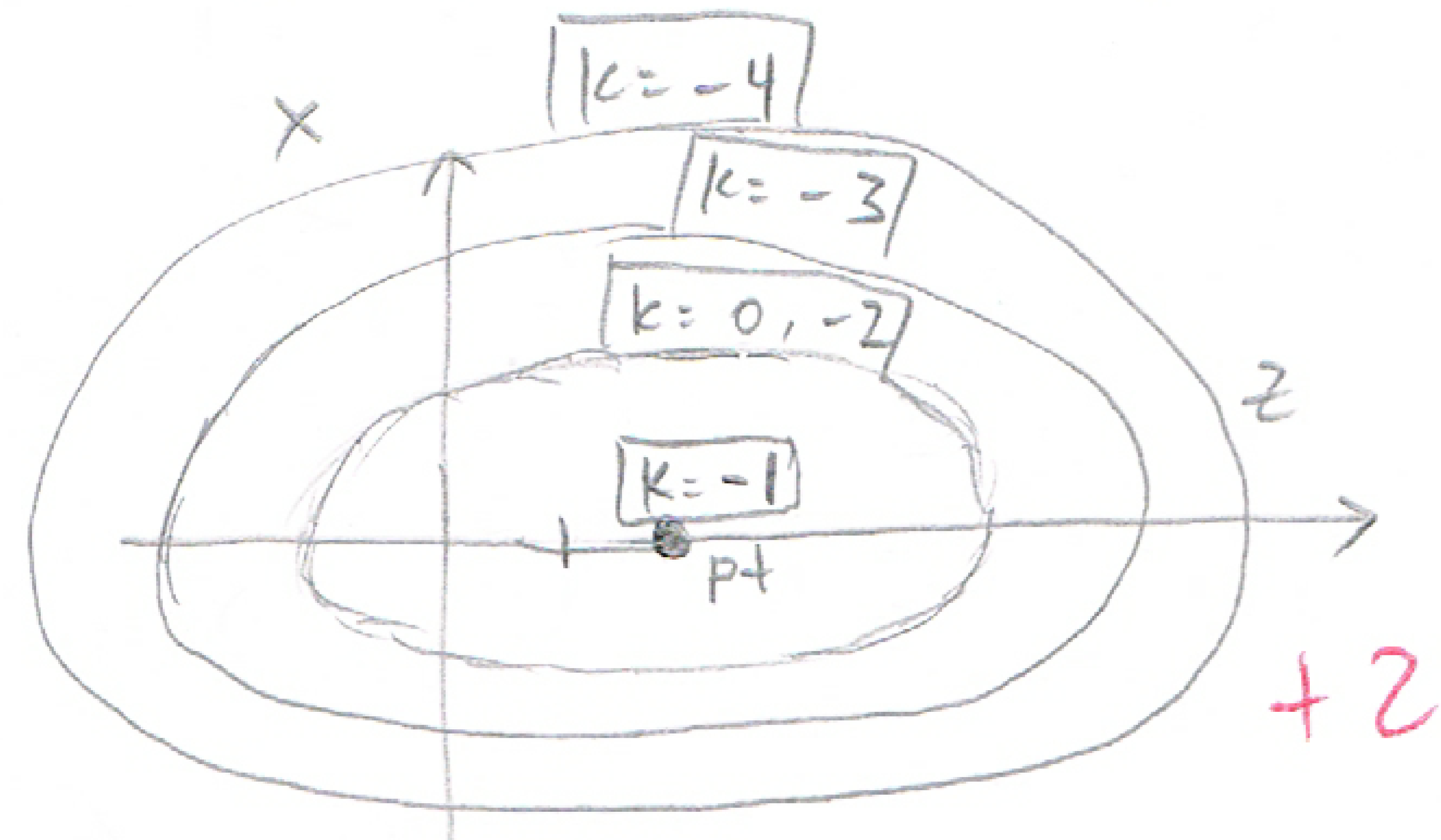
⚠ When $k = \pm 2$, RHS is negative!

slope of Asymptote is 2



+2

(ii) $4x^2 + (z-2)^2 = 12 + 4(y+1)^2$



+2

2. Consider the vector function $\vec{r}(t) = \langle \cos t,\ \sin t,\ t^{3/2} \rangle$.

(a) [3pts] Compute the integral $\int_0^{\pi} \vec{r}(t)$.

+2, as long as answer was a vector still.

$$= \left\langle \sin t,\ -\cos t,\ \frac{2t^{5/2}}{5} \right\rangle \Big|_0^{\pi} =$$

$$= \left\langle 0,\ 1,\ \frac{2\pi^{5/2}}{5} \right\rangle - \langle 0, -1, 0 \rangle$$

$$= \boxed{\left\langle 0,\ 2,\ \frac{2\pi^{5/2}}{5} \right\rangle}$$ +1

(solns)

(b) [3pts] Compute $\vec{r}\,'(t)$. Recall that the curve is $\vec{r}(t) = \langle \cos t,\ \sin t,\ t^{3/2}\ \rangle$.

$$\vec{r}\,'(t) = \langle \underbrace{-\sin t}_{+1},\ \underbrace{\cos t}_{+1},\ \underbrace{\tfrac{3}{2}\sqrt{t}}_{+1} \rangle$$

(c) [3pts] Find the equation of the tangent line at $t = \pi$ (here, $\vec{r}(\pi) = \langle -1, 0, \pi^{3/2} \rangle$).

