

Math 3D Practice Midterm Spring 2017

April 25, 2017

1. Find an explicit solution to the following differential equation:

$$xy' + \left(x + \frac{1}{3}\right)y = e^{-2x}y^{-2}$$

2. Find the solution to the following differential equation (implicit okay):

$$(e^y + 1)^2 e^{-y} + [(e^x + 1)^3 e^{-x}] \frac{dy}{dx} = 0, \quad y(0) = 0$$

3. Find the solution to the following differential equation:

$$y'' - 4y' + 13y = 0, \quad y(0) = -11, \quad y'(0) = 19$$

4. (a) Find the general solution to the following differential equation:

$$y''' + 12y'' + 36y' = 0$$

- (b) Use your work in part (b) to solve the following differential equation:

$$y''' + 12y'' + 36y' = 108x^2$$

5. (a) Find the general solution to the following differential equation:

$$x^2y'' + 7xy' + 5y = 0$$

- (b) Use your work in part (b) to solve the following differential equation:
(Hint: Use Variation of Parameters):

$$x^2y'' + 7xy' + 5y = 2x + x^2$$

6. Find all critical points of the differential equation and plot its phase diagram:

$$\frac{dy}{dt} = -(y-7)^2(y-3)(y+1)$$

7. Determine whether the following sets of functions are linearly independent:

- (a) $\{\sin x, \cos x, x \sin x\}$
- (b) $\{xe^x, x^2e^x, e^{2x}\}$
- (c) $\{\cos^2 x, \sin^2 x, 1\}$

8. A wizard creates gold continuously at the rate of 1 ounce per hour, but an assistant steals it continuously at the rate of 5% of the total amount per hour. Let $W(t)$ be the number of ounces that the wizard has at time t .
- (a) Determine the first-order differential equation that models this scenario.
 - (b) Solve your differential equation for $W(t)$ if $W(0) = 1$.