# Math 3D Laplace Transform Cheat Sheet (Spring 2017)

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## Common Forwards (and hence, Backwards) Transforms on Pg 269

- $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$
- $\mathcal{L}\{e^{-at}\} = \dfrac{1}{s+a}$
- $\mathcal{L}\{\sin(wt)\} = \dfrac{w}{s^2+w^2}$
- $\mathcal{L}\{\cos(wt)\} = \dfrac{s}{s^2+w^2}$
- $\mathcal{L}\{\sinh(wt)\} = \dfrac{w}{s^2-w^2}$
- $\mathcal{L}\{\cosh(wt)\} = \dfrac{s}{s^2-w^2}$
- $\mathcal{L}\{u(t-a)\} = \dfrac{e^{-as}}{s}$
- $\mathcal{L}\{\delta(t-a)\} = e^{-as}$ (Actually on Pg 289)

$u(t-a) = \begin{cases} 1 & t \geq a \\ 0 & t < a \end{cases}$ is the Heaviside Function, and $\delta(t-a) = \begin{cases} 1 & t = a \\ 0 & else \end{cases}$ is the Delta Function.

## Shifting Properties

[We use Capital letters to denote a Laplace Transform of the lowercase lettered function.]

- $\mathcal{L}\{e^{-at}f(t)\} = F(s+a)$ Pg 272
- $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ Pg 276

## Transformations of Derivatives, Integrals, and Convolutions:

Derivatives (Pg 274) :

$$\begin{aligned} \mathcal{L}\{g'(t)\} &= sG(s) - g(0) \\ \mathcal{L}\{g''(t)\} &= s^2G(s) - sg(0) - g'(0) \\ \mathcal{L}\{g'''(t)\} &= s^3G(s) - s^2g(0) - sg'(0) - g''(0) \end{aligned}$$

Extra Derivative Property (Exercise 6.2.7) : $\mathcal{L}\{-tf(t)\} = F'(s)$

Integrals (Pg 279) : $\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \dfrac{1}{s}F(s).$

Convolutions (Pg 283) : $\mathcal{L}\{(f*g)(t)\} = \mathcal{L}\left\{\int_0^t f(y)g(t-y)dy\right\} = F(s)G(s).$