Solutions

Math 3D Quiz 3 Morning - April 27th

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Show all of your work. *There is a question on the back side.*

1. [6pts] Let $\{f(x) = e^x - \cos x,\ g(x) = e^x + \cos x,\ h(x) = \cos x\}$. Is this set linearly independent? If not, find a non-trivial combination satisfying the zero equation.

Main Soln:

Note $f + 2h = g$ +4

(or any similar identity, like $f - g = -2h$).

Then $f + 2h - g = 0$ +1

is a nontrivial combination satisfying the 0-eqn,

↳ L.D. ✓ +1

Less Ideal: Consider $c_1 f + c_2 g + c_3 h = 0$ and its derivatives.

1st derivatives:

$f' = e^x + \sin x$

$g' = e^x - \sin x$

$h' = -\sin x$

2nd derivatives:

$f'' = e^x + \cos x$

$g'' = e^x - \cos x$

$h'' = -\cos x$

$x = 0$: $2c_2 + c_3 = 0$, $\boxed{c_3 = -2c_2}$

derivative, $x=0$: $c_1 + c_2 = 0$, $\boxed{c_1 = -c_2}$

2nd derivative $x=0$: $2c_1 - c_3 = 0$, $\boxed{c_3 = 2c_1 = -2c_2}$

+4

So c_2 = free and $c_1 = -c_2$, $c_3 = -2c_2$, +2

Set $c_2 = 1 \Rightarrow c_1 = -1$, $c_3 = -2$ $\Rightarrow \boxed{-f + g - 2h = 0}$, $\boxed{\text{L.D}}$

2. [6pts] Suppose that we know the exact solution to a 4th order equation is $y = x\sin(2x)$.
(a) Find the differential equation that it came from, assuming the coefficient of $y^{(4)}$ is a 1.
(b) What would be the general solution to this equation?
Hint: The auxiliary equation has roots $\pm 2i$ with multiplicity 2.

(a) $y = x\sin(2x) \Rightarrow$ Roots are $r = \pm 2i$ (mult. 2)

For $r = \pm 2i$ ⇝ comes from a (r^2+4) term. Due to mult. 2,

Aux Eqn is $(r^2+4)^2 = r^4 + 8r^2 + 16$.

↳ Diff. Eqn is $\boxed{y^{(4)} + 8y'' + 16y = 0}$ +4

(b) With roots $\pm 2i$ (mult. 2), the general solution is

$$\boxed{y = C_1 \cos 2x + C_2 \sin 2x + C_3 x\cos 2x + C_4 x \sin 2x}$$ +2

Solns Cont'd

3. [8pts] Find the solution to $2y'' + 2y' - 4y = 0$ with $y(0) = 2$ and $y'(0) = 5$.

Aux Eqn: $2r^2 + 2r - 4 = 0 \iff r^2 + r - 2 = 0$

$(r+2)(r-1) = 0$, $\boxed{r = 1, -2}$ +2

$y_{gen} = c_1 e^{x} + c_2 e^{-2x}$. +2 Need to solve for c_1, c_2

$y'_{gen} = c_1 e^{x} - 2c_2 e^{-2x}$. +1

$y(0) = 2$: $2 = c_1 + c_2$

$y'(0) = 5$: $5 = c_1 - 2c_2$

subtract, +1

$-3 = 3c_2$, $\boxed{c_2 = -1}$

Plug in to 1st eqn (or 2nd),

$2 = c_1 - 1$; $\boxed{c_1 = 3}$ +1

+1

$\boxed{y = 3e^{x} - e^{-2x}}$ is our solution.

Checks:
- $y(0) = 3e^0 - e^0 = 2$ ✓
- $y'(0) = 3e^{x} + 2e^{-2x}\big|_{x=0} = 5$ ✓

Aside: Can solu for c_1, c_2 by solving $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -2 & 5 \end{array}\right]$

$\xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -3 & 3 \end{array}\right] \Rightarrow \boxed{c_2 = -1}, \boxed{c_1 = 3}$.