Practice Final Winter 2016

June 4, 2017

1. Find the solution to the following differential equation (implicit okay):

$$\frac{y^2+1}{x^2+1} + y\frac{dy}{dx} = 0, \ \ y(0) = 0$$

2. Find an explicit solution to the following differential equation:

$$y' + 3y = 2xe^{-3x}$$

3. Find the general solution to the following differential equation:

$$y'' + y' + y = \sin x$$

4. Find a real-valued vector solution to the following systems of differential equations:

   (a)

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & -2 \\ 2 & 5 \end{pmatrix} \mathbf{x}(t).$$

   (b)

$$\mathbf{x}'(t) = \begin{pmatrix} 4 & -1 \\ 13 & 0 \end{pmatrix} \mathbf{x}(t).$$

   Hint: One eigenvalue is $\lambda_1 = 2+3i$ and an eigenvector for λ_1 is $\mathbf{v}_1 = \begin{pmatrix} 2+3i \\ 13 \end{pmatrix}$.

   (c)

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

   Hint: Undetermined coefficients will be easiest for this system.

5. Determine whether the following set of vector-valued functions are linearly independent.

$$\begin{pmatrix} 1 \\ -6 \end{pmatrix} e^{-t}, \begin{pmatrix} 1 \\ -6 \end{pmatrix} e^{-2t}, \begin{pmatrix} 2 \\ -3 \end{pmatrix} e^{-t}$$

6. (a) Solve the following Laplace transforms and inverse transforms (you may refer to the table given).

$$\mathcal{L}^{-1}\left\{\frac{1}{4s^2+1}\right\}$$

   (b)

$$\mathcal{L}\left\{(t-3)^3[u(t-1) - u(t-3)]\right\}$$

(c)

$$\mathcal{L}^{-1}\left\{\frac{5}{(s+1)^6((s+2)^2+25)}\right\}$$

For part c <u>only</u>, you may write your answer as the convolution of <u>two</u> functions.

7. (a) Use Laplace transforms to solve the following initial-value problems for $x(t)$.

$$x'' + 4x' + 3x = 1,\ x(0) = 0,\ x'(0) = 2.$$

(b)

$$x'' + x = \cos(3t),\ x(0) = 1,\ x'(0) = 0.$$

(c)

$$x'' + 2x' + x = \delta(t-3),\ x(0) = 0,\ x'(0) = 1.$$

8. Solve the following differential equations by means of a power series centered at a given x_0. Find the recurrence relation between the coefficients, and give which coefficients are constants.

(a)

$$y'' + x^2 y = 5 + x,\ x_0 = 0$$

(b)

$$y'' - (x+1)y' - y = 0,\ x_0 = -1$$