Solns

Math 3D Quiz 4 Afternoon - May 4th

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Show all of your work. *There is a question on the back side.*

1. (a) [3pts] Find the general solution to $y'' - 5y' + 6y = 0$.

+1

Auxiliary/Characteristic Eqn: $r^2 - 5r + 6 = 0$, $(r-2)(r-3) = 0$

Roots $r = 2, 3$ $\rightsquigarrow$ $\boxed{y_{gen} = c_1 e^{2x} + c_2 e^{3x}}$ +1

+1

(This is y_c for later parts)

(b) [7pts] Find a particular solution to $y'' - 5y' + 6y = e^{3x}$ using Variation of Parameters.

① Identify y_1, y_2: (from part (a)) Let $y_1 = e^{2x}$, $y_2 = e^{3x}$ (order doesn't matter)

+1

② Solve $\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = f(x) \end{cases} \Rightarrow \begin{cases} u_1' e^{2x} + u_2' e^{3x} = 0 \\ 2u_1' e^{2x} + 3u_2' e^{3x} = e^{3x} \end{cases}$ (already in Standard form)

+1

Use either approach:

Cramer: $W = \det\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} = e^{5x}$

$W_1 = \det\begin{vmatrix} 0 & e^{3x} \\ e^{3x} & 3e^{3x} \end{vmatrix} = -e^{6x}$

$W_2 = \det\begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & e^{3x} \end{vmatrix} = e^{5x}$

$\boxed{u_1' = \frac{W_1}{W} = -e^{x} \; ; \; u_2' = \frac{W_2}{W} = 1}$ +2

Eliminate a variable: Do eq. 2 − 2 · eq 1

$\Rightarrow u_2' e^{3x} = e^{3x}$, so $\boxed{u_2' = 1}$

Plug back, $u_1' e^{2x} + e^{3x} = 0$,

$u_1' = -e^{3x}/e^{2x} = -e^{x}$

$\boxed{u_1' = -e^{x} \; ; \; u_2' = 1}$

Either way, $u_1' = -e^{x} \Rightarrow \boxed{u_1 = -e^{x}}$; $u_2' = 1 \Rightarrow \boxed{u_2 = x}$

+1 +1

So $y_p = u_1 y_1 + u_2 y_2 \Rightarrow \boxed{y_p = -e^{3x} + xe^{3x}}$ +1

Soln Contd

(c) [7pts] Now, find a particular solution to $y'' - 5y' + 6y = e^{3x}$ using Undetermined Coefficients.

* First for RHS $= e^{3x}$, we guess $y_p = Ae^{3x}$

* Since $y_c = C_1 e^{2x} + \underline{C_2 e^{3x}}$ from (a), we rescale by x.

↳ Our y_p is of the form $\boxed{y_p = Axe^{3x}}$ +3

Then $y_p' = A(e^{3x} + 3xe^{3x})$, $y_p'' = A(3e^{3x} + 3e^{3x} + 9xe^{3x})$

+1 $= A(6e^{3x} + 9xe^{3x})$ +1

Plugin, $A(6e^{3x} + 9xe^{3x}) - 5A(e^{3x} + 3xe^{3x}) + 6Axe^{3x} = e^{3x}$

Function Types: e^{3x}: $6A - 5A = 1$, $\boxed{A=1}$ +1

xe^{3x}: $9A - 15A + 6A = 0$ ✓ consistent.

So $A=1 \Rightarrow \boxed{y_p = xe^{3x}}$ +1

+1 (d) [1pt] Are your answers in (b) and (c) the same? If not, are both answers valid? Explain.

Not same, but both are valid: (b) has an extra $(-e^{3x})$ piece, a part of y_c, which gets zero-ed out by the derivatives.

(e) [2pts] Reduce the order of the system in (b),(c) into a 1st order matrix vector equation.

Set $u_0 = y$, $u_1 = y'$ $\Rightarrow$ $u_0' = u_1$ +1, $u_1' = 5u_1 - 6u_0 + e^{3x}$ $\Rightarrow$ $\boxed{\begin{bmatrix} u_0 \\ u_1 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} u_0 \\ u_1 \end{bmatrix} + \begin{bmatrix} 0 \\ e^{3x} \end{bmatrix}}$

Then $y'' = 5y' - 6y + e^{3x}$ +1

in other words, $\boxed{\begin{bmatrix} y \\ y' \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix} + \begin{bmatrix} 0 \\ e^{3x} \end{bmatrix}}$