At $t = \pi$, $\vec{r}\,'(\pi) = \langle 0, -1, \frac{3}{2}\sqrt{\pi} \rangle$ +1

so, the tangent is $\boxed{\vec{L}(t) = \langle -1, 0, \pi^{3/2} \rangle + t\langle 0, -1, \tfrac{3}{2}\sqrt{\pi} \rangle}$

or, $\boxed{x = -1,\ y = -t,\ z = \pi^{3/2} + \tfrac{3\sqrt{\pi}}{2}t}$ +2

(d) [3pts] Find the length of the curve from $0 \le t \le 4\pi$. Hint: Reuse part (b).

$$L = \int_0^{4\pi} \sqrt{\sin^2 t + \cos^2 t + \tfrac{9}{4}t}\, dt = \int_0^{4\pi} \sqrt{1 + \tfrac{9}{4}t}\, dt$$ +1

$u = 1 + \frac{9}{4}t$, $du = 9/4\, dt$

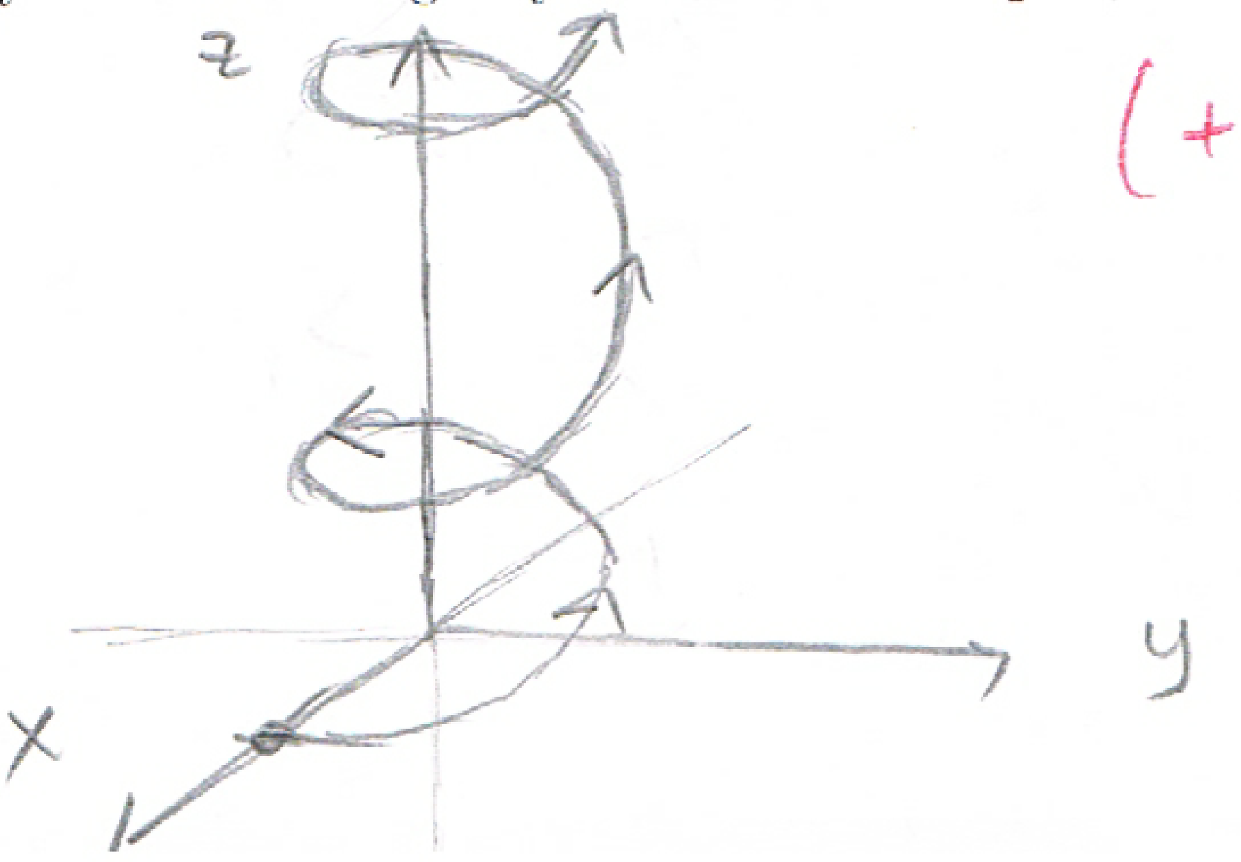
(so, $dt = \frac{4}{9}du$!)

$$= \int_1^{1+9\pi} \sqrt{u} \cdot \frac{4}{9}\, du = \frac{4}{9} \cdot \frac{2u^{3/2}}{3} \Bigg|_1^{1+9\pi}$$ +1

$$= \boxed{\frac{8}{27}\left[(1+9\pi)^{3/2} - 1 \right]}$$ +1

(e) [1pt] **Optional** Sketch the curve $\vec{r}(t) = \langle \cos t,\ \sin t,\ t^{3/2}\ \rangle$ with $t \ge 0$.
If you are missing any credit on this quiz, I will add one point back for a good graph!

(+1)



or,